# SINGLE IMAGE SUPER-RESOLUTION USING ITERATIVE WIENER FILTER

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### ABSTRACT

In this paper, we propose an iterative Wiener filter which can simultaneously perform interpolation and restoration by using nonlocal means to directly model the correlation between the desired high-resolution image and observed low-resolution image. A novel mechanism is proposed to control the decay speed of the correlation function while iteratively updating both estimated correlation and high-resolution image. During the iterations, the image is decomposed into patches with similar intensities at initial iterations and the patches are connected naturally with good convergence. Experimental results show that the proposed algorithm is able to produce natural image structures, and provides better PSNR and visual quality than the state-of-the-art algorithms using the sparse representation and natural image priors.

Index Terms-Iterative Wiener filter, super-resolution

#### **1. INTRODUCTION**

Single image super-resolution reconstruction (SRR) aims to restore a high-resolution (HR) image from an observed low-resolution (LR) images. Let us consider an observation model which describes the image formulation process without considering the additive image noise [1-5]. Throughout this paper, an image is represented in columnwise lexicographically ordered vector notation for convenience. Let **x** represent the observed image, where the size of **x** is  $[n \times 1]$ . We assume that the observed image is the result of uniform decimation **D** and space invariant linear blur **H** performed on a high-resolution image represented by **y**, which has the size of  $[s^2 \cdot n \times 1]$ . Turning the above descriptions into an analytical model, we have

$$\mathbf{x} = \mathbf{D}\mathbf{H}\mathbf{y} \tag{1}$$

where the decimation **D** depends on the magnification factor and the blur **H** is often assumed known or can be approximated due to the knowledge of camera PSF [6].

In the literature, many algorithms were proposed to estimate the desired HR image from an observed LR image. Due to an infinite number of solutions that can formulate the same LR image, SRR is well known to be an ill-posed problem. Reconstructionbased algorithms [1, 6-8] apply natural image priors to regularize the solution to more likely to be the desired image. Training-based algorithms [4-5, 9] use online and offline dictionaries to provide additional information of natural images. Recent algorithms often utilize both natural image priors and dictionary to provide better results [2-3, 10]. Specifically, the algorithm [10] using the adaptive sparse domain and adaptive regularizations shows much better results than the sparse representation [9] and natural image priors [7-8] in terms of PSNR and visual quality. Hence, it is used [10] to compare with our comparison.

In this paper, we propose an iterative algorithm to estimate the desired HR image using the classic Wiener filter. The contribution of this paper is two folded. First, we verify the use of non-local means as the correlation function to directly model the correlation between the desired HR image and observed LR image with successful results, such that the Wiener filter can simultaneously perform interpolation and restoration. Second, we propose a novel mechanism to control the decay speed of the correlation function within the iteration loops of the Wiener filter. At initial iterations, the image is decomposed into rough patches with similar intensities by putting the correlation function to decay quickly. As the Wiener filter iterates, the patches will be connected naturally by lowering the decay speed of the correlation function. As a result, natural image structures will be produced and the proposed algorithm is not dependent on the initial estimation of HR image.

Compared with the available correlation-based algorithms [11-12], the proposed algorithm uses a novel mechanism to iteratively reconstruct the natural image structures by updating the varying correlations. The adaptive Wiener filter [11] uses a wide-sense stationary correlation function to model the correlation between pixels according to the geometric distance. It is good that the Gaussian process regression [12] verifies that the non-local means [13] can model the correlation in either interpolation or restoration process; however this algorithms relies on the initial estimation of the HR image to model the correlation.

Experimental results show that the proposed algorithm provides both better PSNR and visual quality than the available correlation-based algorithms [11-12] and the recently proposed adaptive sparse domain selection algorithm [10]. The rest of the organization of this paper is as follows. Section 2 shows the proposed algorithm with the explanation referring to the classic Wiener filter and the proposed correlation function. Section 3 gives the experimental results and section 4 concludes the paper.

## 2. ITERATIVE WIENER FILTER FOR SR 2.1 Classic Wiener filter

The proposed iterative Wiener filter estimates the desired HR image block-by-block. Let *i* be the block index, and the block size depends on the magnification factor *s*, i.e. block size= $s \times s$ . Consider the classic Wiener filter which minimizes the linear mean squared error, shown below

$$\mathbf{r}_i = \mathbf{R}_i^{-1} \mathbf{P}_i \tag{2}$$

where the filter weight  $\mathbf{W}_i$  is related with the autocorrelation matrix  $\mathbf{R}_i$  for the observation vector and cross-correlation matrix  $\mathbf{P}_i$ for the desired vector and observation vector. To simultaneously perform interpolation and restoration, the desired vector  $\mathbf{y}_i$  is defined as pixels inside a block in the desired HR image  $\mathbf{y}$  and observation vector  $\mathbf{x}_i$  is defined as pixels geometrically closest to the desired vector in the observed LR images  $\mathbf{x}$ . Figure 1 shows an example of these definitions. The Wiener filter estimates the desired vector by

$$\hat{\mathbf{y}}_i = \mathbf{W}_i^T \mathbf{x}_i \tag{3}$$

and the desired image can be estimated by  $\hat{\mathbf{y}} = {\{\hat{\mathbf{y}}_i\}}$ .



Figure 1: An example illustrating the observation vector  $\mathbf{x}_i$  and the desired vector  $\mathbf{y}_i$  when s=3 and size of  $\mathbf{x}_i$  is 16.

### 2.2 Correlation function using non-local means

In this section, we present the proposed correlation function for simultaneous interpolation and restoration. Let us consider the definitions of auto-correlation matrix and cross-correlation matrix

$$\mathbf{R}_i = E\{\mathbf{x}_i \mathbf{x}_i^T\} \text{ and } \mathbf{P}_i = E\{\mathbf{x}_i \mathbf{y}_i^T\}$$
(4)

Due to the normalization of filter weights as in [11], i.e. each column of  $W_i$  is summed to 1, the normalized correlation functions can be used. Let us use the non-local means [13] to model the normalized correlation functions for the correlation matrices

$$r(x_j, x_k) = e^{-E(x_j - x_k)^2 / \sigma}$$
 and  $p(x_j, y_k) = e^{-E(x_j - y_k)^2 / \sigma}$  (5)

where  $\{x_j, x_k\} \in \mathbf{x}_i$  and  $\{y_k\} \in \mathbf{y}_i$ . The expected squared difference between two pixels,  $E(x_j-x_k)^2$ , is computed using surrounding pixels within a local window which has a size of  $m \times m$ . The surrounding pixels of  $\mathbf{x}_i$  and  $\mathbf{y}_i$  are located on the blurred HR image and desired HR image respectively. The variance  $\sigma$  controls the decay speed of the correlation function, which is a crucial parameter of the proposed algorithm.

### 2.3 Varying decay speed of the correlation function

Due to the fact that the proposed correlation function depends on both desired HR image and blurred HR image, we propose an iterative Wiener filter to update both estimated HR image and correlation matrices iteratively as in the classic iterative Wiener filter [15]. During the iterations, a novel mechanism is proposed to vary the decay speed (variance  $\sigma$ ) of the correlation function, such that the iterative Wiener filter can recover the natural image structures and always converges to the same result regardless of the initial estimation.

The algorithm flow of the proposed iterative Wiener filter is shown in Algorithm 1. At the end of an iteration of the Wiener filter, we apply the classic iterative back-projection (IBP) [14] to refine the result to fit the image model in (1). Depending upon the spread of blur **H** and magnification factor *s*, IBP usually converges in less than three iterations, where the factor  $\lambda$  is set to a value less than 1 for stability. The iterative Wiener filter terminates when the variance  $\sigma$  reaches a threshold meeting the desired decay speed of correlation function.

# Algorithm: Wiener(x)

1. Initialization

- (a) Initialize the estimation  $\hat{\mathbf{y}}^{(0)}$  of the desired HR image.
- (b) Create a blurred HR image using  $\mathbf{H}\hat{\mathbf{y}}^{(0)}$
- (c) Set the initial value of variance in (5) to  $\sigma = u$ .
- Iterate on *n* until the value of variance σ exceeds a threshold *T*.
  (a) Compute the correlation matrices in (4) using the estimated HR image ŷ<sup>(n)</sup> and blurred HR image Hŷ<sup>(n)</sup>.
  (b) Use the computed correlation matrices to update the estimated HR image to ŷ<sup>(n+1)</sup>.

(c) Iterative back-projection [14] to refine the estimated HR image  $\hat{\mathbf{v}}^{(n+1)}$  to fit the image model in (1):

$$\hat{\mathbf{y}}^{(n+1)} = \hat{\mathbf{y}}^{(n+1)} - \lambda \mathbf{H}^T \mathbf{D}^T (\mathbf{D} \mathbf{H} \hat{\mathbf{y}}^{(n+1)} - \mathbf{x})$$

(d) Update the blurred HR image using  $\mathbf{H}\hat{\mathbf{y}}^{(n+1)}$ .

(e) Update the variance  $\sigma$  using  $\sigma = u \cdot p^{(n+1) \cdot (n+1)/\alpha}$ .

(f) Repeat 2(a) to 2(e) until termination.

Algorithm 1: Iterative Wiener filter for super-resolution

#### 2.4 Recover the natural image structures

Figure 2 show portions of the Lena image during iterations. Initially, the variance  $\sigma$  is set to a small value which implies that the correlation function decays very quickly, such that the pixels with similar intensities are grouped into patches, as shown in figures 2(c) and 2(d). Such grouping of patches is very rough initially and there are explicit boundary effects (similar to color quantization effect) between difference patches. As the Wiener filter iterates, we increase the variance  $\sigma$  exponentially to increase the correlation between patches, such that the patches will be connected smoothly. By carefully choosing the decay speed initially and the speed of increment of decay speeds during the iterative. Figure 2 shows that the image structures are reconstructed well progressively.

Table 1 shows that the proposed algorithm always converges to the same result regardless of the initial estimations using IBP with nearest neighbor interpolation [14], adaptive Wiener filter [11] and adaptive sparse domain selection [10]. This result confirms that the proposed mechanism is strong at re-synthesizing the image structure at initial iterations and converging it into the natural image structure.

Table 1: PSNR (dB) using different initial HR images

Initial image	Initial	1 <sup>st</sup>	3 <sup>rd</sup>	5 <sup>th</sup>	7 <sup>th</sup>	8 <sup>th</sup>
IBP w/ NN [14]	28.642	30.36	30.696	31.396	32.691	32.721
AWF [11]	31.5	30.67	30.757	31.409	32.695	32.729
ASDS [10]	32.696	30.764	30.78	31.415	32.7	32.73



(a) Original (b) Initial [14] (c) 1<sup>st</sup> iteration



(d) 3<sup>th</sup> iteration (e) 5<sup>th</sup> iteration (f) 8<sup>th</sup> iteration **Figure 2:** Portions of the estimated Lena image using the proposed algorithm at different iterations.

# 2.5 Optimizing the parameters

Due to the iterative scheme, the proposed algorithm has several parameters which are optimized empirically and verified through the cross-validation. The parameters are mostly invariant to the magnification factor *s* and blur **H** (unless otherwise specified) according to our experiments. Table 2 shows the explanation and suggested values of these parameters. Using the suggested values, the proposed algorithm always converges at the  $8^{th}$  iteration.

<b>Table 2:</b> The parameters of the	the proposed algorith	ım.
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Parameters	Suggested
	value
(a) The number of input (size of $\mathbf{x}_i$ ) is the balance of the	16
stability and localization. More inputs can provide a more	
stable result but reduce the local adaptation.	
(b) The window size $(m \times m)$ for the correlation function	$(2s+1) \times (2s+1)$
depends on the magnification factor <i>s</i> because a large	
magnification factor involving more unreliable pixels	
requires a large window to provide an accurate result.	
(c) Initial value (u) of the variance $\sigma$ determines how	2.5
aggressive the image decomposition into patches is at	
initial iterations.	
(d) Power of increment (p) increase the variance $\sigma$ by	e (=2.72)
orders, which can ensure a fast convergence to meet the	
termination threshold T.	
(e) The parameter ( $\alpha$ ) is a complementary parameter of	4.5
the power function to control the increment of variance $\sigma$ .	
(f) The termination threshold ( <i>T</i> ) determines the number	1.5·10 <sup>5</sup>
of iterations.	

Table 3: PSNR (dB) and SSIM [1]	61	using different algorithms
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Images	IBP [14]		AWF [11]		GPR [12]		ASDS [10]		Proposed	
	22.9	0.71	22.9	0.72	22.3	0.66	23.7	0.75	23.9	0.76
Bike	1	6	7	0	9	8	2	8	2	0
House	20.4	0.64	20.3	0.63	20.1	0.60	20.5	0.65	20.7	0.66
	2	2	8	8	- 0	7	1	4	5	4
Game	25.7	0.74	25.7	0.74	25.2	0.70	26.2	0.75	26.1	0.75
	5	0	8	1	5	3	2	6	9	4
Statue	28.7	0.83	28.7	0.84	28.2	0.81	29.3	0.85	29.3	0.85
	3	8	- 9	1	7	6	3	4	8	7
Woman	25.7	0.76	25.7	0.76	25.2	0.71	26.1	0.77	26.2	0.77
	4	3	9	7	5	8	1	7	5	7
Lighthous	23.7	0.75	23.7	0.75	23.4	0.73	23.8	0.76	23.9	0.76
e	6	6	2	3	4	2	9	7	4	7
Parrot	30.5	0.90	30.7	0.91	29.5	0.89	31.4	0.91	31.7	0.91
	6	9	- 0	0	- 0	7	9	8	1	9
	31.3	0.87	31.5	0.88	30.7	0.86	32.7	0.89	32.7	0.89
Lena	3	- 9	- 0	1	7	4	- 0	2	2	2
Average	26.1	0.78	26.2	0.78	25.6	0.75	26.7	0.79	26.8	0.79
	5	- 0	- 0	1	2	1	4	7	6	9

### **3. EXPERIMENTAL RESULTS**

To give the subjective and objective performance evaluation of the proposed algorithm (using the suggested values of parameters in Table 2), extensive experiments have been carried out. The algorithms for comparison include the iterative back projection (IBP) (using bicubic interpolation as the initialization) [14], the adaptive Wiener filter (AWF) [11], the Gaussian process regression (GPR) [12] and the adaptive sparse domain selection (ASDS) [10]. The reference algorithms [11, 14] and the proposed algorithm were implemented using MATLAB, while the codes of the other algorithms [10, 12] were provided by the respective authors. The system for evaluating the algorithms is an Intel i7 950 system.

Eight natural images were used for the evaluation. The images formulate the observed LR images using image model in (1). The magnification factor *s* used is 3 as in [6, 10] and the blur **H** is a  $3\times3$  uniform blur as in [6]. Table 3 shows that the proposed algorithm obtains the highest PSNR and SSIM [16] results among different algorithms. Due to bigger image sizes (around  $512\times512$ ) and some test images (house, game and lighthouse) containing complex structures, the differences of PNSR and SSIM values are

not significant as in [10]. Furthermore, the proposed Wiener filter is readily extended to the parallelization by computing blocks by blocks independently, as explained in [11].

Subjective evaluations are shown by the portions of the estimated images in figures 3. In figure 3, the proposed algorithm always produces the most natural image structures compared with the original image. ASDS [10] produces some jags and aliased effects around the edges while AWF [11] and GPR [12] generally cannot handle edges well although the PSNR values of AWF [11] are not far lower than that of the proposed algorithm. The results generally agree with the SSIM values in Table 3, while the image structures look natural and most pleasure, by using the proposed algorithm.

#### 4. CONCLUSION AND DISCUSSION

In this paper, we propose a new iterative Wiener filter for the single image super-resolution. A novel mechanism is proposed to control the decay speed of the correction function during iterations. During the iterations, the image is decomposed into patches with similar intensities at initial iterations and the patches are connected naturally when convergence. Experimental results show that the proposed algorithm can produce natural image structures with high fidelity, such that it obtains the highest PSNR and visual quality among the state-of-the-art algorithms using some natural images.

One possible future direction is to exploit the adaptive size of input  $(\mathbf{x}_i)$  and adaptive window size  $(m \times m)$  according to local image statistics. We have tried to adaptively select the inputs (within different local regions) which have the highest correlations with desired pixels using the correlation function in (5); however, very limited PSNR improvements are shown, i.e. 0.01 dB. This result may confirm that a small number of local inputs are sufficient to reconstruct the image structures. Hence, extending the proposed algorithm to multi-frame super-resolution is not straightforward or not by just adding inputs from other frames. Furthermore, in order to consider additive noise in the image model in (1), one may consider changing the variance of the nonlocal means function (i.e. correlation function) as in some denoising [13, 17] and super-resolution algorithms [6]. Although a systematic study of optimal parameters in case of the noisy situation and the restoration only situation are not included in this paper, the proposed algorithm is indeed extendable to these applications.

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Figure 3: Portions of the estimated test images using different algorithms.