PARAMETRIC MULTICHANNEL ADAPTIVE SIGNAL DETECTION: EXPLOITING PERSYMMETRIC STRUCTURE

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ABSTRACT

This paper considers a *parametric* approach for adaptive multichannel signal detection, where the disturbance is modeled by a multichannel auto-regressive (AR) process. Motivated by the fact that a symmetric antenna geometry usually yields a persymmetric structure on the covariance matrix of disturbance, a new persymmetric AR (PAR) modeling for the disturbance is proposed and, accordingly, a persymmetric parametric adaptive matched filter (Per-PAMF) is developed. The developed Per-PAMF, while allowing a simple implementation like the traditional PAMF, extends the PAMF by developing the maximum likelihood (ML) estimation of unknown nuisance (disturbance-related) parameters under the persymmetric constraint. Numerical results show that the Per-PAMF provides significantly better detection performance than the conventional PAMF and other non-parametric detectors when the number of training signals is limited.

***Index Terms*—** Multichannel signal processing, adaptive matched filter, maximum likelihood estimation, persymmetry, multichannel auto-regressive process.

1. INTRODUCTION

Multichannel adaptive signal detection against strong spatially and temporally colored disturbances has been encountered in many applications, e.g., wireless communications, hyperspectral imaging, and medical imaging. Traditional techniques are limited for practical applications due to their excessive training requirement and high computational complexity. For example, the covariance-matrix-based detectors, e.g., Kelly's generalized likelihood ratio test (GLRT) [1], the adaptive matched filter (AMF) [2], and the adaptive coherence estimator (ACE) [3], need $K \geq JN$ training signals to ensure a full-rank estimate of the disturbance covariance matrix and have to invert the $JN \times JN$ covariance matrix, where J denotes the number of antennas and N denotes the number of pulses.

Among other techniques, a class of *parametric detectors* provide an efficient way to simultaneously mitigate the training requirement and reduce the computational complexity [4–7] (and reference therein). By modeling the disturbance as a multichannel auto-regressive (AR) process, the parametric detectors decompose the jointly spatio-temporal whitening of the covariance-matrix-based detectors into successive temporal whitening and spatial whitening. As a well-known parametric detector, the parametric AMF (PAMF) is simple to implement and has been verified to provide better performance with reduced computational complexity than the non-parametric counterpart, i.e., the AMF, especially when $K \ll JN$.

In addition to the assumption of the multichannel AR process, the aim of this paper is to further improve the traditional PAMF in terms of training-signal efficiency by exploiting the structural information about the disturbance covariance matrix, i.e., the persymmetry. In [8], Nitzberg shows that the efficiency of usage of training signals is improved by up to a factor of two by utilizing persymmetry. Other adaptive detection schemes explicitly taking into account the persymmetry have been proposed in [9] and, more recently, [10–13]. The results in the above references show that exploiting the persymmetric property improves the detection performance in terms of the training efficiency and enhances the robustness in terms of the constant false alarm rate (CFAR).

In this paper, we take advantages of both the multichannel-AR-based PAMF and the underlying persymmetry to propose a persymmetric PAMF (Per-PAMF). The Per-PAMF is developed in a two-step procedure. In detail, a non-adaptive parametric matched filter (PMF) is first introduced by assuming the knowledge of the nuisance parameters and, then, the Per-PAMF is developed from the PMF by replacing the nuisance parameters by their maximum likelihood (ML) estimates from training signals under the persymmetric constraint. The Per-PAMF is numerically compared to the conventional PAMF and the results show that the Per-PAMF has slightly better performance than the PAMF when the number of training signals is sufficient, while it significantly outperforms the the PAMF in cases with extremely limited training signals.

The remainder of the paper is organized as follows. Section 2 contains the signal model and introduces the persymmetric AR modeling for the disturbance. The Per-PAMF detector is derived in Section 3. Numerical results with two distinct datasets are provided in Section 4. The conclusion is finally drawn in Section 5.

2. SIGNAL MODEL

The problem of interest is to decide one of the following two hypotheses is true:

$$\begin{aligned} H_0 &: \mathbf{x}_0 = \mathbf{d}_0, \\ H_1 &: \mathbf{x}_0 = \alpha\mathbf{s} + \mathbf{d}_0, \end{aligned} \tag{1}$$

where $\mathbf{x}_0$ is the $JN \times 1$ test signal, $\mathbf{s}$ is the *known* space-time steering vector which is a Kronecker product between the space and temporal steering vectors, i.e.,$\mathbf{s} = \mathbf{s}_d \otimes \mathbf{s}_s$, α is an *unknown* complex-valued amplitude, and $\mathbf{d}_0$ is the disturbance which is assumed to be complex Gaussian vector with zero-mean and unknown covariance matrix $\mathbf{R}$, i.e., $\mathbf{d}_0 \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$. In addition, there are K target-free independent and identically distributed (i.i.d.) training signals $\mathbf{x}_k = \mathbf{d}_k, k = 1, \cdots, K$ with distribution $\mathbf{d}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$, which is also independent of the test signal. Moreover, the disturbance in both test and training signals can be modeled as a multichannel AR process [4]. Specifically, let $\mathbf{d}_k(n) \in \mathbb{C}^{J\times 1}, n = 0, 1, \ldots, N-1$, denote the N non-overlapping temporal segments of $\mathbf{d}_k$, i.e., $\mathbf{d}_k \triangleq$

$\left[\mathbf{d}_k^T(0), \mathbf{d}_k^T(1), \cdots, \mathbf{d}_k^T(N-1)\right]^T$. The multichannel AR modeling is mathematically described by

$$\mathbf{d}_k(n) = -\sum_{p=1}^{P} \mathbf{A}^H(p)\mathbf{d}_k(n-p) + \boldsymbol{\varepsilon}_k(n), \quad (2)$$

where $\boldsymbol{\varepsilon}_k(n) \sim \mathcal{CN}(0, \mathbf{Q})$ is the J-channel temporally white but spatially colored Gaussian driving noise with $\mathbf{Q}$ denoting the *unknown* $J \times J$ spatial covariance matrix, and $\{\mathbf{A}(p)\}_{p=1}^{P}$ denote the *unknown* $J \times J$ AR coefficient matrices.

In this paper, we consider a case frequently encountered in practice, where the systems use a symmetric antenna configuration (symmetrical with respect to its phase center) and transmit a set of pulses of equal duration [8]. For example, the widely used uniform linear array with a constant pulse repetition frequency (PRF) is such a system. The structured antenna array configurations and constant PRF cause the spatio-temporal covariance matrix $\mathbf{R}$ to be *persymmetric-block-Toeplitz*. To exploit this structure information of the covariance matrix $\mathbf{R}$, a new multichannel AR model by incorporating the persymmetric property is introduced below; in addition to (2), we impose two more assumptions (**AS1** and **AS2**) about the covariance matrix $\mathbf{Q}$ of the driving noise and the AR coefficient matrices $\mathbf{A}(p), p = 1, \cdots, P$:

$$\textbf{AS1}: \quad \mathbf{Q} = \mathbf{E}\mathbf{Q}^*\mathbf{E}, \quad (3)$$

where $[\cdot]^*$ denotes the complex conjugate and $\mathbf{E}$ denotes the exchange matrix which is one at the anti-diagonal elements and zeros otherwise, and

$$\textbf{AS2}: \quad \mathbf{A}(p) = \mathbf{E}\mathbf{A}^*(p)\mathbf{E}. \quad (4)$$

The covariance matrix of the above persymmetric AR (PAR) process can be verified to be persymmetric-block-Toeplitz[1] and, hence, the PAR modeling (of (2), (3) and (4)) provides a parametric way to approximate the disturbance with a persymmetric-block-Toeplitz covariance matrix $\mathbf{R}$.

3. PERSYMMETRIC PARAMETRIC ADAPTIVE MATCHED FILTER

The Per-PAMF is developed in a two-step approach: 1) find the GLRT when the nuisance parameters $\mathbf{A}$ and $\mathbf{Q}$ are assumed both known; 2) then replace $\mathbf{A}$ and $\mathbf{Q}$ by their ML estimates from training signals $\mathbf{x}_k, k = 1, \cdots, K$, under the persymmetric constraint.

3.1. PMF — the GLRT with Known $\mathbf{A}$ and $\mathbf{Q}$

When $\mathbf{A}$ and $\mathbf{Q}$ are both known, the GLRT reduces to the nonadaptive PMF [7]

$$T_{\text{PMF}} = \frac{\left|\sum\limits_{n=P}^{N-1} \tilde{\mathbf{s}}^H(n)\mathbf{Q}^{-1}\tilde{\mathbf{x}}_0(n)\right|^2}{\sum\limits_{n=P}^{N-1} \tilde{\mathbf{s}}^H(n)\mathbf{Q}^{-1}\tilde{\mathbf{s}}(n)} \quad (5)$$

where $\tilde{\mathbf{s}}$ and $\tilde{\mathbf{x}}_0$ are, respectively, the temporally whitened steering vector and test signal obtained with the *true* temporal correlation matrices $\mathbf{A}(p), p = 1, \cdots, P$,

$$\tilde{\mathbf{s}}(n) = \mathbf{s}(n) + \sum_{p=1}^{P} \mathbf{A}^H(p)\mathbf{s}(n-p), \quad (6)$$

$$\tilde{\mathbf{x}}_0(n) = \mathbf{x}_0(n) + \sum_{p=1}^{P} \mathbf{A}^H(p)\mathbf{x}_0(n-p). \quad (7)$$

In the case of unknown $\mathbf{A}$ and $\mathbf{Q}$, the above PMF cannot be implemented and, therefore, we need to replace $\mathbf{A}$ and $\mathbf{Q}$ with their ML estimates under the persymmetric constraints of (3) and (4).

3.2. Persymmetric ML Estimate of $\mathbf{Q}$

The ML estimates of $\mathbf{A}$ and $\mathbf{Q}$ under the persymmetric constraint (denoted as the PML estimates) are obtained from training signals only. According to the signal model, the joint likelihood function of training signals $\mathbf{x}_k, k = 1, \cdots, K$, can be written as

$$p(\mathbf{x}_1, \cdots, \mathbf{x}_K; \mathbf{A}, \mathbf{Q}) = \left[\frac{1}{\pi^J |\mathbf{Q}|} e^{-\operatorname{tr}\left(\mathbf{Q}^{-1}\boldsymbol{\Gamma}_0\right)}\right]^{K(N-P)},$$

where

$$K(N-P)\boldsymbol{\Gamma}_0 = \sum_{k=1}^{K}\sum_{n=P}^{N-1} \boldsymbol{\varepsilon}_k(n)\boldsymbol{\varepsilon}_k^H(n), \quad (8)$$

with definitions

$$\boldsymbol{\varepsilon}_k(n) = \mathbf{x}_k(n) + \sum_{p=1}^{P} \mathbf{A}^H(p)\mathbf{x}_k(n-p). \quad (9)$$

Alternatively, $K(N-P)\boldsymbol{\Gamma}_0$ can be rewritten as

$$K(N-P)\boldsymbol{\Gamma}_0 = \hat{\mathbf{R}}_{xx} + \mathbf{A}^H\hat{\mathbf{R}}_{yx} + \hat{\mathbf{R}}_{yx}^H\mathbf{A} + \mathbf{A}^H\hat{\mathbf{R}}_{yy}\mathbf{A}, \quad (10)$$

where

$$\mathbf{A} \triangleq \left[\mathbf{A}^H(1), \mathbf{A}^H(2), \cdots, \mathbf{A}^H(P)\right]^H, \quad (11)$$

$$\hat{\mathbf{R}}_{xx} = \sum_{k=1}^{K}\sum_{n=P}^{N-1} \mathbf{x}_k(n)\mathbf{x}_k^H(n), \quad (12)$$

$$\hat{\mathbf{R}}_{yy} = \sum_{k=1}^{K}\sum_{n=P}^{N-1} \mathbf{y}_k(n)\mathbf{y}_k^H(n), \quad (13)$$

$$\hat{\mathbf{R}}_{yx} = \sum_{k=1}^{K}\sum_{n=P}^{N-1} \mathbf{y}_k(n)\mathbf{x}_k^H(n), \quad (14)$$

and $\mathbf{y}_k(n)$ is a regression vector of $\mathbf{x}_k(n)$:

$$\mathbf{y}_k(n) \triangleq \left[\mathbf{x}_k^T(n-1), \mathbf{x}_k^T(n-2), \cdots, \mathbf{x}_k^T(n-P)\right]^T. \quad (15)$$

By exploiting the persymmetric property of $\mathbf{Q}$, i.e., (3), we have

$$\operatorname{tr}\left(\mathbf{Q}^{-1}\boldsymbol{\Gamma}_0\right) = \operatorname{tr}\left(\mathbf{Q}^{-1}\frac{\boldsymbol{\Gamma}_0 + \mathbf{E}\boldsymbol{\Gamma}_0^*\mathbf{E}}{2}\right), \quad (16)$$

which leads to $\ln p \propto -\ln|\mathbf{Q}| - \frac{1}{2}\operatorname{tr}\left(\mathbf{Q}^{-1}\left[\boldsymbol{\Gamma}_0 + \mathbf{E}\boldsymbol{\Gamma}_0^*\mathbf{E}\right]\right)$. Taking the derivative of $\ln p$ with respect to $\mathbf{Q}$ and equating the results to zero produces the ML estimates of $\mathbf{Q}$ as

$$\hat{\mathbf{Q}}_{\text{PML}} = \frac{1}{2}\left(\boldsymbol{\Gamma}_0 + \mathbf{E}\boldsymbol{\Gamma}_0^*\mathbf{E}\right). \quad (17)$$

As a result, $\ln p \propto -\ln\left|\frac{1}{2}\left(\boldsymbol{\Gamma}_0 + \mathbf{E}\boldsymbol{\Gamma}_0^*\mathbf{E}\right)\right|$. Therefore, the persymmetric ML estimate of $\mathbf{A}$ is equivalent to minimizing the determinant of $K(N-P)\left(\boldsymbol{\Gamma}_0 + \mathbf{E}\boldsymbol{\Gamma}_0^*\mathbf{E}\right)/2$.

[1]The proof is skipped here due to the page limit.

3.3. Persymmetric ML Estimate of $\mathbf{A}$

Note that the new variable $\mathbf{A}$ of (11) stacks $\mathbf{A}(p)$ in a column wise. From (4), $\mathbf{A}$ has the following property

$$\mathbf{A} = \mathbf{E}_B \mathbf{A}^* \mathbf{E}, \tag{18}$$

where $\mathbf{E}_B = \mathbf{I}_P \otimes \mathbf{E}$ with $\mathbf{I}_P$ denoting a $P \times P$ identity matrix.

From (10), $K(N-P)(\mathbf{\Gamma}_0 + \mathbf{E}\mathbf{\Gamma}_0^* \mathbf{E})$ can be expressed as

$$\begin{aligned}
&K(N-P)(\mathbf{\Gamma}_0 + \mathbf{E}\mathbf{\Gamma}_0^*\mathbf{E}) \overset{(a)}{=} \left[\hat{\mathbf{R}}_{xx} + \mathbf{E}\hat{\mathbf{R}}_{xx}^*\mathbf{E}\right] \\
&+ \left[\mathbf{A}^H \hat{\mathbf{R}}_{yx} + \mathbf{E}(\mathbf{A}^H \mathbf{E}_B \mathbf{E}_B \hat{\mathbf{R}}_{yx})^* \mathbf{E}\right] \\
&+ \left[\hat{\mathbf{R}}_{yx}^H \mathbf{A} + \mathbf{E}(\hat{\mathbf{R}}_{yx}^H \mathbf{E}_B \mathbf{E}_B \mathbf{A})^* \mathbf{E}\right] \\
&+ \left[\mathbf{A}^H \hat{\mathbf{R}}_{yy} \mathbf{A} + \mathbf{E}(\mathbf{A}^H \mathbf{E}_B \mathbf{E}_B \hat{\mathbf{R}}_{yy} \mathbf{E}_B \mathbf{E}_B \mathbf{A})^* \mathbf{E}\right] \\
&\overset{(b)}{=} \left[\hat{\mathbf{R}}_{xx} + \mathbf{E}\hat{\mathbf{R}}_{xx}^*\mathbf{E}\right] + \mathbf{A}^H \left[\hat{\mathbf{R}}_{yx} + \mathbf{E}_B \hat{\mathbf{R}}_{yx}^* \mathbf{E}\right] \\
&+ \left[\hat{\mathbf{R}}_{yx}^H + \mathbf{E}(\hat{\mathbf{R}}_{yx}^H)^* \mathbf{E}_B\right] \mathbf{A} + \mathbf{A}^H \left[\hat{\mathbf{R}}_{yy} + \mathbf{E}_B \hat{\mathbf{R}}_{yy}^* \mathbf{E}_B\right] \mathbf{A},
\end{aligned}$$

where (a) has used the fact that $\mathbf{E}_B \mathbf{E}_B = \mathbf{I}_{JP}$, and (b) is due to (18) and $\mathbf{E}_B^H = \mathbf{E}_B$. Denote

$$\hat{\mathbf{R}}_{xx,P} = \left(\hat{\mathbf{R}}_{xx} + \mathbf{E}\hat{\mathbf{R}}_{xx}^*\mathbf{E}\right)/2, \tag{19}$$

$$\hat{\mathbf{R}}_{yx,P} = \left(\hat{\mathbf{R}}_{yx} + \mathbf{E}_B\hat{\mathbf{R}}_{yx}^*\mathbf{E}\right)/2, \tag{20}$$

$$\hat{\mathbf{R}}_{yy,P} = \left(\hat{\mathbf{R}}_{yy} + \mathbf{E}_B\hat{\mathbf{R}}_{yy}^*\mathbf{E}_B\right)/2. \tag{21}$$

As a result, $K(N-P)(\mathbf{\Gamma}_0 + \mathbf{E}\mathbf{\Gamma}_0^*\mathbf{E})/2$ can be rewritten as

$$\begin{aligned}
&K(N-P)(\mathbf{\Gamma}_0 + \mathbf{E}\mathbf{\Gamma}_0^*\mathbf{E})/2 \\
=&\hat{\mathbf{R}}_{xx,P} + \mathbf{A}^H \hat{\mathbf{R}}_{yx,P} + \hat{\mathbf{R}}_{yx,P}^H \mathbf{A} + \mathbf{A}^H \hat{\mathbf{R}}_{yy,P} \mathbf{A} \\
=&\left(\mathbf{A}^H + \hat{\mathbf{R}}_{yx,P}^H \hat{\mathbf{R}}_{yy,P}^{-1}\right) \hat{\mathbf{R}}_{yy,P} \left(\mathbf{A}^H + \hat{\mathbf{R}}_{yx,P}^H \hat{\mathbf{R}}_{yy,P}^{-1}\right)^H \\
&+ \left(\hat{\mathbf{R}}_{xx,P} - \hat{\mathbf{R}}_{yx,P}^H \hat{\mathbf{R}}_{yy,P}^{-1} \hat{\mathbf{R}}_{yx,P}\right).
\end{aligned} \tag{22}$$

Since $\hat{\mathbf{R}}_{yy,P}$ is nonnegative definite and the remaining term does not depend on $\mathbf{A}$, it follows that

$$\begin{aligned}
(\mathbf{\Gamma}_0 + \mathbf{E}\mathbf{\Gamma}_0^*\mathbf{E})/2 &\geq (\mathbf{\Gamma}_0 + \mathbf{E}\mathbf{\Gamma}_0^*\mathbf{E})/2 \Big|_{\mathbf{A} = -\hat{\mathbf{R}}_{yx,P}^H \hat{\mathbf{R}}_{yy,P}^{-1}} \\
&= \frac{\hat{\mathbf{R}}_{xx,P} - \hat{\mathbf{R}}_{yx,P}^H \hat{\mathbf{R}}_{yy,P}^{-1} \hat{\mathbf{R}}_{yx,P}}{K(N-P)},
\end{aligned} \tag{23}$$

which implies that

$$\hat{\mathbf{A}}_{\text{PML}} = -\hat{\mathbf{R}}_{yy,P}^{-1} \hat{\mathbf{R}}_{yx,P}, \tag{24}$$

and the persymmetric ML estimate of $\mathbf{Q}$ of (17) reduces to

$$\hat{\mathbf{Q}}_{\text{PML}} = \frac{\hat{\mathbf{R}}_{xx,P} - \hat{\mathbf{R}}_{yx,P}^H \hat{\mathbf{R}}_{yy,P}^{-1} \hat{\mathbf{R}}_{yx,P}}{K(N-P)}. \tag{25}$$

Interestingly, the persymmetric ML estimates of $\mathbf{A}$ and $\mathbf{Q}$ at (24) and (25) turn out to be, respectively, a persymmetric version of the standard ML estimates i.e.,

$$\hat{\mathbf{A}}_{\text{ML}} = -\hat{\mathbf{R}}_{yy}^{-1} \hat{\mathbf{R}}_{yx}, \quad \hat{\mathbf{Q}}_{\text{ML}} = \frac{\hat{\mathbf{R}}_{xx} - \hat{\mathbf{R}}_{yx}^H \hat{\mathbf{R}}_{yy}^{-1} \hat{\mathbf{R}}_{yx}}{K(N-P)}. \tag{26}$$

3.4. Per-PAMF

By replacing $\mathbf{A}$ and $\mathbf{Q}$ with the persymmetric ML estimates of $\mathbf{A}$ and $\mathbf{Q}$ in (5), we obtained the Per-PAMF as

$$T_{\text{Per-PAMF}} = \frac{\left|\sum_{n=P}^{N-1} \hat{\tilde{\mathbf{s}}}_P^H(n) \hat{\mathbf{Q}}_{\text{PML}}^{-1} \hat{\tilde{\mathbf{x}}}_{0,P}(n)\right|^2}{\sum_{n=P}^{N-1} \hat{\tilde{\mathbf{s}}}_P^H(n) \hat{\mathbf{Q}}_{\text{PML}}^{-1} \hat{\tilde{\mathbf{s}}}_P(n)} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{\text{Per-PAMF}} \tag{27}$$

where $\gamma_{\text{Per-PAMF}}$ is a threshold corresponding to a preset probability of false alarm, and $\hat{\tilde{\mathbf{s}}}_P$ and $\hat{\tilde{\mathbf{x}}}_{0,P}$ are obtained from (6) and (7), respectively, with the persymmetric ML estimate of $\mathbf{A}$ in (24), and $\hat{\mathbf{Q}}_{\text{PML}}$ is given by (25).

From (27), it is seen that the Per-PAMF inherits the reduced computational complexity of the conventional PAMF by performing successively a temporal whitening following by a spatial whitening, in contrast to the computationally intensive joint spatio-temporal whitening of the covariance matrix based approach (e.g., the AMF [2]). On the other hand, it further improves robustness of the parameter estimation by exploiting the underlying structure of the disturbance covariance matrix via the persymmetric ML estimates of the AR coefficient matrices $\mathbf{A}$ and the spatial covariance matrix $\mathbf{Q}$.

4. NUMERICAL EVALUATION

In this section, simulation results are provided to demonstrate the efficiency of the proposed Per-PAMF in the training-limited case, e.g., $K \ll JN$. The disturbance signal $\mathbf{d}_k$ is generated as a multichannel second-order PAR process ($P = 2$) with a AR coefficient $\mathbf{A}$ and a spatial covariance matrix $\mathbf{Q}$ satisfying (4) and (3). The signal-to-interference-plus-noise ratio (SINR) is defined as

$$\text{SINR} = |\alpha|^2 \mathbf{s}^H \mathbf{R}^{-1} \mathbf{s}, \tag{28}$$

where $\mathbf{R}$ is the corresponding covariance matrix of the PAR-modeled disturbance. The steering vector $\mathbf{s}$ is generated with a normalized spatial frequency $f_s = 0.2$ and a normalized Doppler frequency is $f_d = 0.2$, respectively. The simulated performance is obtained by using at least 10000 Monte Carlo trials for the probability of false alarm $P_f = 0.01$. Performance comparisons are made among the non-parametric AMF [2], the non-parametric persymmetric AMF (Per-AMF) [13], the conventional PAMF [4], and the clairvoyant matched filter (MF) [2]. Particularly, the simulated scenario uses $J = 5$ antenna elements and $N = 11$ pulses, while the number of training signals are, respectively, $K = 2$ and $K = 8$.

Fig. 1 shows the probability of detection versus the SINR with comparably sufficient training signals, e.g. $K = 8$. In this case, the performance gain of the Per-PAMF over the conventional PAMF is marginal since both parametric detectors have enough training signals to obtain good estimates of the unknown parameters. Meanwhile, both parametric detectors, i.e., the PAMF and Per-PAMF with $K = 8$ training signals, show better detection performance than the non-parametric covariance matrix based AMF and Per-AMF with $K = 2JN = 110$ training signals.

In Fig. 2, the number of training signals is reduced to $K = 2$. As shown in Fig. 2, the performance gap between the Per-PAMF and the PAMF increases to about 5 dB when $P_d = 0.8$, as the traditional PAMF gives much worse performance with $K = 2$ training signals. In other words, with only $K = 2$ training signals, the conventional PAMF cannot obtain reliable estimates of unknown parameters, e.g., $\mathbf{A}$ and $\mathbf{Q}$, which leads to performance degradation, while the Per-PAMF has better efficiency of using training signals for unknown

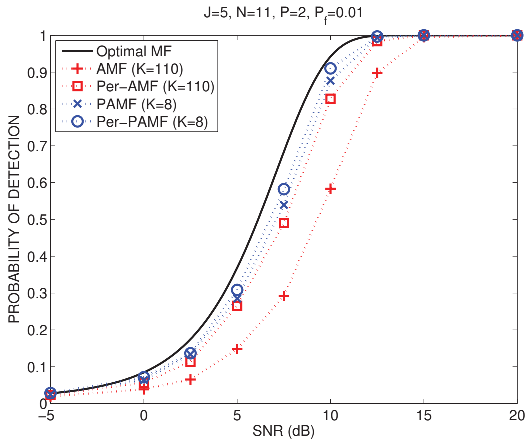

Fig. 1. Probability of detection versus SINR for $K = 8$ when $J = 5$, $N = 11$, $P = 2$, and $P_f = 0.01$.

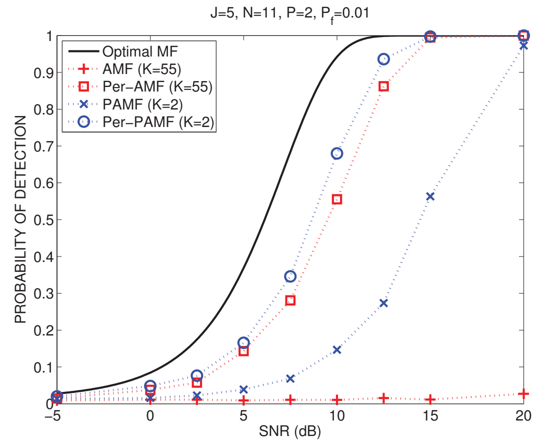

Fig. 2. Probability of detection versus SINR for $K = 2$ when $J = 5$, $N = 11$, $P = 2$, and $P_f = 0.01$.

parameter estimation and thus maintains its performance even with only $K = 2$ training signals. The Per-PAMF with $K = 2$ training signals is also better than its non-parametric Per-AMF with $K = 55$ training signals and much better than the non-parametric AMF with $K = 55$ training signals which fails to detect the target signal.

5. CONCLUSION

This paper extends the conventional PAMF by exploiting the structure properties of the disturbance covariance matrix. The developed Per-PAMF shares the same detection statistics as the conventional PAMF but utilizes the structure information through the estimation of the unknown AR coefficient matrices $\mathbf{A}$ and spatial covariance matrix $\mathbf{Q}$. Numerical results have verified that exploiting the persymmetric information allows a reduction in the number of training signals for the detection and, hence, yields better detection performance than the conventional PAMF as well as the covariance matrix based detectors when training signals are limited.

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