COLOCATED MIMO RADAR TRANSMIT BEAMFORMING USING ORTHOGONAL WAVEFORMS

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ABSTRACT

Multiple-input-multiple-output (MIMO) radar transmit beamforming mainly relies on designing transmitted signals with an appropriate covariance matrix. The signals can be designed using a two-step method which first optimizes the covariance matrix and then searches for the signals accordingly. A more efficient way is to synthesize transmitted signals by designing a weight matrix given a set of orthogonal waveforms, which makes use of both MIMO waveform diversity and phased-array transmit gain. In this paper, we propose a method to design the transmit beampattern by solving a semidefinite programming (SDP) problem. Then the eigen-decomposition of the optimal covariance matrix yields the weight matrix. Therefore it is called the SDP-EIG method. As a result, the overall transmitted waveforms are obtained more simply and efficiently, and the number of orthogonal signals required to form a desired beam reaches its minimum.

Index Terms— MIMO Radar, transmit beamforming, transmit energy focusing, transmitted signal synthesis.

1. INTRODUCTION

Multiple-input-multiple-output (MIMO) radar systems have been intensively studied in the recent decade. Compared to conventional phased-array radars, MIMO radars with orthogonal transmitted waveforms enjoys the advantage of waveform diversity but has drawbacks in terms of signal-to-noise ratio (SNR) loss [1]. To both preserve the waveform diversity benefits and enjoy the advantages of phased-array radar, new configurations of MIMO radar transmit antennas such as the phased-MIMO radar and hybrid MIMO phased-array radar, have been proposed in [2], [3], [4]. The signal strategies discussed in these works transmit orthogonal waveforms while each of them is transmitted from multiple antennas (subarrays) to form a phased-array beam.

Besides the techniques and algorithms developed based on the variations of array subaperturing and signal choices, MIMO radar transmit beampforming, also known as MIMO radar transmit energy focusing, has received much attention recently [5], [6]. Such an idea of array processing at the transmit mode aims to improve the direction-of-arrival (DOA) estimation by focusing the transmit beampattern to a pre-assigned angle section within which multiple targets are assumed to exist. As a result, the transmit beampattern will not be flat as conventional MIMO radar. With an appropriate design, the performance can be improved compared to the shape-fixed transmit beampattern of phased-array radar.

The covariance matrix of transmitted signals is a key parameter deciding the array's spatial response. In this paper we do not follow the popular notion of MIMO radar waveforms' orthogonality. Thus the covariance matrix can be arbitrary chosen between identity matrix and all one matrix, which has been done in [5]. The idea in [5] is a two-step optimization problem. First, the covariance matrix is optimized such that the squared error between the transmit beampattern and the desired one is minimized. Second, the signals are optimized using the obtained covariance matrix. in [6], a new transmitted signal model is proposed, in which each orthogonal waveform is weighted and transmitted from the array instead of a single antenna, respectively. In other words, each antenna transmit a linear combination of all the orthogonal signals. This scenario introduces a weight matrix, and can be referred to as an electronic hybrid MIMO phased-array radar compared to its physical counterparts proposed in [2], [3], [4].

In this paper, we formulate a semidefinite programming (SDP) problem inspired by the work in [5] and [6], and find the optimal weight matrix via eigen-decomposition. Thus the method is regarded as SDP-EIG method. The waveform design problem boils down to the optimization of the weight matrix. Compared to [5], the optimization of waveforms is removed, which simplifies the problem. Compared to [6], we formulate an alternative optimization problem which can be efficiently solved using standard optimization tools. Simulations demonstrate the advantages of the proposed method, and reveal the relation between the transmit beampattern design for MIMO radar and that for the conventional phased-array. The superiority of the former is addressed. Both ideal and real (practical) signals are used in the simulations.

2. TRANSMIT BEAMFORMING SIGNAL MODEL

Consider a MIMO radar system with colocated N_T transmit antennas and N_R receive antennas. The K orthogonal unity energy signals are stacked in an $N \times K$ matrix $\mathbf{S}$, ($K \leq N_{\mathrm{T}}$):

$$\mathbf{S} = \left[\begin{array}{cccc} \mathbf{s}_0 & \mathbf{s}_1 & \cdots & \mathbf{s}_{K-1} \end{array} \right], \tag{1}$$

where N is the length of the signals. Suppose the targets are from far field and the transmit and receive array are uniform linear arrays (ULA) sharing the same target angle with inter element spacing of half of the carrier wavelength, and the angle is counted counterclockwise with the broadside corresponding to $90°$. The transmit and receive steering vector are:

$$\mathbf{e}_{\mathrm{T}}(\theta) = \left[\begin{array}{cccc} 1 & e^{-j\pi\cos\theta} & \cdots & e^{-j\pi(N_{\mathrm{T}}-1)\cos\theta} \end{array} \right]^T, \tag{2}$$

$$\mathbf{e}_{\mathrm{R}}(\theta) = \left[\begin{array}{cccc} 1 & e^{-j\pi\cos\theta} & \cdots & e^{-j\pi(N_{\mathrm{R}}-1)\cos\theta} \end{array} \right]^T, \tag{3}$$

where $\{\cdot\}^T$ is the transpose operator. The transmitted signal is a linear combination of the orthogonal signals, which is modeled as an $N_{\mathrm{T}} \times N$ matrix $\mathbf{S}_{\mathrm{T}}$:

$$\mathbf{S}_{\mathrm{T}} = \sum_{k=0}^{K-1} \mathbf{c}_k \mathbf{s}_k^T = \mathbf{C}\mathbf{S}^T, \tag{4}$$

where $\mathbf{C}$ is the $N_{\mathrm{T}} \times K$ weight matrix whose kth column is $\mathbf{c}_k$ ($k \in [0,\ K-1]$). The transmitted signal energy is:

$$E_{\mathrm{T}} = \mathrm{tr}\left\{\mathbf{S}_{\mathrm{T}}\mathbf{S}_{\mathrm{T}}^H\right\} = \mathrm{tr}\left\{\mathbf{C}\mathbf{C}^H\right\}, \tag{5}$$

where $\{\cdot\}^H$ is the transpose conjugate operator. To make the total transmit energy equal to that of the conventional MIMO radar, we set $\mathrm{tr}\left\{\mathbf{C}\mathbf{C}^H\right\} = N_{\mathrm{T}}$. Note that this constraint is a relaxed version of $\|\mathbf{c}_k\|^2 = 1$ in [6]. This relaxation offers more flexibility of designing $\mathbf{C}$ while preserving the energy constraint. The transmit beampattern $P_{\mathrm{T}}(\theta)$ and overall beampattern after matched filtering, $P_{\mathrm{MF}}(\theta)$, are:

$$\begin{aligned} P_{\mathrm{T}}(\theta) &= \mathbf{e}_{\mathrm{T}}^H(\theta)\,\mathbf{S}_{\mathrm{T}}\mathbf{S}_{\mathrm{T}}^H\mathbf{e}_{\mathrm{T}}(\theta) \\ &= \mathbf{e}_{\mathrm{T}}^H(\theta)\,\mathbf{C}\mathbf{C}^H\mathbf{e}_{\mathrm{T}}(\theta), \end{aligned} \tag{6}$$

$$\begin{aligned} P_{\mathrm{MF}}(\theta) &= \mathrm{tr}\left\{\mathbf{S}_{\mathrm{R}}\hat{\mathbf{S}}_{\mathrm{R}}^H\right\} \\ &= \mathrm{tr}\left\{\mathbf{e}_{\mathrm{R}}(\theta_0)\,\mathbf{e}_{\mathrm{T}}^T(\theta_0)\,\mathbf{C}\mathbf{C}^H\mathbf{e}_{\mathrm{T}}^*(\theta)\,\mathbf{e}_{\mathrm{R}}^H(\theta)\right\}, \end{aligned} \tag{7}$$

where $\{\cdot\}^*$ is the conjugate operator and θ_0 is the target angle (a single target is present). $\mathbf{S}_{\mathrm{R}}$ is the $N_{\mathrm{R}} \times N$ matrix of received signals. Note that both the transmit and overall beampattern are related to $\mathbf{C}$. Hence, by using the same set of orthogonal signals, we can modify the beampatterns by appropriately choosing the weight matrix $\mathbf{C}$, which cannot be fulfilled in conventional MIMO radar scenario.

3. PROBLEM FORMULATION

Suppose we want the energy of transmit beam uniformly focusing within a range sector denoted as $\mathbf{\Theta}$, according to which the desired transmit beampattern, denoted as $P_d(\theta)$, is preassigned. The transmit beamforming problem is interpreted as the convex optimization problem below:

$$\begin{aligned} \min_{\mathbf{C}} \quad & \sum_{\theta} \left| P_d(\theta) - \mathbf{e}_{\mathrm{T}}^H(\theta)\,\mathbf{C}\mathbf{C}^H\mathbf{e}_{\mathrm{T}}(\theta) \right|^2 \\ \text{s.t.} \quad & \mathrm{tr}\left\{\mathbf{C}\mathbf{C}^H\right\} = N_{\mathrm{T}}. \end{aligned} \tag{8}$$

Note that (8) is a 4th order nonlinear problem, in which $\mathbf{C}$ is difficult to be solved. Instead of directly solving $\mathbf{C}$, we propose to solve $\mathbf{R_C}$ first, where $\mathbf{R_C} = \mathbf{C}\mathbf{C}^H$, i.e.,

$$\begin{aligned} \min_{\mathbf{R_C}} \quad & \sum_{\theta} \left| P_d(\theta) - \mathbf{e}_{\mathrm{T}}^H(\theta)\,\mathbf{R_C}\mathbf{e}_{\mathrm{T}}(\theta) \right|^2 \\ \text{s.t.} \quad & \mathrm{tr}\left\{\mathbf{R_C}\right\} = N_{\mathrm{T}}, \\ & \mathbf{R_C} \succeq 0, \end{aligned} \tag{9}$$

where $\succeq$ is matrix inequality operator (if $\mathbf{R_C} \succeq 0$, then $\mathbf{R_C}$ is positive semidefinite). Note that (9) is not equivalent to (8). In other words, $\mathbf{C}$ may not necessarily be the unique solution given $\mathbf{R_C}$. It is worth noting that (9) is mathematically equivalent to the optimization problem formulated in [5]. However, [5] aims to optimize the covariance matrix of transmitted signals followed by another optimization problem for waveform design, whereas here we already have the signals at hand and (9) is to optimize the covariance form of the weight matrix. In [5], barrier method incorporating Newton's method is proposed to iteratively calculate the solution. The problem can be more efficiently solved using `CVX` SDP tools (see [7]).

An intuitive way to generate $\mathbf{C}$ given $\mathbf{R_C}$ is based on the eigen-decomposition of $\mathbf{R_C}$ [8]. In fact, any $\mathbf{C}$ with N_{T} rows and arbitrary columns that satisfies $\mathbf{C}\mathbf{C}^H = \mathbf{R_C}$ forms the same transmit beampattern. Thus,

$$\mathbf{C} \triangleq \mathbf{U}\sqrt{\mathbf{\Lambda}}, \tag{10}$$

where $\mathbf{U}$ and $\mathbf{\Lambda}$ are the matrix of eigenvectors and the diagonal matrix of the eigenvalues of $\mathbf{R_C}$, respectively. Note that the number of non-zero columns in $\mathbf{C}$ defined in (10)is determined by the the rank of $\mathbf{R_C}$. This property is important as it implies the minimal number of orthogonal signals required to form the desired transmit beam, i.e,

$$K_{\min} = \mathrm{rank}\left(\mathbf{R_C}\right). \tag{11}$$

Assume $\mathrm{rank}\left(\mathbf{R_C}\right) = R < N_{\mathrm{T}}$, by discarding the $N_{\mathrm{T}} - R$ zero columns of $\sqrt{\mathbf{\Lambda}}$, (10) is modified to yield the optimal weight matrix $\tilde{\mathbf{C}}$ of dimension $N_{\mathrm{T}} \times R$:

$$\tilde{\mathbf{C}} \triangleq \mathbf{U}\sqrt{\tilde{\mathbf{\Lambda}}}. \tag{12}$$

Since the portion discarded are all zeros, the total transmit beam energy is preserved. As a result, By designing an appropriate weight matrix $\tilde{\mathbf{C}}$, which is embodied in designing an appropriate covariance matrix $\mathbf{R_C}$, we can achieve a desired transmit beampattern using the least orthogonal waveforms. This method is regarded as the SDP-EIG method. Simulations will demonstrate the above points in the following section.

4. SIMULATION RESULTS

We first assume $K = N_{\mathrm{T}}$ and solve the SDP problem. Then $K_{\min}$ is obtained by discarding the columns of $\sqrt{\tilde{\mathbf{\Lambda}}}$ corresponding to zeros eigenvalues. We also implement the discrete prolate spheroidal sequence (DPSS) based method proposed in [6] for comparison. Then we study the performance under real situation where the orthogonal waveforms designed in [9] for enhanced MIMO radar delay-Doppler estimation are considered. The parameters are set as follows: $N_{\mathrm{T}} = 10$, a wide and a narrow angle range section are used respectively for the comparison under different energy focusing objectives, $\mathbf{\Theta}_1 \in [65°,\ 115°]$, $\mathbf{\Theta}_2 \in [85°,\ 95°]$. The angular resolution is $0.5°$. $P_d(\theta)$ is designed as a rectangular function whose value outside $\mathbf{\Theta}$ is 1. The value imposed within $\mathbf{\Theta}$, denoted as P_0 ($P_0 \gg 1$, $P_0 = 10^2$ corresponds to 20 dB), highly affects the slope of transition band of the spatial frequency response. A higher value of P_0 pulls the transmit main beam edge down faster than a lower value does. Hence we choose P_0 lower for $\mathbf{\Theta}_1$ and higher for $\mathbf{\Theta}_2$, accordingly.

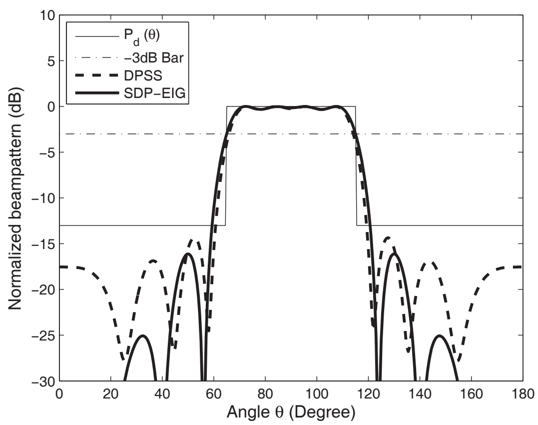

Fig. 1. Beampatterns using $\mathbf{\Theta}_1$, $P_0 = 20$.

The transmit beampattern comparison between SDP-EIG method and DPSS based method for the wide mainbeam case is shown in Fig.1, where $P_0 = 20$ (13 dB), $\mathrm{rank}\,(\mathbf{R_C}) = 4$, and the eigenvalues are 1.901, 2.667, 2.702, and 2.730. The DPSS method [6] forms a matrix $\mathbf{A}$ by summing up the the outer product of $\mathbf{e}_{\mathrm{T}}(\theta)$ at each angle. Then the eigenvectors corresponding to several largest eigenvalues form the weight matrix. In this example, the nonzero eigenvalues of $\mathbf{A}$ are 15.91, 93.73, 200.4, 229.7, 234.2, and 235.0, and the eigenvalues chosen are the largest 4 of them. The two methods have nearly identical mainbeams, where the normalized energy within the section of interest is strictly lower bounded by -3 dB. Both the mainbeam peak areas are nearly flat. Note that the SDP-EIG pattern is better than the DPSS one in terms of the reduction of sidelobe peaks.

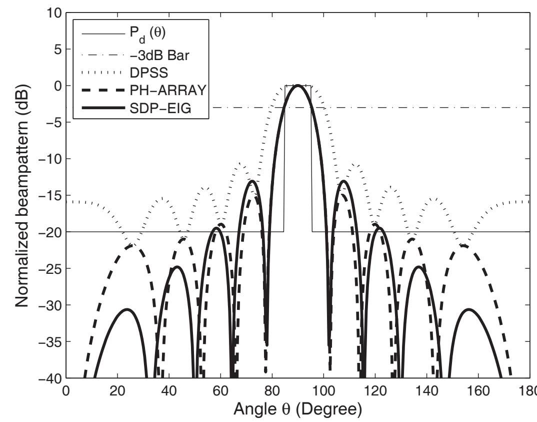

Fig. 2. Beampatterns using $\mathbf{\Theta}_2$, $P_0 = 10^2$.

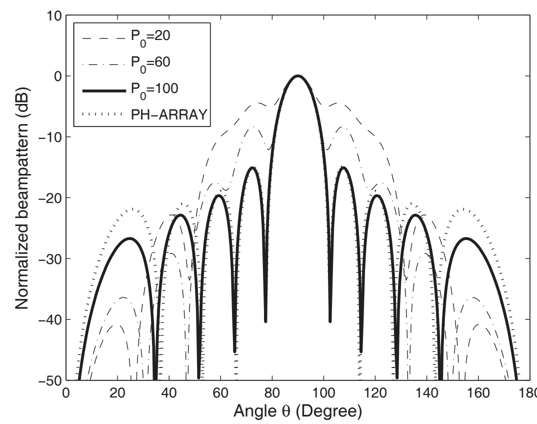

Fig. 3. SDP-EIG Beampatterns using $\mathbf{\Theta}_2$ and different P_0.

A more strict situation for designing a narrow focusing section is demonstrated in Fig.2, where $P_0 = 10^2$ (20 dB). In this case, $\mathrm{rank}\,(\mathbf{R_C}) = 1$ and the only eigenvalue is 10, which is equal to the energy constraint. The eigenvalues of $\mathbf{A}$ is 1.612, 37.58 and 170.8, and the eigenvectors corresponding to the largest two are chosen. Note that the SDP-EIG method always enjoys the advantage of sidelobe reduction compared to DPSS method. This is because in DPSS method there is no desired pattern $P_d(\theta)$ to be fitted. Note that the peak of DPSS within the section of interest is flat whereas in SDP-EIG it is not, but the -3 dB bound property is still preserved. Thus if the flat energy peak needs to be strictly preserved (better environment), we propose to use DPSS weight matrix. if the system suffers more from noise and interference, we propose to use SDP-EIG weight matrix.

Another important point to address is that $\mathrm{rank}\,(\mathbf{R_C}) = 1$ implies the system boils down to a phased-array structure, whereas the phase shift between each antenna may not be a constant like the typical phased-array radar. We further ob-

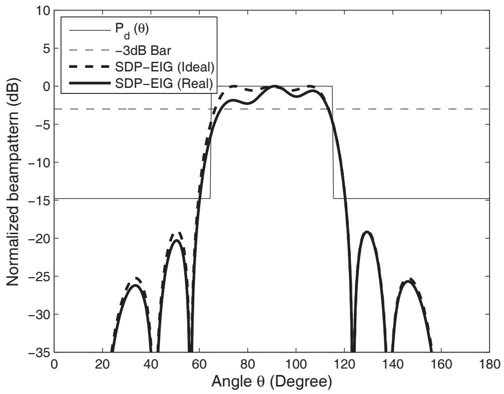

Fig. 4. SDP-EIG Beampattern using $\mathbf{\Theta}_1$ and Real Signals.

serve in Fig.3 that in a design of P_0 approaching infinity, the beampattern approaches that of the phased-array, or slightly better, still in terms of sidelobe reduction. As the SDP-EIG method finally results in a phased-array radar, we can conclude that if we want to form a narrow beam (say $10°$ or less) towards an angle section of interest, the simplest and efficient way is still using phased-array radar. The advantage of SDP-EIG is reflected when we want flexibly and efficiently design a wider transmit beampattern within which the mainlobe covers all the targets that are likely to exist.

Finally, we study the performance considering the real (practical) signals generated in [9]. The 4 Costas array coding (CAC) orthogonal waveforms, with coding dimension $M = 17$, have a MIMO ambiguity function sidelobe peak upper bounded by -10 dB, and all sidelobes are widely distributed in delay-Doppler plain. Fig.4, where $P_0 = 30$, shows that although the peak platform is degraded due to the imperfection of the covariance matrix of the orthogonal signals, it still lies above -3 dB level within the section of interest. Thus the signals designed for enhanced MIMO radar delay-Doppler resolution are also applicable for MIMO transmit beamforming.

5. CONCLUSION AND DISCUSSION

Inspired by the signal model in [6] and the problem formulation in [5], we propose the SDP-EIG method to design transmit beampattern for colocated MIMO radars. Instead of directly solving the weight matrix $\mathbf{C}$, we propose to solve $\mathbf{R_C}$ first and then find $\tilde{\mathbf{C}}$ based on eigen-decomposition of $\mathbf{R_C}$ and column reduction manipulations. We analyze the performance of SDP-EIG method in relation with the DPSS method and conventional phased-array. Simulation shows that to design a narrow transmit beam, SDP-EIG boils down to a phased-array system, and there is a tradeoff between flat peak requirement and sidelobe reduction issues. The advantage of our proposed method is reflected from the problem of designing a wide transmit beam, where both the waveform diversity and the phased-array beam property can be efficiently combined. The SDP-EIG and the DPSS offer a nearly identical flat mainbeam, and the SDP-EIG has substantially lower sidelobes, which could be considered as a sidelobe free beampattern. The simulation using real (practical) signals further verifies the performance our proposed method.

Note that the desired beampattern $P_d(\theta)$ strongly affects the optimization performance. In this paper, we only use a rectangular function to specify it. It is worth to think about designing a more appropriate $P_d(\theta)$ by incorporating some features of $\mathbf{e}_{\mathrm{T}}^{H}(\theta)\,\mathbf{R_C}\mathbf{e}_{\mathrm{T}}(\theta)$ which may further reduce the squared error. In addition, it is intuitive to consider the transmit beamforming issues for a phased-array radar using the same weighting strategy, and analyze its performance in relation with the issues discussed in this work.

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