SURE-FAST BILATERAL FILTERS

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ABSTRACT

Edge-preserving smoothing is widely used in image processing and bilateral filtering is one way to achieve it. Bilateral filter is a nonlinear combination of domain and range filters. Implementing the classical bilateral filter is computationally intensive, owing to the nonlinearity of the range filter. In the standard form, the domain and range filters are Gaussian functions and the performance depends on the choice of the filter parameters. Recently, a constant time implementation of the bilateral filter has been proposed based on raised-cosine approximation to the Gaussian to facilitate fast implementation of the bilateral filter. We address the problem of determining the optimal parameters for raised-cosine-based constant time implementation of the bilateral filter. To determine the optimal parameters, we propose the use of Stein's unbiased risk estimator (SURE). The fast bilateral filter accelerates the search for optimal parameters by faster optimization of the SURE cost. Experimental results show that the SURE-optimal raised-cosine-based bilateral filter has nearly the same performance as the SURE-optimal standard Gaussian bilateral filter and the Oracle mean squared error (MSE)-based optimal bilateral filter.

Index Terms— SURE, Bilateral filter, Raised-cosine approximation.

1. INTRODUCTION

Spatial (domain) filtering is a well known image denoising technique. Linearity and spatial invariance properties of the domain filter enable fast Fourier transform (FFT)-based implementations of spatial filtering. However, as a consequence of spatial invariance, such filters do not preserve edges as averaging is performed over homogeneous regions as well as regions with discontinuities with the same kernel. This drawback is alleviated by the edge-preserving bilateral filter, first proposed by Tomasi and Manduchi [1]. Elad showed that the bilateral filter can be derived as the solution to the optimal denoising problem in the Bayesian framework [2]. The bilateral filter has found applications in image processing, computer graphics and computer vision, for denoising [3], demosaicing [4] and optical-flow estimation [5].

The bilateral filter ϕ is a combination of the domain filter and range filter, that is,

$$\phi_{\mathbf{k},\mathbf{m}}(y_{\mathbf{k}}, y_{\mathbf{m}}) = w_{\mathbf{k}-\mathbf{m}} r(y_{\mathbf{k}} - y_{\mathbf{m}}), \tag{1}$$

wherein the domain filter $w_{\mathbf{k}-\mathbf{m}}$ is based on the geometric proximity between the pixel of interest at spatial coordinate $\mathbf{k}$ and a nearby pixel at coordinate $\mathbf{m}$. It is symmetric, non-negative and assigns coefficients that fall off with decreasing geometric proximity, thus, effectively localizing the averaging operation to a neighbourhood $\mathcal{N}$ of $\mathbf{k}$. The range filter $r(y_{\mathbf{k}} - y_{\mathbf{m}})$ measures the similarity between the intensity of the pixel of interest $y_{\mathbf{k}}$ and that of its neighbour $y_{\mathbf{m}}$. It is symmetric, non-negative and assigns higher weights to pixels that are photometrically similar to the pixel of interest than the dissimilar ones. The general expression for the bilateral filter output is given by

$$\hat{x}_{\mathbf{k}} = \frac{\sum\limits_{\mathbf{m}\in\mathcal{N}_{\mathbf{k}}} w_{\mathbf{k}-\mathbf{m}} r(y_{\mathbf{k}} - y_{\mathbf{m}}) y_{\mathbf{m}}}{\sum\limits_{\mathbf{m}\in\mathcal{N}_{\mathbf{k}}} w_{\mathbf{k}-\mathbf{m}} r(y_{\mathbf{k}} - y_{\mathbf{m}})}. \tag{2}$$

In (2), the denominator is a normalizing factor that preserves the local mean of the image $\mathbf{y}$.

The nonlinearity of r makes the implementation of (2) computationally intensive for real-time applications since FFT-based acceleration is not possible. To address this problem, Porikli proposed a constant time O(1) implementation of the bilateral filter (for arbitrary spatial kernels) using polynomial range kernels [6]. Yang et al. [7] proposed an O(1) algorithm for arbitrary range and spatial kernels by extending the bilateral filtering method of Durand et al. [8]. Their algorithm is based on a piecewise-linear approximation of the bilateral filter obtained by quantizing the range kernel. Recently, Chaudhury et al. [9] have proposed a new implementation of the bilateral filter in constant time by employing a raised-cosine range kernel. The motivation behind using the family of raised-cosines given in (3) is that, in addition to qualifying as valid range kernels, they also closely approximate the Gaussian function [9]:

$$r(y_{\mathbf{k}} - y_{\mathbf{m}}) = \left[\cos\Big(\frac{\gamma}{\rho\sqrt{L}}(y_{\mathbf{k}} - y_{\mathbf{m}})\Big)\right]^{L}. \tag{3}$$

The raised-cosine kernel is non-negative when its argument takes values between $-\pi/2$ and $\pi/2$. In (3), the normalizing constant γ constrains the argument of the raised-cosine between $-\pi/2$ and $\pi/2$. It is given by $\gamma = \pi/2T$, where $[0, T]$ is the dynamic range of the image $\mathbf{y}$. The parameters L and ρ of the raised-cosine depend on the standard deviation σ_r of the Gaussian to be approximated. ρ is given by $\rho = \gamma\sigma_r$. If $\sigma_r > \gamma^{-2}$, a large value is chosen for L, otherwise, $L = \rho^{-2}$.

To optimally smooth an image in the presence of noise using the fast bilateral filter, it becomes necessary to optimally choose the parameters as well, an aspect that is addressed in this paper. Peng and Rao [10] proposed a risk minimization approach using SURE for standard Gaussian bilateral filtering. We propose SURE to compute the optimal parameters of the fast bilateral filter in [9] in constant time and compare the performance of SURE-optimal fast bilateral filter with that of Oracle MSE-based optimal bilateral filter and SURE-optimal Gaussian bilateral filter.

The paper is organized as follows. We provide the problem statement in Section 2. In Section 3, we provide the SURE background and the SURE calculations for the standard bilateral filter, the Gaussian bilateral filter and the raised-cosine bilateral filter. To validate the accuracy of SURE derived for the fast bilateral filter and its closeness to the MSE, we present our experimental results in Section 4. Concluding remarks are given in Section 5.

2. PROBLEM FORMULATION

We consider a digital image $\mathbf{x}$ (vector representation of an image) corrupted by additive white Gaussian noise $\mathbf{b}$ of zero-mean and $\sigma^2\mathrm{I}$ covariance matrix. The noisy image $\mathbf{y}$ is given by $\mathbf{y}=\mathbf{x}+\mathbf{b}$. We would like to obtain a denoised image $\hat{\mathbf{x}}$ that is a close estimate of the original image. The bilateral filter is chosen as the denoising function. The MSE is used to quantify the closeness of the bilaterally filtered image to the ground truth. The MSE is defined as

$$\mathrm{MSE}(\hat{\mathbf{x}}) = \mathcal{E}\{\|\hat{\mathbf{x}}-\mathbf{x}\|^2\} = \frac{1}{N}\|\hat{\mathbf{x}}-\mathbf{x}\|^2,$$

where N denotes the total number of pixels in the image $\mathbf{x}$. To obtain the best bilaterally filtered output, we have to find the optimal bilateral filter. This can be accomplished by computing the denoised output for various parameters of the bilateral filter and selecting the parameter that minimizes the MSE. In practical cases, the Oracle MSE cannot be computed as we do not have access to the original image $\mathbf{x}$. SURE is an unbiased estimate of the MSE that depends on the noisy image. Therefore, we propose the use of SURE to obtain the optimal parameters of the bilateral filter.

3. COMPUTING THE OPTIMAL PARAMETERS OF THE BILATERAL FILTER USING SURE

3.1. Theoretical background for SURE

We recall the following multidimensional version of the Stein's lemma [11, 12] to replace the MSE by an unbiased estimate that depends on the noisy image $\mathbf{y}$.

Lemma 1: For an N-dimensional function $\hat{\mathbf{x}}$ with $\mathcal{E}\{|\partial\hat{x}_{\mathbf{k}}/\partial y_{\mathbf{k}}|\} < \infty$ $\forall\mathbf{k}$, under additive white Gaussian noise assumption, the expressions $\hat{\mathbf{x}}^T\mathbf{x}$ and $\hat{\mathbf{x}}^T\mathbf{y} - \sigma^2\,\mathrm{div}_{\mathbf{y}}(\hat{\mathbf{x}})$ have the same expectation.

By applying Lemma 1 to the expression of MSE, the following expression for SURE is obtained [12]:

$$\mathrm{SURE}(\hat{\mathbf{x}}) = \frac{1}{N}\|\hat{\mathbf{x}}-\mathbf{y}\|^2 + \frac{2\sigma^2}{N}\mathrm{div}_{\mathbf{y}}(\hat{\mathbf{x}}) - \sigma^2. \tag{4}$$

SURE is an unbiased estimate of the MSE, and therefore, the optimal parameters in the minimum MSE sense can be approximated by minimizing SURE.

3.2. SURE for the general bilateral filter

In the case of bilateral filtering, the term $\frac{1}{N}\|\hat{\mathbf{x}}-\mathbf{y}\|^2$ in (4) represents the average squared error between the bilateral filter output $\hat{\mathbf{x}}$ and the noisy image $\mathbf{y}$. The divergence term $\mathrm{div}_{\mathbf{y}}(\hat{\mathbf{x}})$ is given by

$$\mathrm{div}_{\mathbf{y}}(\hat{\mathbf{x}}) = \sum_{\mathbf{k}\in\mathcal{I}} \frac{\partial\hat{x}_{\mathbf{k}}}{\partial y_{\mathbf{k}}}, \tag{5}$$

where $\mathcal{I}$ represents the whole range of pixel coordinates of the image. The differential of the bilateral filter output with respect to the noisy image is given by

$$\frac{\partial\hat{x}_{\mathbf{k}}}{\partial y_{\mathbf{k}}} = \frac{1}{W_{\mathbf{k}}}\Big(\sum_{\mathbf{m}\in\mathcal{N}_{\mathbf{k}}} \frac{\partial\phi_{\mathbf{k},\mathbf{m}}(y_{\mathbf{k}},y_{\mathbf{m}})}{\partial y_{\mathbf{k}}} y_{\mathbf{m}} + \phi_{\mathbf{k},\mathbf{k}}(y_{\mathbf{k}},y_{\mathbf{k}}) - \hat{x}_{\mathbf{k}} \sum_{\mathbf{m}\in\mathcal{N}_{\mathbf{k}}} \frac{\partial\phi_{\mathbf{k},\mathbf{m}}(y_{\mathbf{k}},y_{\mathbf{m}})}{\partial y_{\mathbf{k}}}\Big), \tag{6}$$

$$\text{where } W_{\mathbf{k}} = \sum_{\mathbf{m}\in\mathcal{N}_{\mathbf{k}}} \phi_{\mathbf{k},\mathbf{m}}(y_{\mathbf{k}},y_{\mathbf{m}}) \quad \text{and} \quad \phi_{\mathbf{k},\mathbf{k}}(y_{\mathbf{k}},y_{\mathbf{k}}) = 1.$$

SURE for the general bilateral filter can be computed by substituting (2) and (5) in (4).

$$\mathrm{SURE}(\hat{\mathbf{x}}) = \frac{1}{N}\|\hat{\mathbf{x}}-\mathbf{y}\|^2 - \sigma^2 + \sum_{\mathbf{k}\in\mathcal{I}} \frac{1}{W_{\mathbf{k}}}\left(\sum_{\mathbf{m}\in\mathcal{N}_{\mathbf{k}}} \frac{\partial\phi_{\mathbf{k},\mathbf{m}}(y_{\mathbf{k}},y_{\mathbf{m}})}{\partial y_{\mathbf{k}}} y_{\mathbf{m}} + 1 - \hat{x}_{\mathbf{k}} \sum_{\mathbf{m}\in\mathcal{N}_{\mathbf{k}}} \frac{\partial\phi_{\mathbf{k},\mathbf{m}}(y_{\mathbf{k}},y_{\mathbf{m}})}{\partial y_{\mathbf{k}}}\right).$$

3.3. SURE for the Gaussian bilateral filter

The classical Gaussian bilateral filter employs Gaussian range and domain kernels. It is given by

$$\phi_{\mathbf{k},\mathbf{m}}(y_{\mathbf{k}},y_{\mathbf{m}}) = \exp\Big(\frac{-\|\mathbf{k}-\mathbf{m}\|^2}{2\sigma_d^2}\Big)\exp\Big(\frac{-|y_{\mathbf{k}}-y_{\mathbf{m}}|^2}{2\sigma_r^2}\Big). \tag{7}$$

Taking the derivative of (7) with respect to $y_{\mathbf{k}}$, we get that

$$\frac{\partial\phi_{\mathbf{k},\mathbf{m}}(y_{\mathbf{k}},y_{\mathbf{m}})}{\partial y_{\mathbf{k}}} = \phi_{\mathbf{k},\mathbf{m}}(y_{\mathbf{k}},y_{\mathbf{m}})\Big(\frac{y_{\mathbf{m}}-y_{\mathbf{k}}}{\sigma_r^2}\Big). \tag{8}$$

Using (5), (6), and (8), we can express the divergence term as

$$\mathrm{div}_{\mathbf{y}}(\hat{\mathbf{x}}) = \sum_{\mathbf{k}\in\mathcal{I}}\left(\frac{\sum_{\mathbf{m}\in\mathcal{N}_{\mathbf{k}}} \phi_{\mathbf{k},\mathbf{m}}(y_{\mathbf{k}},y_{\mathbf{m}})y_{\mathbf{m}}^2}{\sigma_r^2 W_{\mathbf{k}}} + \frac{1}{W_{\mathbf{k}}} - \frac{\hat{x}_{\mathbf{k}}^2}{\sigma_r^2}\right). \tag{9}$$

On substituting (9) in (4) , we get that

$$\begin{aligned}\mathrm{SURE}(\hat{\mathbf{x}}) &= \frac{1}{N}\|\hat{\mathbf{x}}-\mathbf{y}\|^2 - \sigma^2 + \frac{2\sigma^2}{N}\sum_{\mathbf{k}\in\mathcal{I}}\frac{1}{W_{\mathbf{k}}} \\ &+ \frac{2\sigma^2}{N\sigma_r^2}\sum_{\mathbf{k}\in\mathcal{I}}\left(\frac{\sum_{\mathbf{m}\in\mathcal{N}_{\mathbf{k}}} \phi_{\mathbf{k},\mathbf{m}}(y_{\mathbf{k}},y_{\mathbf{m}})y_{\mathbf{m}}^2}{W_{\mathbf{k}}} - \hat{x}_{\mathbf{k}}^2\right).\end{aligned}$$

Finding the SURE-optimal Gaussian bilateral filter can be accomplished by minimizing SURE over several values of the parameters σ_d and σ_r. Computation of SURE involves computing $\hat{\mathbf{x}}$ and $\mathrm{div}_{\mathbf{y}}(\hat{\mathbf{x}})$. $\hat{\mathbf{x}}$ and $\mathrm{div}_{\mathbf{y}}(\hat{\mathbf{x}})$ cannot be computed in constant time due to the non-linearity of the Gaussian range kernel, thus, making the process of finding the optimal Gaussian bilateral filter computationally intensive.

3.4. SURE for the bilateral filter with Gaussian domain kernel and raised-cosine range kernel

Writing $\cos(\theta) = \left(e^{\mathrm{j}\theta} + e^{-\mathrm{j}\theta}\right)/2$ and applying the binomial theorem, the raised-cosine kernel in (3) is expressed as

$$r(y_{\mathbf{k}}-y_{\mathbf{m}}) = \sum_{l=0}^{L} 2^{-L}\binom{L}{l}\exp\left(\frac{\mathrm{j}\gamma}{\rho\sqrt{L}}(2l-L)(y_{\mathbf{k}}-y_{\mathbf{m}})\right). \tag{10}$$

The domain filter used in [9] is $w_{\mathbf{k}-\mathbf{m}} = \exp\left(\frac{-\|\mathbf{k}-\mathbf{m}\|^2}{2\sigma_d^2}\right)$.
$$(11)$$

The bilateral filter output is obtained by substituting (10) and (11) in (2), that is,

$$\hat{x}_{\mathbf{k}} = \frac{\sum_{l=0}^{L} d_{\mathbf{k}}(l)\bar{g}_{\mathbf{k}}(l)}{\sum_{l=0}^{L} d_{\mathbf{k}}(l)\bar{h}_{\mathbf{k}}(l)}, \tag{12}$$

$$\begin{aligned} \text{where } d_{\mathbf{k}}(l) &= 2^{-L}\binom{L}{l}\exp\left(\frac{\mathrm{j}\gamma}{\rho\sqrt{L}}(2l-L)y_{\mathbf{k}}\right), \\ h_{\mathbf{k}}(l) &= \exp\left(-\frac{\mathrm{j}\gamma}{\rho\sqrt{L}}(2l-L)y_{\mathbf{k}}\right), \text{ and} \\ g_{\mathbf{k}}(l) &= h_{\mathbf{k}}(l)y_{\mathbf{k}}. \end{aligned}$$

In (12), $\bar{g}_{\mathbf{k}}(l)$ and $\bar{h}_{\mathbf{k}}(l)$ are obtained by filtering $g_{\mathbf{k}}(l)$ and $h_{\mathbf{k}}(l)$, respectively, with a Gaussian kernel of mean zero and variance σ_d^2. An O(1) algorithm is used to compute the filtered outputs $\bar{g}_{\mathbf{k}}(l)$ and $\bar{h}_{\mathbf{k}}(l)$. In addition to the O(1) complexity, the $2L$ different filtered outputs in (12) can be computed in parallel to further accelerate the speed of computation of the bilaterally filtered image.

From (1), (10), and (11), the derivative of ϕ with respect to y is written as

$$\frac{\partial \phi_{\mathbf{k},\mathbf{m}}(y_{\mathbf{k}}, y_{\mathbf{m}})}{\partial y_{\mathbf{k}}} = w_{\mathbf{k}-\mathbf{m}} \sum_{l=0}^{L} c_{\mathbf{k}}(l) \exp\left(-\frac{\mathrm{j}\gamma}{\rho\sqrt{L}}(2l-L)y_{\mathbf{m}}\right), \tag{13}$$

where $c_{\mathbf{k}}(l) = \frac{\mathrm{j}\gamma}{\rho\sqrt{L}}(2l-L)d_{\mathbf{k}}(l)$.

Using (5) and (6), the divergence term is computed as

$$\mathrm{div}_{\mathbf{y}}(\hat{\mathbf{x}}) = \sum_{\mathbf{k}\in\mathcal{I}} \frac{\partial \hat{x}_{\mathbf{k}}}{\partial y_{\mathbf{k}}} = \sum_{\mathbf{k}\in\mathcal{I}} \frac{\sum_{l=0}^{L} c_{\mathbf{k}}(l)\bar{g}_{\mathbf{k}}(l) + 1 - \hat{x}_{\mathbf{k}} \sum_{l=0}^{L} c_{\mathbf{k}}(l)\bar{h}_{\mathbf{k}}(l)}{\sum_{l=0}^{L} d_{\mathbf{k}}(l)\bar{h}_{\mathbf{k}}(l)}. \tag{14}$$

On substituting (14) in (4), we obtain the expression for SURE as

$$\begin{aligned} \mathrm{SURE}(\hat{\mathbf{x}}) &= \frac{1}{N}\|\hat{\mathbf{x}}-\mathbf{y}\|^2 - \sigma^2 \\ &+ \frac{2\sigma^2}{N} \sum_{\mathbf{k}\in\mathcal{I}} \frac{\sum_{l=0}^{L} c_{\mathbf{k}}(l)\bar{g}_{\mathbf{k}}(l) + 1 - \hat{x}_{\mathbf{k}} \sum_{l=0}^{L} c_{\mathbf{k}}(l)\bar{h}_{\mathbf{k}}(l)}{\sum_{l=0}^{L} d_{\mathbf{k}}(l)\bar{h}_{\mathbf{k}}(l)}. \end{aligned} \tag{15}$$

The computation of SURE involves computing the bilateral filter output $\hat{\mathbf{x}}$ and the divergence term $\mathrm{div}_{\mathbf{y}}(\hat{\mathbf{x}})$. In [9], it has been proven that the bilateral filter output $\hat{\mathbf{x}}$ (12) is a constant time O(1) implementation. From (14), we see that computing $\mathrm{div}_{\mathbf{y}}(\hat{\mathbf{x}})$ involves performing linear convolution on pointwise transforms $g_{\mathbf{k}}(l)$ and $h_{\mathbf{k}}(l)$ with the Gaussian domain kernel. An O(1) algorithm can be used to compute the filtered outputs $\bar{g}_{\mathbf{k}}(l)$ and $\bar{h}_{\mathbf{k}}(l)$. Further, the $2L$ different convolution outputs in (15) can be computed in parallel to accelerate the speed of computation of SURE. A constant time O(1) implementation of (15) is thus possible.

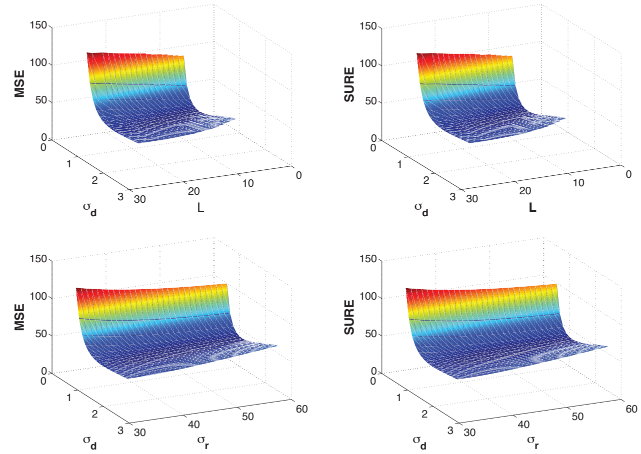

Fig. 1: (in color, in electronic version) Comparison of the MSE and SURE plots for Lenna image: (a) and (b) correspond to the fast bilateral filter, whereas (c) and (d) correspond to the Gaussian bilateral filter. SURE in (b) and (d) closely approximates the MSE in (a) and (c), respectively. Also, SURE for the fast bilateral filter (b) and SURE for the Gaussian bilateral filter (d) match closely.

4. EXPERIMENTS

The main objective of our experiments is to examine if the SURE follows the MSE closely for the fast and Gaussian bilateral filters and to observe how closely SURE for the fast bilateral filter approximates that of the Gaussian bilateral filter. We choose a grayscale Lenna image of size 512 $\times$ 512. A noisy realization of the image is obtained by adding zero-mean white Gaussian noise of standard deviation $\sigma = 14.34$ to obtain a peak signal to noise ratio (PSNR) of 25 dB. The image was denoised using the fast and Gaussian bilateral filters for different parameter settings. In each case, the MSE and SURE were computed. The results are shown in Figure 1. We observe that the SURE approximates the MSE accurately for the fast as well as the Gaussian bilateral filter. The cost functions seem to be well behaved to enable parameter search using optimization techniques. We have used gradient descent technique to compute the optimal parameters.

The fast bilateral filter is parameterized by three parameters namely σ_d, ρ, and L. In our experiments, we observed that restricting the search for σ_r such that $\sigma_r < \gamma^{-2}$ gave satisfactory results and from [9], we have $\rho = \frac{1}{\sqrt{L}}$. Thus, the raised-cosine kernel used to approximate the Gaussian range kernel can be effectively controlled by the parameter L alone. As a result, the fast bilateral filter is parameterized by σ_d and L. The Oracle MSE and SURE were computed over the parameters L and σ_d for the fast bilateral filter. The parameter L was varied from 8 to 26 in steps of 1 and σ_d from 0.5 to 2.5 in steps of 0.125. The optimal parameters for MSE and SURE turned out to be the same: $L = 12$ and $\sigma_d = 1.25$. We show in Figure 2, the original Lenna image, a noisy image of PSNR 19 dB and the image denoised using the SURE-optimal Gaussian and fast bilateral filters. In Figure 3, a noisy Cameraman image of PSNR 22.03 dB and the SURE-optimal fast and Gaussian filtered images are shown. We infer that the SURE-optimal Gaussian and raised-cosine bilateral filters have nearly the same performance since the PSNR values of the filtered images in Figure 2 and Figure 3 are close enough.

(a) (b) (c) (d) (e)

Fig. 2: (a) Original image, (b) Noisy image (PSNR 19 dB), (c) Denoised image obtained with SURE-optimal Gaussian bilateral filter (PSNR 28.98 dB), (d) Denoised image obtained with SURE-optimal fast bilateral filter (PSNR 28.80 dB), (e) (Color online) Comparison of Oracle MSE and SURE for fast bilateral filter and Gaussian bilateral filter using optimal parameter settings. The y-axis in (e) is in linear scale.

(a) (b) (c) (d) (e)

Fig. 3: (a) Original image, (b) Noisy image (PSNR 22.03 dB), (c) Denoised image obtained with SURE-optimal Gaussian bilateral filter (PSNR 30.83 dB), (d) Denoised image obtained with SURE-optimal fast bilateral filter (PSNR 30.66 dB), (e) (Color online) Comparison of Oracle MSE and SURE for fast bilateral filter and Gaussian bilateral filter using optimal parameter settings. The y-axis in (e) is in linear scale.

We next vary the PSNR and compare the optimal Oracle MSE versus the optimum provided by SURE for both Gaussian and raised-cosine bilateral filters. The results are illustrated in Figure 2(e) and Figure 3(e). We observe that the optimum values obtained by minimizing SURE agree well with those obtained by minimizing Oracle MSE for a wide range of PSNR values.

5. CONCLUSION

We have proposed a technique for choosing the optimal parameters for raised-cosine-based fast bilateral filtering by minimizing the SURE cost. SURE being an unbiased estimate of the MSE, the parameters that minimize SURE have been found to be nearly optimal in the minimum MSE sense. We have derived SURE expressions for the fast bilateral filter and verified that SURE for the fast bilateral filter closely approximates SURE for the standard Gaussian bilateral filter. We experimentally validated that the derived SURE and the Oracle MSE have local minima for the same parameters of the fast bilateral filter. Optimal parameter estimation by the proposed SURE technique turned out to be fast as well, owing to the constant time implementation of the bilateral filter [9] and the divergence term.

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