

PEAK-PULSE DETECTION ERROR PROBABILITY FOR RFID SAW-TAGS WITH PULSE POSITION CODING

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°GVR Trade SA, Ch. du Vignoble 31C, 2022 Bevaix, Switzerland

ABSTRACT

We discuss the code reading error probability (EP) in the radio frequency identification (RFID) surface acoustic wave tags with pulse position coding and peak-pulse detection. EP is found in the most general form assuming M groups of codes with N slots each and allowing individual signal-to-noise ratios (SNRs) in each slot. We show that if the RFID tag is designed such that the spurious responses are attenuated on more than 20 dB below On-pulses, then EP can be achieved at the level of 10^{-8} (one false per 10^8 readings) with $\text{SNR} > 17$ dB for any reasonable M and N .

Index Terms— radio frequency identification, surface acoustic wave, code reading error probability

1. INTRODUCTION

Radio frequency identification (RFID) tags employing surface acoustic waves (SAWs) have found industrial applications requiring highly reliable ID [1]. Although a prototype, the familiar optical character recognition known as *barcode*, has been in use since 1960s, the first RFID SAW-tags were invented in 1970s [2] and used since 1980s [3]. In early designs, binary coding was implemented [4] similarly to binary amplitude shift keying. Today, SAW tag products employ mostly the pulse position coding (PPC) [1]: the total time delay is divided into groups of slots with one On-pulse in each group [2, 5]. We find such a design having 2 groups with 4 slots each in [6, 7]. A 5 digit decimal number (10 slots) design is described in [8] and the design having 6 groups with 16 slots each addressed in [9].

In modern RFID SAW tags with PPC, the identification is provided employing the peak-pulse detection algorithms [9]. Increased the code length, the problem thus arises similarly to that in digital communications: the longer the code is the larger the code reading *error probability* (EP) is expected. Since EP strongly depends on noise and the reader power, the effective reader range is also limited. Although the demand for small EP naturally arises from customers, the complete theory of errors in RFID SAW tags is still not addressed. Note that attempts to specify EP were made in [11] via the threshold and in [9] via the peak-pulse detection.

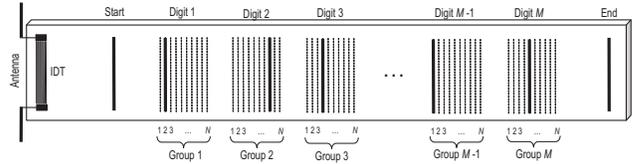


Fig. 1. A generalized structure of the RFID SAW-tag with pulse position coding.

A generalized structure of the RFID SAW-tag with PPC is sketched in Fig. 1. Here, equal slots of time duration comparable to $1/B$ (B is the tag frequency bandwidth, in Hz) are allocated for responses and the center of each slot is dashed. Only a unique response (On-pulse) provided by a single reflector is allowed in each group of slots. The groups are separated by additional “guard” slots. The code-reflector array is commonly designed such that the responses have near equal amplitudes at the reader. The reader time scale is calibrated with two reflectors “Start” and “End”. A typical time response of a tag manufactured to have 6 groups with 16 slots and allowing for about 1.7×10^6 different codes (24 bits) is shown in Fig. 2.

Given the RFID SAW tag (Fig. 1), we show below an exact formula for the code reading EP allowing for individual signal-to-noise ratios (SNRs) in all On- and Off-pulses.

2. CODE READING ERROR PROBABILITY

The EP can be determined by the ratio of the number of false readings to the total number of readings. In the m th group of slots, EP can be defined by the ratio of false reading of the On-pulse to the total number of reading.

2.1. Error in a Single Group

Consider the m th group of N slots (Fig. 1). Designers try to create a system such that signal is always larger than noise. Nature, in turn, may act in an opposite direction so that On-pulse can suddenly finds itself below noise.

Suppose that the RF On-pulse representing the n th slot in m th group arrives at the reader with the peak power $2S_{n,m}$.

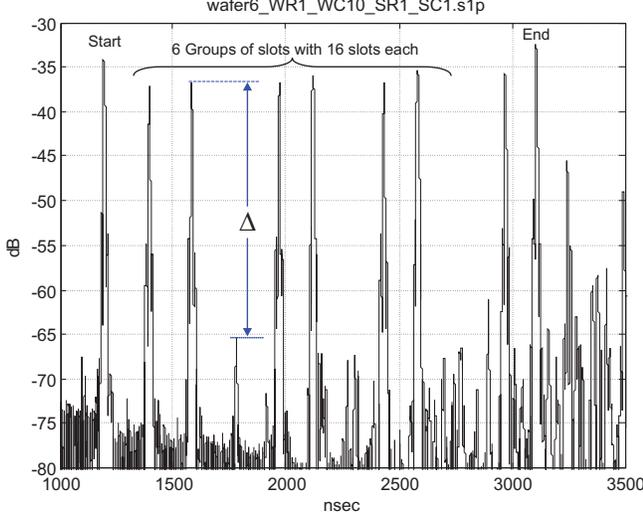


Fig. 2. Typical time response of a SAW-tag with PPC manufactured to have 6 groups with 16 slots each. “Start” and “End” pulses are used to calibrate time scale.

It is then contaminated by narrowband noise [10] having the variance $\sigma_{n,m}^2$ and becomes noisy with the peak-envelope $V_{n,m} \geq 0$ [11, 12]. We thus introduce the SNR $\gamma_{n,m}$ and normalized peak-envelope $z_{n,m}$ as, respectively,

$$\gamma_{n,m} = \frac{S_{n,m}}{\sigma_{n,m}^2}, \quad (1)$$

$$z_{n,m} = \frac{V_{n,m}}{\sqrt{2\sigma_{n,m}^2}}. \quad (2)$$

For further convenience, replace On-pulse to the last N th slot, because its location does not affect errors. Normal functioning of the m th group is implied if On-pulse in the N th slot exceeds Off-pulses in the remaining ones. The events when each Off-pulse does not exceed On-pulse can be depicted as $\bar{C}_{1,m}, \bar{C}_{2,m}, \dots, \bar{C}_{N-1,m}$. Then the probability of simultaneous occurring of these events can be written as

$$P(\bar{\mathbf{C}}_m | \mathbf{\Upsilon}_m, z_{N,m}), \quad (3)$$

where $\bar{\mathbf{C}}_m = [\bar{C}_{1,m}, \bar{C}_{2,m}, \dots, \bar{C}_{N-1,m}]$ and $\mathbf{\Upsilon}_m = [\gamma_{1,m}, \gamma_{2,m}, \dots, \gamma_{N,m}]$, under the condition that $z_{N,m}$ and $\mathbf{\Upsilon}_m$ are given. In turn, the probability that at least one Off-pulse exceeds On-pulse (event \tilde{C}_m) can be found with

$$P(\tilde{C}_m | \mathbf{\Upsilon}_m, z_{N,m}) = 1 - P(\bar{\mathbf{C}}_m | \mathbf{\Upsilon}_m, z_{N,m}). \quad (4)$$

We can also assign the probability that On-pulse has the envelope $z_{N,m}$ (event D_m), given $\gamma_{N,m}$, as

$$P(z_{N,m} | \gamma_{N,m}) = P(D_m | \gamma_{N,m}). \quad (5)$$

On the other hand, when we speak about On-pulse we think that it has $z_{N,m}$ with small tolerance $\Delta z_{N,m} \ll 1$.

Because $z_{N,m}$ is fully determined by the probability density function (pdf) $p(z_{N,m} | \gamma_{N,m})$ of the On-pulse envelope, $P(z_{N,m} | \gamma_{N,m})$ can thus alternatively be represented as

$$P(z_{N,m} | \gamma_{N,m}) \cong p(z_{N,m} | \gamma_{N,m}) \Delta z_{N,m}. \quad (6)$$

EP \mathcal{P}_m in the m th group can now be defined by simultaneous occurring of \tilde{C}_m and D_m , meaning that at least one Off-pulse exceeds On-pulse. Supposing that these events are independent and utilizing the above-given relationships, we arrive at

$$\mathcal{P}_{em}(z_{N,m}, \gamma_{N,m}) \cong [1 - P(\bar{\mathbf{C}}_m | \mathbf{\Upsilon}_m, z_{N,m})] \times p(z_{N,m} | \gamma_{N,m}) \Delta z_{N,m}. \quad (7)$$

To get rid of $z_{N,m}$, integrate it out in $\mathcal{P}_{em}(z_{N,m}, \gamma_{N,m})$. EP for the m th group thus becomes

$$\mathcal{P}_{em}(\gamma_{N,m}) = \int_0^\infty [1 - P(\bar{\mathbf{C}}_m | \mathbf{\Upsilon}_m, x)] p(x | \gamma_{N,m}) dx. \quad (8)$$

Typically, noise in slots is uncorrelated and thus the events $\bar{C}_{1,m}, \bar{C}_{2,m}, \dots, \bar{C}_{N-1,m}$ are uncorrelated as well. Accepting this, assigning the opposite events when Off-pulse exceeds On-pulse as $C_{1,m}, C_{2,m}, \dots, C_{N-1,m}$, and using the relationships

$$P(\bar{\mathbf{C}}_m | z_{N,m}) = \prod_{n=1}^{N-1} [1 - P(C_{n,m} | \gamma_{n,m}, z_{N,m})], \quad (9)$$

where $\mathbf{C}_m = [C_{1,m}, C_{2,m}, \dots, C_{N-1,m}]$, we come up with the most general form of EP,

$$\mathcal{P}_{em}(\mathbf{\Upsilon}_m) = \int_0^\infty \left\{ 1 - \prod_{n=1}^{N-1} [1 - P(C_{n,m} | \gamma_{n,m}, x)] \right\} \times p(x | \gamma_{N,m}) dx, \quad (10)$$

suitable for the m th group of slots with the individual SNRs in On-pulse and Off-pulses.

2.2. Total Error

Let us get back to Fig. 1 and assign the event of normal functioning of the m th group as \bar{A}_m , meaning that On-pulse exceeds all Off-pulses. Otherwise, the event is A_m . The probability of successful code reading can be defined by the probability of simultaneous normal functioning of all of the groups as $P(\bar{A}_1 \bar{A}_2 \dots \bar{A}_M)$.

The reader makes mistakes if error occurs at least in one of the groups. Because errors in groups can appear independently, the code reading EP can be defined as

$$\mathcal{P}_e = 1 - \prod_{m=1}^M [1 - P(A_m)]. \quad (11)$$

Here $P(A_m)$ can be substituted with EP (10) in the m th group and we finally have the code reading EP for the SAW-tag (12) (see next page).

$$\mathcal{P}_e(\Upsilon_m, M, N) = 1 - \prod_{m=1}^M \left\{ 1 - \int_0^{\infty} \left\{ 1 - \prod_{n=1}^{N-1} [1 - P(C_{n,m}|\gamma_{n,m}, x)] \right\} p(x|\gamma_{N,m}) dx \right\}. \quad (12)$$

3. GAUSSIAN MODEL

In narrowband Gaussian noise environment, the normalized envelope z of On-pulse has the Rice pdf [11]

$$p(z_{n,m}, \gamma_{n,m}) = 2z_{n,m} e^{-z_{n,m}^2 - \gamma_{n,m}} I_0(2z_{n,m} \sqrt{\gamma_{n,m}}), \quad (13)$$

where $I_0(x)$ is the modified Bessel function of the first kind and zeroth order. This pdf becomes Rayleigh's,

$$p(z_{n,m}) = 2z_{n,m} e^{-z_{n,m}^2}, \quad (14)$$

for Off-pulses formed only by noise. However, it follows from Fig. 2 that some of Off-pulses can suffer of residual reflections so that, most generally, (13) should be used to describe responses of both On- and Off-pulses.

By (13), the probability $P(C_{n,m}|\gamma_{n,m}, z_{N,m})$ can be rewritten as

$$P(C_{n,m}|\gamma_{n,m}, z_{N,m}) = Q_1(\sqrt{2\gamma_{n,m}}, \sqrt{2}z_{N,m}), \quad (15)$$

where $Q_1(a, b)$ is the generalized Marcum Q -function of first order. Substituting in (12) $P(C_{n,m}|\gamma_{n,m}, x)$ with (15) and $p(z_{n,m}, \gamma_{N,m})$ with (13) gives us EP for the Gaussian noise.

3.1. Basic case

The case of zero SNRs in all Off-pulses and non-zero equal SNRs in all On-pulses can be said to be basic or ideal for RFID SAW tagging. This case implies Rice's pdf (13) for all On-pulses and Rayleigh's (14) for all Off-pulses with $z = z_{n,m}$ and $\gamma = \gamma_{n,m}$. That gives us

$$p(z, \gamma) = 2ze^{-z^2 - \gamma} I_0(2z\sqrt{\gamma}), \quad (16)$$

$$P(C_{n,m}|z) = 2 \int_z^{\infty} xe^{-x^2} dx = e^{-z^2} \quad (17)$$

and transforms (12) to (18) (see next page) still having no closed form. Originally, (18) was published in [9].

3.2. Limiting EP

Let us suppose that the SAW-tag responses at the reader with equal SNRs in all On-pulse and zeroth SNRs in all Off-pulses. EP of code reading in such a tag can be calculated employing (18). Because variations in the amplitudes of both On- and Off-pulses are ignored along with the possible excursions in some Off-pulses, we call this measure the *limiting code reading EP*, as corresponding to minimum errors.

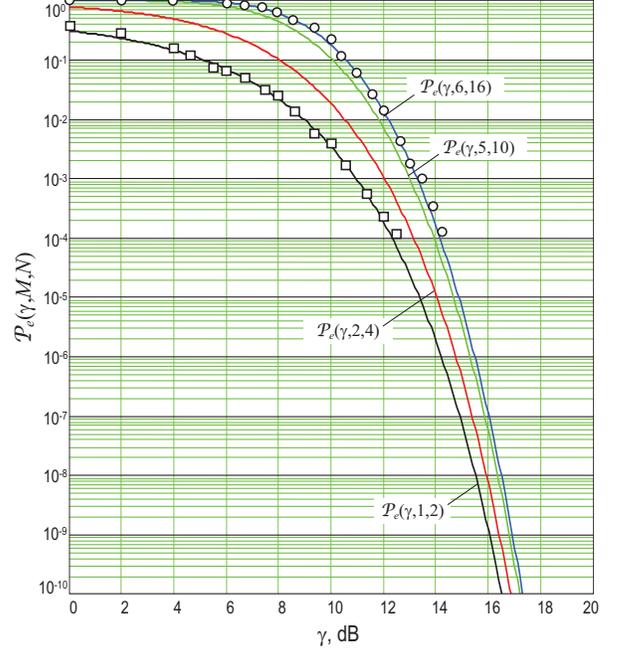


Fig. 3. Code reading EP for RFID SAW-tags. Simulation is depicted with \circ and \square , provided 2×10^5 readings for each SNR.

The case of $M = 1$ and $N = 2$ is limiting. It gives us the EP lower bound

$$\mathcal{P}_e(\gamma, 1, 2) = \frac{1}{2} e^{-\frac{\gamma}{2}} Q_1(\sqrt{\gamma}, 0) \quad (19)$$

that cannot be crossed in any design of SAW-tags with PPC.

Figure 3 illustrates (19) as a function of SNR. It also shows several limiting EPs computed by (18) for tags discussed in [2, 6–8] and having $M > 1$ and $N > 2$. Observing this figure, one can deduce that the limiting EP of 10^{-3} (one false per 1000 reading) can be achieved with SNR of just 14 dB for all reasonable M and N , provided equal white Gaussian noise in all Off-pulses.

4. READER RANGE

The reader range r (distance between the antenna and SAW-tag) can now be determined employing the radar equation [13]:

$$r = \frac{\lambda}{4\pi} \sqrt[4]{\frac{P_{t,reader} G_{reader}^2 G_{tag}^2}{L_{total} P_{r,reader}}}, \quad (20)$$

where λ is the electromagnetic wavelength, $P_{t,reader}$ and $P_{r,reader}$ are the signal power transmitted and received by

$$\mathcal{P}_e(\gamma, M, N) = 1 - \left\{ 1 - 2e^{-\gamma} \int_0^{\infty} \left[1 - \left(1 - e^{-x^2} \right)^{N-1} \right] x e^{-x^2} I_0(2x\sqrt{\gamma}) dx \right\}^M. \quad (18)$$

the reader, respectively, G_{reader} and G_{tag} are the reader and tag antenna gains, respectively, and L_{total} is the total loss of energy in the SAW-tag system including the propagation losses and SAW attenuation in the tag.

The receiver signal power can be expressed as $P_{r,\text{reader}} = P_{\text{noise}} \gamma_{\text{on}}(\mathcal{P}_e)$, where $P_{\text{noise}} = kT_0 B_r F$ is the thermal noise power, in which k is Boltzmann's constant, T_0 is absolute temperature, B_r is the reader system bandwidth in Hz, and F is the noise figure. Here, $\gamma_{\text{on}}(\mathcal{P}_e)$ is the SNR in On-pulses at the receiver detector. Note that $\gamma_{\text{on}}(\mathcal{P}_e)$ is specified by (18) or Fig. 3 for the EP \mathcal{P}_e required.

In white Gaussian noise, SNR can be reduced by the factor of N with multiple reading if one integrates N received pulses over time [13]. Replacing γ_{on} with γ_{on}/N and rewriting (20) in terms of the signal integration time T_{int} gives us the reader range as a function of the allowed EP,

$$r(\mathcal{P}_e) = \frac{\lambda}{4\pi} \sqrt[4]{\frac{P_{t,\text{reader}} G_{\text{reader}}^2 G_{\text{tag}}^2 T_{\text{int}}}{L_{\text{total}} k T_0 F \gamma_{\text{on}}(\mathcal{P}_e)}}, \quad (21)$$

in which the typical values are: $G_{\text{reader}} = 12$ dBi, $G_{\text{tag}} = 6$ dBi, $T_0 = 300$ K, $F = 5$ dB [14], and $L_{\text{total}} = 40$ dB. For these values and $P_{t,\text{reader}} = 10$ mW, the reader range varies from 5 m to 20 m [2] depending on the reader algorithm, T_{int} , and several other factors. Analyzing (21), one can deduce that, in order to guarantee very low EP at a given power emitted by the reader antenna, it needs obtaining $\gamma_{\text{on}} > 20$ dB. For this value of SNR, the reader range must be decreased by the factor of about 3 with respect to the maximal distance associated with visibly high EP of reader operation.

5. CONCLUSIONS

We have addressed the code reading EP for RFID SAW tag systems with PPC employing the peak-pulse detection algorithms. The most general conclusion is the following. If the RFID tag is designed such that the spurious responses are attenuated on more than 20 dB below On-pulses, as in Fig. 2, then EP of code reading over peak-pulse detection can be achieved at the level of 10^{-8} (one false per 10^8 readings) with $\text{SNR} > 17$ dB for any reasonable M and N .

In addition, the authors would like to thank Dr. V. Kalinin and Mr. B. Lulin ("Avangard" Ltd., St. Petersburg, Russia) for the given opportunity to use measurements of the SAW-tag time responses.

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