RELAXATION OF PARTICLE IMAGE VELOCIMETRY BASED ON SINGLE AUTOCORRELATION OF FILTERED MOTION BLURRING

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ABSTRACT

In this article, a technique to relax particle image velocimetry (PIV) based on the analysis of motion blurring of particles is proposed in order to reduce the frame rate required for measurement of particle velocities. Another advantage is the relaxation of the requirements of the measurement setup while retaining the principle of measurement analysis. This technique is based on convolutions with 5x5 spatial filters that can be computed in linear time, independently from the contents of the image. Due to multi-scale processing, the technique adapts its parameters autonomously, according to the image and particle properties to provide accurate results. The accuracy of the results is guaranteed by integrating the filter parameters, the measured angle and eventually the measured size of the particles into a linear model. The low complexity and high parallelizability of this method enable the online measurement of particles' velocities.

Index Terms— particle image velocimetry, motion blur, autocorrelation, spatial filter, multi scale representation

1. INTRODUCTION

Optical flow measurements [1] enable the detection of motion by analyzing sequences of ordered images. The technique utilizes the fact that motion causes a displacement of intensities from one image to the other (formula 1). Here, an intensity I at the position x, y and at the time t is expected to be equal to the intensity at the position x + dx, y + dy at time t + dt. A common method to determine optical flow is cross-correlation of image data, where the position of the peak in the correlation results reveals the direction and the value of the displacement.

$$I(x, y, t) = I(x + dx, y + dy, t + dt)$$

$$(1)$$

In measurement engineering, a technique similar optical flow measurement is particle image velocimetry (PIV). PIV is a method to measure particle velocities in natural sciences and engineering based on cross-correlation of the data of two images. Thus, in order to capture mostly the same particles in both images, short delays between the capturing of images in the microsecond range are required for fast particle velocities which can be as high as up to 600 m/s in some manufacturing processes [2]. Such short delays between the capturing of two images require special imaging systems which complicates the measurement setup of PIV.

In this paper, a filter technique is presented that relaxes the requirements of PIV measurement setups by exploiting the motion blurring of particles that is captured in a single image with an arbitrary imaging system. After that, the start and end positions of each motion blurring are filtered and the filtered image data is autocorrelated. Here, autocorrelation results show a peak similar to cross-correlation of PIV which reveals the direction and value of displacement.

In order to highlight the start and end of motion blurring but remove the motion blurring, the corner detectors Laplacian of gaussian, FAST and Harris [3] have been investigated. Here, all investigated techniques work for very thin particles, whereas Harris achieves the best results. However, for particles with average or big size, a loss of accuracy due to an offset was observed. This offset [4] is caused by the fact that a particle projection is circular but feature detectors highlight the edge of its motion blurring. The effect is shown in Fig. 1, where (a) and (b) are short exposure captions of the particle, (c) the long exposure image and (d) the filterd motion blurring. Here, an offset can be observed between the start and end with Harris (d) and the center of the particle positions (a) and (b).

The approach presented in this paper applies simple kernel filters but compensates for the impact of the particle size on the measured velocity. This is achieved measuring its size based on the highlighted width of the particle(e). An autonomous-validation method that is taking advantage of multi-scale-representation, is proposed in order to determine convenient parameters for the kernel filters. The method is evaluated on a 10.000 image test series and a benchmark of the technique for various signal-to-noise ratio and contrast settings demonstrates a high performance for typical application scenarios.

The remainder of this paper is organized as follows: In the second section, the method is presented and in Section 3, the robustness is investigated. Then, experimental results are shown in Section 4 and finally, Section 5 concludes this paper.



Fig. 1. Impact of the size on measured velocity.

2. SINGLE FRAME PARTICLE IMAGE VELOCIMETRY

The technique to relax PIV is based on filtering the motion blurring of objects or particles that is captured in one single image. After capturing, a filtering process is applied on the image data. Then, the filtered image data that contains the start and end positions of the motion blurring, is autocorrelated. Finally, a search for local maxima in the autocorrelation results enables to determine the velocity of the particles.

Figure 2 shows the results of the filter process (b) and of the autocorrelation (c) for an exemplary particle motion blurring (a). Here, the filtering does not only detect the start and end position of motion blurring but also highlights the width of the blurring. Accordingly, the autocorrelation results show multiple peaks which are the basis for a velocity vector and the size vector. Then, a linear regression model is applied in order to compensate for the impact of the particle size on the calculated particle velocities.



Fig. 2. Filter and autocorrelation results.

2.1. Filter process and autocorrelation

The filter results shown in Figure 2 are achieved by applying Sobel filter kernels [5] that are supplemented by a suppression factor s [4], as shown in Formula 2 and 3.

$$F_x^+ = \begin{pmatrix} 1 & 2 & -s & -2 & -1 \\ 4 & 8 & -s & -8 & -4 \\ 6 & 12 & 0 & -12 & -6 \\ 4 & 8 & -s & -8 & -4 \\ 1 & 2 & -s & -2 & -1 \end{pmatrix}, F_x^- = \begin{pmatrix} -1 & -2 & -s & 2 & 1 \\ -4 & -8 & -s & 2 & 1 \\ -6 & -12 & 0 & 12 & 6 \\ -4 & -8 & -s & 8 & 4 \\ -1 & -2 & -s & 2 & 1 \end{pmatrix}$$
(2)

$$F_{y}^{+} = \begin{pmatrix} -1 & -4 & -6 & -4 & -1 \\ -2 & -8 & -12 & -8 & -2 \\ -s & -s & 0 & -s & -s \\ 2 & 8 & 12 & 8 & 2 \\ 1 & 4 & 6 & 4 & 1 \end{pmatrix}, F_{y}^{-} = \begin{pmatrix} 1 & 4 & 6 & 4 & 1 \\ 2 & 8 & 12 & 8 & 2 \\ -s & -s & 0 & -s & -s \\ -2 & -8 & -12 & -8 & -2 \\ -1 & -4 & -6 & -4 & -1 \end{pmatrix}$$
(3)

Here, the effect of the parameter s is to suppress the motion blur while maintaining the start and end position of the blurring. Figure 3 shows the filter results for different settings of the suppression factor. Subfigure (a) shows an image of a motion blurred particle and Subfigure (b) represents filter results with no suppression factor (s = 0). Subfigure (c) shows good selection of suppression factor and for Subfigure (d), the value is too high.



Fig. 3. Influence of the suppression factor.

Algorithm 1 shows the steps of the technique and Figure 4 the corresponding filter results. First, the image is convolved with F_x^- and F_x^+ (line 2) and the results are combined into one image which highlights the start and end position of motion blurring. Then, in line 3, F_y^- and F_y^+ are convolved with the previous results in order to determine the diameter of the blurring and those results are binarized in order suppress the impact of low intensity filter responses caused by noise (line 4). The same steps are applied again on the image (line 6 to 8) but with a reversed filter order. The results (lines 4 and 8) are then combined (line 10) to implement measurements that are invariant to the angle of the motion blurring, and the importance of this step is shown in the last two columns of Figure 4.

Input: I Image (N,M) **Output**: \overrightarrow{v} : Velocity Vector ; \overrightarrow{s} : Size Vector **Data**: I_x, I_y, I_{yx}, I_{xy} : Grayscale Image (N,M) B_{yx}, B_{xy}, R : Binary Image (N,M) C: Autocorrelation Matrix (M+N-1,M+N-1) P_1, P_2, P_3 : Point 1 begin $I_x = (F_x^+ * I) + (F_x^- * I)$ 2
$$\begin{split} I_{yx} &= (F_y^+ * I_x) + (F_y^- * I_x) \\ B_{yx} &= Binarize(I_{yx}, 255 \to 1, 0...254 \to 0) \end{split}$$
3 4 5 $I_y = (F_y^+ * I) + (F_y^- * I)$ 6
$$\begin{split} I_{xy} &= (F_x^+ * I_y) + (F_x^- * I_y) \\ B_{xy} &= Binarize(I_{xy}, 255 \to 1, 0...254 \to 0) \end{split}$$
7 8 9 10 $R = B_{yx} \circ B_{xy}$ % Hadamard multiplication C = Autocorrelation(R)11 12 $P_1 = argMax_1(C)$ 13 $P_2 = argMax_2(C)$ 14 $P_3 = argMax_3(C)$ 15 16 17 18 19 $\overrightarrow{v} = \overrightarrow{P_1 P_3}; \overrightarrow{s} = \overrightarrow{P_1 P_2}$ 20 end 21 end

Algorithm 1: Algorithm to estimate the velocity.

After autocorrelation (line 11), the three highest local maxima are selected. Theory guarantees that the global maximum is located in the middle of the autocorrelation results. Then, the position of the second and third maxima represent the particles' size vector and velocity vector after a normalization step. The vector with the highest norm is assumed to represent the velocity, the other is considered as size vector. For very thin motion blurring, a size vector may not be detected. In those particular cases, only the velocity vector is calculated and the width of the blurring is set to 1 pixel. The computational complexities of this technique and of PIV are equal since both are dominated by correlation, the complexity of which is $O(n \log n)$.



Fig. 4. Working principle of the filtering technique.

2.2. Autonomous validation of the suppression factor

A well working value of the suppression factor s that was introduced with the filter kernels in the previous subsection, depends on image noise, contrast, particle size. For high speed real-time measurements of industrial process such as thermal spraying, image constraints remain similar for up to hundreds of subsequent frames. Thus, the suppression factor is validated once and then the same value is used for sequences of frames and displacement measurements. Also, the validation can be applied from time to time to verify the accuracy of the results. Due to the unfrequent application of the validation, the computational effort of determining the suppression factor is less significant than the computational effort of the measurement technique.

This autonomous validation is based on a multi-scale approach [6]. The method is applied on the original image and on the same half-sized image. The results are compared by evaluating the conditions in relations 4 and 5. If both conditions are met, the measured values are similar. In those cases, the results are accepted and discarded in any other case. Acceptable values are found by employing a binary search.

$$C_1 = \left(\frac{|g_{\rho} - 2r_{\rho}|}{g_{\rho}} < \epsilon_1\right) \tag{4}$$

$$C_2 = (|g_\theta - r_\theta| < \epsilon_2) \tag{5}$$

2.3. Linear Regression Models

In order to compensate the impact of the particle size on the measured particle velocity (see Section 1), two linear models [7] were generated. The first model 6 is applied, when no size is measured, which occurs on very thin particle motion blurring.

$$v_m = \alpha \ g_\rho + \beta \ g_\theta + \gamma \ s + \delta \tag{6}$$

On 10.000 generated images [2], the model gives the values in relation 7. The traditional matrix approach is applied to minimize the relation 6 with the least squares.

$$v_m = 1.0051 g_{\rho} + 0.1192 g_{\theta} + -0.0173 s - 1.750$$
(7)

If a size is measured, another model is applied that includes the measured size s_{ρ} .

$$v_m = \alpha \ g_\rho + \beta \ g_\theta + \gamma \ s + \lambda s_\rho + \delta \tag{8}$$

The accuracy of the filter technique in combination with the regression model was evaluated on an other set of 10.000 images. After applying the model, the error does not exceed one pixel for both sets. The parameters of the validation technique were set to $\epsilon_1 = 0.1$ and $\epsilon_2 = 5^{\circ}$.

Results show a rate of false positives of 1% in cases of low particle brightness, low contrast ratio and high noise levels. The same benchmark was applied to the technique based on Harris operator which results in a similar rate of false positives of around 1%.

2.4. Optimization of Computations

In this subsection, three optimizations are discussed that reduce the computational time of the technique:

1. Due to the fact that columns 1,2,4 and 5 of F_x^+ are the inverse of the columns 1,2,4 and 5 of F_x^- and row 3 is the same for both filters, F_x^+ and F_x^- can be based on almost the same calculations. First, $F_x^+ * I$ is calculated with $F_x^+ = (A \ B \ C)$, with A, B, C defined in formula 9. Then, the different components of this decomposition of the relation can be processed with $F_x^- = (-A \ B \ -C)$, which requires only two additional subtractions. The same is true for F_y^+ and F_y^- and as a consequence, the computational effort for mask filtering is halved.

$$A = \begin{pmatrix} 1 & 2 \\ 4 & 8 \\ 6 & 12 \\ 4 & 8 \\ 1 & 2 \end{pmatrix}, B = \begin{pmatrix} -s \\ -s \\ 0 \\ -s \\ -s \end{pmatrix}, C = \begin{pmatrix} -2 & -1 \\ -8 & -4 \\ -12 & -6 \\ -8 & -4 \\ -2 & -1 \end{pmatrix}$$
(9)

2. Given that autocorrelation of real data is a symmetric function, only the half of the autocorrelation is calculated.

3. Many operations of the filter technique, such as lines 2 to 8 of Algorithm 1 can be processed in parallel in order to utilize parallel hardware resources and speed up computation time.

3. ROBUSTNESS ANALYSIS

In this Section, the impact of angled motion blurring (Subsection 1) and of crossing particles (Subsection 2) on the measurement results is investigated.

3.1. Angle Invariance

In contrast to pure Sobel filter, whereon the presented technique is based, an advantage of the technique is its robustness against arbitrary angles of motion blurring.

Figure 5 shows motion blurring and filter results for different angles (top: 25° , middle: 45° , bottom: 60°). The original image can be seen in (a), the result of line 4 of Algorithm 1 is presented in (b), line 8 in (c). The global filter result is shown in (d) and (e) represents the autocorrelation results. Here, the symmetry of the filtering phase guarantees that results are the same on rotated images for all those cases.



Fig. 5. Overview of the technique for various angles of motion blurring.

3.2. Crossing Particles

Crossing particles (Fig. 6 (a)) can occur in manufacturing processes such as thermal spraying. In contrast to Harris or FAST, which often only detect the region where particles are crossing, the technique presented here provides the filter results (b) with start and end of particles. The contribution to the overall results is nearly the average of the velocities of the two crossing particles (c).



Fig. 6. Illustration of the technique applied on crossing particles.

4. EXPERIMENTAL RESULTS

Signal-to-noise-ratios (SNR) are calculated based on formula 10 with μ_{Signal} and $\mu_{Background}$ that are respectively the average brightness of the particle and background. σ_{Noise} is the standard deviation of the brightness of the particle.

$$SNR = 20 \cdot log\left(\frac{\mu_{Signal} - \mu_{Background}}{\sigma_{Noise}}\right) \tag{10}$$

In order to measure the stability of the presented method, various SNR were tested by generating 100 images with different angles and velocities. The results (Figure 7) show a detection rate of 100% for $SNR \geq 13 \ dB$ which is acceptable for typical PIV applications. Still, 75% of the results are considered valid for a SNR of 11 dB and 40% for a SNR of 10 dB.



Fig. 7. Detection rate in dependence of SNR.

Regarding the autonomous validation of the suppression factor value, the results given in Table 1 were measured based on the image shown in Fig. 2 with preset values $p_{\rho} = 30 \ px$ and $p_{\theta} = 15^{\circ}$.

The suppression factors and conditions that are considered valid are marked in bold. In those cases, the error for validated results after applying the linear model is less than a third of pixel. If no maximum was found, the cell is left blank.

Table 1. Validation of s and results for thermal spraying simulation.

s	$g_{ ho}$	$g_{ heta}$	$r_{ ho}$	r_{θ}	C_1	C_2	v_m	$ v_m - p_\rho $
1			10.86	86.0				
2	15.50	0.73	12.97	12.7	0.67	9.97	14.2	15.8
3	14.77	1.38	15.53	11.7	1.10	88.8	13.6	16.4
4	31.09	14.4	15.25	12.5	0.02	1.90	30.2	0.24
5	30.90	15.2	14.53	13.7	0.06	1.51	30.0	0.03
6	30.66	15.1	14.95	16.1	0.02	1.00	29.8	0.33
7	30.60	15.0	14.54	15.9	0.05	0.94	29.9	0.31
8	29.81	15.3					28.9	1.14
9	3.14	90.0					1.56	28.4

Finally, the filter technique was evaluated on examples of real images of thermal spray process observations. In Figure 8, the image that contains the motion blurring of particles with high velocity (a) has been filtered (b) and autocorrelated (c). Based on those autocorrelation results, a size and velocity vector were found. The calculated velocity was confirmed by a visual inspection of the image (a) that contains the blurring of the particles.

5. CONCLUSION

The filter technique presented in this article provides a relaxation for particle velocimetry measurements based on particle image velocimetry in terms of frame rate and measurement setup requirements. In order to validate the measurement results and filter parameters, a multi-scale approach with a rate of false positives of less



Fig. 8. Application of the method: thermal spray processes.

than one percent is applied. In contrast to FAST and Harris detector, the presented technique compensates for the impact of the particle size on the measured velocities based on linear regression models and processes crossing particles.

Experimental results show that the method is stable for signalto-noise ratios (SNR) above 13 dB and a detection rate of 75 percent was determined for SNR of 11 dB. The error in those cases is less than 1 pixel, which is acceptable for many manufacturing applications such as thermal spraying.

6. NOMENCLATURE

- s: suppression factor
- s_{ρ} : norm of the size vector for full resolution
- g_{ρ} : norm of the velocity vector for full resolution
- g_{θ} : angle of the velocity vector for full resolution
- r_{ρ} : norm of the velocity vector for reduced resolution
- r_{θ} : angle of the velocity vector for reduced resolution
- p_{ρ} : norm of preset particle velocity
- p_{θ} : angle of preset particle velocity
- v_m : model for norm of velocity vector

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