# A NOVEL CURVELET DOMAIN SPECKLE SUPPRESSION METHOD FOR SAR IMAGES

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## ABSTRACT

This paper introduces a novel Bayesian method for speckle suppression of SAR images. We first analyze the logarithmic transform of the original image by means of the curvelet transform that handles image edges more efficiently than wavelet transform. In a recent work [1], we have shown that due to the statistical properties of the curvelet subbands of SAR images, they can be modelled by two-dimensional Generalized Autoregressive Conditional Heteroscedastic (2D-GARCH) model. Here, we employ a generalization of 2D-GARCH model, called 2D-GARCH Generalized Gaussian (2D-GARCH-GG), to these coefficients. This model preserves the appropriate properties of 2D-GARCH for modeling the curvelet coefficients while extends the dynamic formulation of 2D-GARCH model. Consequently, we design a maximum a-posteriori (MAP) estimator for estimating the clean image curvelet coefficients. Finally, we compare our proposed method with other denoising methods, and quantify the achieved performance improvement.

*Index Terms*— Curvelet transform, MAP estimation, 2D-GARCH-GG model, Synthetic Aperture Radar, Speckle.

### 1. INTRODUCTION

Synthetic aperture radar (SAR) plays a more important role in gathering information with its all time and all weather imaging capability and its high spatial resolution, but the multiplicative speckles produced by coherent imaging mechanics seriously affect the further understanding and interpretation to SAR images. Therefore, the first step to process SAR images is to restrain the speckles. Classical despeckling algorithms such as the Lee filter [2], perform well in despeckling, whereas they found several difficulties when the aim is to simultaneously despeckle and preserve the details of the SAR images. Therefore, other approaches have been proposed based on considering multiresolution decomposition of the input image and applying a different nonlinear adaptive filter to each resolution layer. Wavelet transform has been used extensively in noise reduction of SAR images [1, 3].

Despite considerable success, wavelets are far from being universally effective. Because of wavelets poor orientation selectivity, they do not represent higher-dimensional singular-

ities effectively. Curvelet is a new multiscale transform with strong directional character in which elements are highly anisotropic at fine scales, with effective support shaped according to the parabolic scaling principle [4]. These properties are particularly suitable for SAR datasets which is a map of microwave reflectivity in the form backscatterer from image area. Curvelet denoising procedures consist of three main steps: i) Calculate the curvelet transform of the noisy image, ii) Manipulating the curvelet coefficients and iii) Compute the inverse transform using the modified coefficients. Loosely speaking, two major techniques used in curvelet denoising are the thresholding technique and the Bayesian estimation shrinkage technique. Thresholding methods have two main drawbacks: i) the choice of the threshold, arguably the most important denoising parameter, is made in an ad-hoc manner; and ii) the specific distribution of the signal and noise may not be well matched at different scales [3]. Hence, we propose a novel Bayesian estimation shrinkage technique in the curvelet domain. For a Bayesian estimation process to be successful, the correct choice of priors for curvelet coefficients is certainly a very important factor. Previously, we have shown that the curvelet subbands of SAR images are heteroscedastic, with significantly non-Gaussian statistics that are best described by two-dimensional GARCH (2D-GARCH) model [1]. Energized by the success of 2D-GARCH model in curvelet domain, we tried to generalize 2D-GARCH model and provide better adapted alternatives. Therefore, in this paper, we describe an extension of 2D-GARCH model called 2D-GARCH-GG. One restriction of 2D-GARCH model is that the distribution of innovation is Gaussian. But, 2D-GARCH-GG doesn't suffer from this restriction and allows the innovation to be generalized Gaussian that contains the Gaussian distribution as a special case. 2D GARCH-GG model provides more flexibility which results in better characterization of curvelet subbands and improved despecking results. This paper is organized as follows: In Section 2, we introduce the 2D-GARCH-GG model. Section 3 is dedicated to describing the speckle suppression problem and our proposed scheme. In section 3.1., we design a new curvelet domain processor to despeckle SAR images based to using 2D-GARCH-GG model. The experimental results are presented in section 4. Finally, concluding remarks are given in section 5.

#### 2. 2D-GARCH-GG MODEL

2D-GARCH-GG model is an extension of 2D-GARCH model and includes it as a special case.  $y_{ij}$  follows a pure 2D-GARCH-GG $(p_1, p_2, q_1, q_2)$  model if  $E(y_{ij}) = 0$  and

$$y_{ij} = \sqrt{h_{ij}\varepsilon_{ij}} \tag{1}$$

$$h_{ij} = \alpha_0 + \sum_{k\ell \in \Lambda_1} \alpha_{k\ell} y_{i-k,j-\ell}^2 + \sum_{k\ell \in \Lambda_2} \beta_{k\ell} h_{i-k,j-\ell} (2)$$

where  $\Lambda_1 = \{k\ell \mid 0 \leq k \leq q_1, 0 \leq \ell \leq q_2, (k\ell) \neq (0,0)\},\$  $\Lambda_2 = \{k\ell \mid 0 \leq k \leq p_1, 0 \leq \ell \leq p_2, (k\ell) \neq (0,0)\}, \text{ and } \varepsilon_{ij}$ is a two dimensional iid generalized Gaussian stochastic process, i.e.,  $\varepsilon_{ij} \sim GG(s,p) \Rightarrow f_{s,p}(\varepsilon_{ij}) = \frac{e^{-|\varepsilon_{ij}/s|^p}}{Z(s,p)}$ . GG(s, p) stands for generalized Gaussian distribution with parameters s and p and  $Z(s,p) = 2(s/p)\Gamma(1/p)$ . In 2D-GARCH model,  $\varepsilon_{ij}$  is a normal process. Since, the Generalized Gaussian is an adaptive distribution and includes the normal as a special case, along with many other distributions, 2D-GARCH-GG includes 2D-GARCH model. Let the information set  $\psi_{ij}$  be defined as

 $\psi_{ij} = \{\{z_{i-k,j-\ell}\}_{k,\ell\in\Lambda_1}, \{h_{i-k,j-\ell}\}_{k,\ell\in\Lambda_2}\}, y_{ij} \text{ is condi$ tionally distributed as:

$$f(y_{ij}|\psi_{ij},\Gamma) = \frac{1}{\sqrt{h_{ij}}} \frac{e^{-|\frac{y_{ij}}{\sqrt{h_{ijs}}}|^p}}{Z(s,p)}$$
(3)

If we encounter a process such as  $z_{ij}$  that  $E(z_{ij}) \neq 0$ , we use 2D-GARCH-GG( $p_1, p_2, q_1, q_2$ ) regression model. This model is obtained by letting  $y_{ij}$  be the innovation in a twodimensional linear regression,  $y_{ij} = z_{ij} - \underline{r}_{ij}^T \underline{b}$  where  $r_{ij}$  is a vector of explanatory variables and  $\underline{b}$  is a vector of unknown parameters. So, to find the conditional distribution of  $z_{ij}$ , it is sufficient to replace  $y_{ij}$  with  $z_{ij} - \underline{r}_{ij}^T \underline{b}$  in (3) We use MLE to estimate the parameters of 2D-GARCH-GG

model  $\Gamma = \{ \{ \alpha_0, \alpha_{01}, \cdots, \alpha_{q_1q_2}, \beta_{01}, \cdots, \beta_{p_1p_2} \}, \underline{b}, \{s, p\} \}.$ 

### 3. SPECKLE SUPPRESSION

In this section, our goal is the design of a MAP estimator that recovers the signal component of the curvelet coefficients in noisy images by using a 2D-GARCH-GG model. An approximate speckle noise model [3] is formulated as  $I(x,y) = S(x,y)\eta(x,y)$ , where x and y are variables of spatial locations,  $(x, y) \in \Re^2$ , S(x, y) is a noise-free original image, to be recovered, I(x, y) is a noisy observation of S(x, y), and  $\eta(x, y)$  is multiplicative noise. A functional block diagram of the proposed denoising method is shown in Fig. 1. Atfirst, to separate the noise from the original image, we take a logarithmic transform on the both sides of (3), and we have  $\log(I(x, y)) = \log(S(x, y)) + \log(\eta(x, y))$ . Then, we employ curvelet transform since this transform is more suitable than the classical multiresolution methods. Curvelet transform accurately represents smooth functions using only a few nonzero coefficients, and which also accurately represent edges using only a few nonzero coefficients. Since,



Fig. 1. Block diagram of the proposed algorithm for speckle suppression.

the detailed explanation of curvelet transform has been considered in many papers, we don't repeat it. The interested reader can refer to [4] for details of discrete curvelet transform(DCT). Applying curvelet transform to the logarithm of SAR image, we have:

$$I_{(j,\ell)}(k_1,k_2) = S_{(j,\ell)}(k_1,k_2) + \eta_{(j,\ell)}(k_1,k_2).$$
(4)

where  $I_{(j,\ell)}(k_1, k_2)$ ,  $S_{(j,\ell)}(k_1, k_2)$ , and  $\eta_{(j,\ell)}(k_1, k_2)$  represents the curvelet transform of  $\log(I(x, y))$ ,  $\log(S(x, y))$ , and  $\log(\eta(x, y))$  respectively.  $j, \ell$ , and  $k = (k_1, k_2)$  are scale, orientation, and translation parameters as defined in the curvelet transform. To simplify the notation, in the following parts we ignore the index  $j, \ell$ . In this step, the problem is estimating the curvelet coefficients of original image  $S(k_1, k_2)$  from the noisy coefficients  $I(k_1, k_2)$ .

To design a MAP estimator, we should statistically model the signal and noise terms in curvelet domain. Studying statistical properties of signal component in the curvelet domain in [1] demonstrates that the curvelet coefficients of log-transformed SAR image (LTSI) deviate from the normal distribution, and conditional heteroscedasticity exists in them [1]. Now, we use 2D-GARCH-GG model. This model preserves the favorbale properties of 2D-GARCH model, so it is compatible with curvelet coefficients of LTSIs. Also, 2D-GARCH-GG introduces additional flexibility in the model formulation in comparison with 2D-GARCH model, which results in better characterization of SAR images subbands and improved restoration in noisy environments. The Gaussianity assumption for the log-transformed speckle  $\eta(x, y)$  is confirmed in many papers, such as [1, 3]. Also, our processor employs the curvelet transform which, through the central limit theorem, drives the noise curvelet coefficients to approximate a Gaussian distribution, hence, we assume Gaussian distribution for the curvelet coefficients of noise component  $\eta(k_1, k_2)$ . At the first step, using maximum likelihood method, we estimate parameters of 2D-GARCH-GG model, i.e.  $\Gamma = \{ \alpha_0, \alpha_{01}, \ldots, \alpha_{01} \}$  $\cdots, \alpha_{q_1q_2}, \beta_{01}, \cdots, \beta_{p_1p_2} \}$ ,  $\underline{b}, \{s, p\} \}$ . Referring to (4), we use 2D-GARCH-GG model for the curvelet coefficients of LTSI, i.e.  $S(k_1, k_2)$ , but we should estimate the model parameters given the noisy observation  $I(k_1, k_2)$ . Hence, the likelihood function is formulated as:

$$LF(\Gamma) = \prod_{k_1, k_2 \in \Phi} P(I(k_1, k_2) | \psi_{k_1, k_2}, \Gamma), \qquad (5)$$



**Fig. 2.** left to right: original aerial image, noisy image, image denoised using the proposed method.

where we have (6) to compute  $P(I(k_1, k_2)|\psi_{k_1,k_2}, \Gamma)$ . Since,  $S(k_1, k_2)$  is modeled as 2D-GARCH-GG, we have:

$$P(S(k_1,k_2)|\psi_{k_1,k_2},\Gamma) = \frac{1}{\sqrt{h_{k_1,k_2}}} \frac{e^{-|\frac{S(k_1,k_2)-\Sigma_{k_1,k_2}^{-b}}{\sqrt{h_{k_1,k_2}s}}|^p}}{Z(s,p)} \quad (7)$$

The conditional distribution of  $I(k_1, k_2)$ , given  $S(k_1, k_2)$  is similar to  $\eta(k_1, k_2)$  distribution and we use Gaussian distribution for curvelet coefficients of noise component  $\eta(k_1, k_2)$ . So, we can express:

$$P(I(k_1, k_2)|\psi_{k_1, k_2}, \Gamma, S(k_1, k_2)) =$$
(8)

$$\frac{1}{\sqrt{2\pi}\sigma_n} e^{\frac{-(I(k_1,k_2)-S(k_1,k_2))^2}{2\sigma_\eta^2}} \tag{9}$$

where  $\sigma_{\eta}^2$  denotes the variance of  $\eta(k_1, k_2)$ . Substituting (7) and (8) in (6) and then in (5), we can obtain (11). The parameter set ( $\Gamma$ ) can be calculated by maximizing the likelihood function in (11). Also, we can estimate the parameter set ( $\Gamma$ ) from denoised version of image using a simple denoising method such as median filter and then employ the parameter set in the following steps. Now, we can design and implement a MAP estimator for  $S(k_1, k_2)$ ,  $\hat{S}(k_1, k_2)$ , given the noisy observation,  $I(k_1, k_2)$ , parameter set ( $\Gamma$ ), and  $\psi_{k_1, k_2}$ as described in (12). Substituting (7) and (8) in equation (12), we can obtain equations (13) and (14) to estimate the noise free curvelet coefficients. After MAP estimation, we perform the IDCT. Finally, after adjusting the mean, we apply the exponential transformation to obtain the denoised SAR image.

#### 4. EXPERIMENTAL RESULTS

To validate the effectiveness of the proposed sparse speckle suppressing algorithm, we tested it on the synthetic SAR images as well as real SAR images. The use of synthetic SAR data allows objective performance assessment in detail preservation and in speckle reduction efficiency. SAR images modelled in section 3, so we started by first degrading the an aerial speckleless image with different levels of lognormal synthetic speckle that is a realistic speckle noise model in SAR images [1, 3]. The aerial test image, corresponding simulated radar textured image, and the despeckled image using the proposed method have been shown in Fig. 2. Also, to demonstrate the performance improvement in utilizing the proposed strategy, we compare the results of our proposed method with the results of the other despeckling methods including Lee filter, adaptive thresholding in the wavelet domain method [6], curvelet thresholding, and Bayesian denoising in the curvelet domain based on 2D-GARCH model [1]. In order to evaluate the result of filters quantitatively, we use three parameters MSE, signal-to-noise (S/MSE) ratio and  $\beta$  as defined in [3]. S/MSE in the case of multiplicative noise corresponds to the classical SNR in the case of additive noise. The parameter  $\beta$  is also considered for edge preservation that should be close to unity for an optimal effect of edge preservation. The results for the aerial image shown in Fig. 2 are summarized in table 1. In this table, the best value in each row is bold. The results reported in table 1 verify the efficiency of the proposed method in comparison with other despeckling methods both in speckle suppression and edge preservation. Finally, we apply the proposed denoising technique on tree

 Table 1. Image enhancement measure obtained by several denoising methods at different levels of multiplicative noise.

	Noisy	Adaptive	Curvelet	curvelet	Propo.
		thresh.	thresh.	GARCH	
MSE	170.803	132.391	144.304	98.424	94.431
S/MSE	21.493	22.599	22.225	23.887	24.067
β	.9063	.9338	.9153	.9469	.9474
MSE	135.748	115.087	118.984	96.368	83.333
S/MSE	22.490	23.207	23.063	23.978	24.610
β	.9210	.9407	.9279	.9467	.9523
MSE	104.076	104.593	99.396	97.968	75.448
S/MSE	23.644	23.623	23.844	23.907	25.041
β	.9385	.9458	.9395	.9442	.9570

actual SAR images and visually study the merit of the proposed scheme on these images. The first test image, SAR image 1, is acquired by Computer Vision and Remote Sensing group of Berlin University of Technology. The second test image, SAR image 2, is acquired by America's Air Force. These two images and the corresponding denoised images us-

$$LF(\Gamma) = \prod_{k_1, k_2 \in \Phi} P(I(k_1, k_2) | \psi_{k_1, k_2}, \Gamma)$$
(10)

$$= \prod_{k_1,k_2 \in \Phi} \int \frac{1}{\sqrt{2\pi h_{k_1,k_2}}} e^{-\frac{(I(k_1,k_2) - S(k_1,k_2))^2}{2\sigma_N^2} - |\frac{S(k_1,k_2) - z_{k_1,k_2}^T b}{\sqrt{h_{k_1,k_2}s}}|^p} dS(k_1,k_2)$$
(11)

$$\hat{S}(k_1, k_2) = \arg \max_{\hat{S}(k_1, k_2)} \left( P(S(k_1, k_2) \left| \psi_{k_1, k_2}, \Gamma \right) P(I(k_1, k_2) \left| S(k_1, k_2), \psi_{k_1, k_2}, \Gamma \right) \right)$$
(12)

$$\hat{S}(k_1, k_2) = \arg \max_{\hat{S}(k_1, k_2)} \left\{ \frac{1}{\sqrt{2\pi h_{k_1, k_2}}} e^{-\frac{(I(k_1, k_2) - S(k_1, k_2))^2}{2\sigma_\eta^2} - |\frac{S(k_1, k_2) - r_{k_1, k_2}^L b}{\sqrt{h_{k_1, k_2}s}} |^p} \right\}$$
(13)

$$= \arg \min_{\hat{S}(k_1,k_2)} \left\{ \frac{(I(k_1,k_2) - S(k_1,k_2))^2}{2\sigma_{\eta}^2} + \left| \frac{S(k_1,k_2) - \underline{r}_{k_1,k_2}^T \underline{b}}{\sqrt{h_{k_1,k_2}} s} \right|^p \right\}$$
(14)



**Fig. 3**. Results of the proposed speckle suppression method. top to bottom : SAR image 1, and SAR image 2. left to right : Noisy SAR image, Images denoised using proposed method.

ing the proposed method have been shown in Fig. 3. It is clear from this figure that the proposed method is really successful in reducing speckle and also preserving the edges.

## 5. CONCLUSION

We introduced a novel Bayesian speckle suppression method for SAR images. The logarithm of image was analyzed by means of the curvelet transform that handles image edges more efficiently than wavelet transform. We described 2D-GARCH-GG model as an extension of 2D-GARCH model. This model involves the good properties of 2D-GARCH model in capturing heavy tailed marginal distribution and the intrascale dependencies of curvelet coefficients. Also it provides more flexibility which results in better characterization of curvelet subbands. Then, we designed a new MAP processor to estimate the noise free curvelet coefficients based on 2D-GARCH-GG model. The experimental results verified the good performance of the proposed approach.

## 6. REFERENCES

- M. Amirmazlaghani, and H. Amindavar, "Two novel bayesian multiscale approaches for speckle suppression in SAR images, *IEEE Trans. on Geosci and Remote Sensing*, VOL. 47, NO. 7, pp. 2980–2993, June. 2010.
- [2] J. S. Lee, "Refined Filtering of Image Noise Using Local Statistics", *Computer Graphic and Image Processing*, 15, pp. 380-389, 1981.
- [3] A. Achim, A. Bezerianos, and P. Tsakalides, "SAR image denoising via Bayesian wavelet shrinkage based on heavy tailed modeling", *IEEE Trans. on Geosci and Remote Sensing*, VOL. 41, NO. 8, pp. 1773-1784, August 2003.
- [4] E.J.Candes, L.Demanet, and D.L.Donoho, "Fast Discrete Curvelet Transforms", Applied and Computational Mathematics, California Institute of Technology,pp.1 43, 2005.
- [5] T. Bollerslev, "Generalized autoregressive conditional heteroscedasticity," *Journal of Econometrics*, VOL. 31, pp. 307-327, 1986.
- [6] D. Gnanadurai, and V. Sadasivam, "An Efficient Adaptive thresholding technique for wavelet based image denoising", *International Journal of Signal Processing*, VOL. 2, 2005.