A BILATERAL FILTER IN GRADIENT DOMAIN

Zhengguo Li, Jinghong Zheng, Zijian Zhu, Shiqian Wu, Susanto Rahardja

Signal Processing Department, Institute for Infocomm Research, 1 Fusionopolis Way, Singapore

ABSTRACT

In this paper, a bilateral filter in gradient domain is first proposed. It is then applied to study detail enhancement via multi-light images and noise reduction of differently exposed low dynamic range images. These two applications show that the proposed filter can be applied to extract fine details from a set of images simultaneously and to provide flexibility for noise reduction from selected areas of an image.

1. INTRODUCTION

Extraction of fine details is required by many applications, such as fusion of differently exposed low dynamic range (LDR) images [1], de-noising of images [2], tone mapping of high dynamic range (HDR) images [3], detail enhancement via multi-light images [4], and so on. The fine details can be either noise, e.g., a random pattern with zero mean, or texture, such as a repeated pattern with regular structure. There are many results on the extraction of fine details from a single input image, such as the total variation based method in [5], the half quadratic optimization based method in [2], the quadratic optimization based method in [3], the bilateral filter [6, 7], etc. However, when there are multiple input images, using these methods directly would be complex because each input image needs to be decomposed individually. Recently, Li et al. [8] proposed a quadratic optimization based framework which can be adopted to extract fine details from a set of images simultaneously. The quadratic optimization problem is solved by using an iterative method which could be still complex for mobile devices with limited computational resources. Given the increasing popularity of multi-shot imaging and the intense response from Canon, Fuji, Nikon, Olympus and Ricoh with their latest digital camera for multi-shot capability, it is desirable to provide a better solution that extracts optimally captured details from a number of individual photographs and integrates them together to form a better image.

In this paper, a bilateral filter in gradient domain is introduced to extract fine details from a vector field. The proposed filter is based on the following two observations: 1) the details of an image are usually captured by its luminance variations; and 2) it is easy to generate the gradient field of its luminance, but more complex to calculate an image for a given vector field via solving the Possion Equation [10]. The proposed bilateral filter is adopted to study two examples: detail enhancement via multi-light images and denoising of differently exposed LDR images. It is shown from these applications that 1) it is easier to use the proposed filter to extract the desired fine details from a set of images; and 2) it is more flexible to use the proposed filter to remove noise from an image in the sense that the proposed filter can be used to reduce noise from selected areas of an image.

2. BILATERAL FILTER IN GRADIENT DOMAIN

In this section, the bilateral filter in [6] is extended to the gradient domain. For a given image $Z(i,j)$, the bilateral filter in [6] is

$$\hat{Z}_b(i,j) = \sum_{(k,l)\in\Omega_r(i,j)} \frac{w_1(Z,i,j,k,l)}{\sum\limits_{(k,l)\in\Omega_r(i,j)} w_1(Z,i,j,k,l)} Z(k,l), \quad (1)$$

where $\Omega_r = \{(k,l) \| k-i| \le r, |l-j| \le r\}$, $w_1(Z,i,j,k,l)$ is $\exp(-\frac{(k-i)^2+(l-j)^2}{\sigma_s^2})\exp(-\frac{(Z(i,j)-Z(k,l))^2}{\sigma_r^2})$, σ_s and σ_r are two parameters.

Let $\hat{Z}_d(i,j)$ denote $(Z(i,j) - \hat{Z}_b(i,j))$, it follows that

$$\hat{Z}_d(i,j) = \sum_{(k,l)\in\Omega_r(i,j)} \frac{w_1(Z,i,j,k,l)}{\sum\limits_{(k,l)\in\Omega_r(i,j)} w_1(Z,i,j,k,l)} (Z(i,j)-Z(k,l)). \quad (2)$$

Define the gradient field of image $Z(i,j)$ as

$$\nabla Z_x(i,j) = Z(i,j+1)-Z(i,j), \nabla Z_y(i,j) = Z(i+1,j)-Z(i,j),$$

and a vector field $\vec{G}(\nabla Z,i,j,k,l)$ as

$$G_y(\nabla Z,i,j,k,l) = \begin{cases} \sum\limits_{q=k}^{i-1} \nabla Z_y(q,j); & \text{if } i \ge k \\ -\sum\limits_{q=i}^{k-1} \nabla Z_y(q,j); & \text{otherwise} \end{cases},$$

$$G_x(\nabla Z,i,j,k,l) = \begin{cases} \sum\limits_{q=l}^{j-1} \nabla Z_x(k,q); & \text{if } j \ge l \\ -\sum\limits_{q=j}^{l-1} \nabla Z_x(k,q); & \text{otherwise} \end{cases}.$$

$\hat{Z}_d(i,j)$ can also be computed as

$$\hat{Z}_d(i,j) = \frac{\sum\limits_{(k,l)\in\Omega_r(i,j)} [w_2(\nabla Z,i,j,k,l) \sum\limits_{q\in\{x,y\}} G_q(\nabla Z,i,j,k,l)]}{\sum\limits_{(k,l)\in\Omega_r(i,j)} w_2(\nabla Z,i,j,k,l)}, \quad (3)$$

where $w_2(\nabla Z,i,j,k,l)$ is the product of $\exp(-\frac{(k-i)^2+(l-j)^2}{\sigma_s^2})$ and $\exp(-\frac{(\sum\limits_{q\in\{x,y\}} G_q(\nabla Z,i,j,k,l))^2}{\sigma_r^2})$.

Replacing the gradient field ∇Z by a vector field $\vec{V}(i,j) = (V_x(i,j), V_y(i,j))$, a bilateral filter in gradient domain is obtained as follows:

$$\hat{V}_d(i,j) = \frac{\sum_{(k,l)\in\Omega_r(i,j)} [w_2(V,i,j,k,l) \sum_{q\in\{x,y\}} G_q(V,i,j,k,l)]}{\sum_{(k,l)\in\Omega_r(i,j)} w_2(V,i,j,k,l)}. \tag{4}$$

It is shown in Equations (1)-(3) that the proposed filter is equivalent to the bilateral filter in [6] when the vector field $\vec{V}(i,j)$ is the gradient field of an image $Z(i,j)$. Therefore, the bilateral filter in [6] is a special case of the proposed filter.

In the end of this section, we show that the trilateral filter in [16] can be extended in a similar way. For a given vector field $\vec{V}(i,j) = (V_x(i,j), V_y(i,j))$, another vector field $U(i,j)$ is defined as

$$U(i,j) = \frac{\sum_{(k,l)\in\Omega_r(i,j)} [w_3(V,i,j,k,l)V(k,l)]}{\sum_{(k,l)\in\Omega_r(i,j)} w_3(V,i,j,k,l)}, \tag{5}$$

where $w_3(V,i,j,k,l)$ is $\exp(-\frac{(k-i)^2+(l-j)^2}{\sigma_s^2})\exp(-\frac{\|V(k,l)-V(i,j)\|^2}{\sigma_r^2})$. Two functions $\Phi(U,V,i,j,k,l)$ and $\Psi(U,i,j,k,l)$ are then given as

$$\Phi = \sum_{q\in\{x,y\}} G_q(V,i,j,k,l) - U_x(i,j)(k-i) - U_y(i,j)(l-j),$$

$$\Psi = \begin{cases} 1; & \text{if } \|U(k,l) - U(i,j)\| < \xi \\ 0; & \text{otherwise} \end{cases},$$

where ξ is a threshold [16].

A trilateral filter in gradient domain is obtained as follows:

$$\hat{V}_d(i,j) = -\frac{\sum_{(k,l)\in\Omega_r(i,j)} [w_4(U,V,i,j,k,l)\Phi(U,V,i,j,k,l)]}{\sum_{(k,l)\in\Omega_r(i,j)} w_4(U,V,i,j,k,l)},$$

where $w_4(U,V,i,j,k,l)$ is the product of $\exp(-\frac{(k-i)^2+(l-j)^2}{\sigma_s^2})$, $\exp(-\frac{\Phi^2(U,V,i,j,k,l)}{\sigma_r^2})$ and $\Psi(U,i,j,k,l)$.

3. DETAIL ENHANCEMENT VIA MULTI-LIGHT IMAGES

Multi-light images that capture the same scene under different lighting positions can collectively provide a much more detailed description of the scene than a single image. The information lost in one image can be complemented by the information from other images. This feature of multi-light images can be applied to enhance the shape and surface details of a scene [4]. In this section, the proposed bilateral filter is adopted to extract fine details from multi-light images simultaneously.

Let input images be denoted as $Z_k(i,j)(1 \le k \le N)$ and their luminance components denoted as $Y_k(i,j)(1 \le k \le N)$. As the contents of an image are usually indicated by the intensity variation, the maximal gradient at each location around the input images is usually used for the construction of guidance field. However, shading information from different images could also be contained in the guidance field as a shadow often has large intensity variations at its boundary. As a result, shadow regions will cause artifacts in the final image. To remove the intensity variation caused by shadow edges from the guidance field, a shadow detection approach can be adopted to divide an image into shadow and non-shadow areas; the gradients near shadow edges are not included in the guidance field [9]. However, it can be complex to detect shadows for each input image. Here, the simple method in [8] is adopted to determine the vector field $\vec{V}(i,j)$. The value of $V_x(i,j)$ is

$$\begin{cases} g_x(k^{x,2}(i,j),i,j), & \text{if } \left|\nabla Y_{k^{x,1},x}(i,j)\right| > 3\left|\nabla Y_{k^{x,2},x}(i,j)\right| \\ g_x(k^{x,1}(i,j),i,j), & \text{otherwise} \end{cases},$$

and the value of $V_y(i,j)$ is

$$\begin{cases} g_y(k^{y,2}(i,j),i,j), & \text{if } \left|\nabla Y_{k^{y,1},y}(i,j)\right| > 3\left|\nabla Y_{k^{y,2},y}(i,j)\right| \\ g_y(k^{y,1}(i,j),i,j), & \text{otherwise} \end{cases},$$

$k^{i,1}(i,j)$ and $k^{i,2}(i,j)$ $(i = x, y)$ denote the indices of two images with the largest gradient and the second largest gradient at position (i,j) along direction x and y, respectively. The functions $g_x(k,i,j)$ and $g_y(k,i,j)$ are defined as

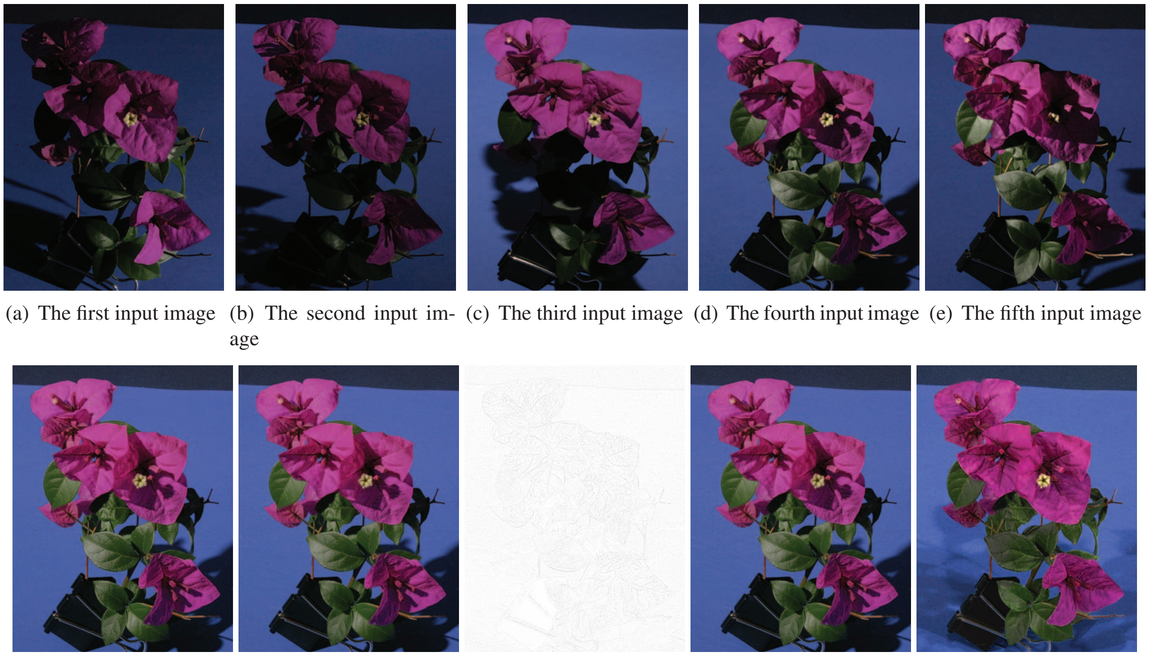
$$g_x(k,i,j) = w_5(Y_k(i,j))w_5(Y_k(i+1,j))\nabla Y_{k,x}(i,j),$$
$$g_y(k,i,j) = w_5(Y_k(i,j))w_5(Y_k(i,j+1))\nabla Y_{k,y}(i,j),$$

and the weighting function $w_5(z)$ is

$$w_5(z) = \begin{cases} z/30; & \text{if } z \le 30 \\ (255-z)/30; & \text{if } z > 225 \\ 1; & \text{if } 225 \ge z > 30 \end{cases}. \tag{6}$$

Due to the space limitation, only one set of images is tested. The values of r, σ_s and σ_r are chosen as 2, 4, and 0.5, respectively. Input images are given in Figures 1(a)-1(e) and image size is 750x975. The case that there is only one input image is first compared with the case that there are multi-light input images. It is shown in Figs. 1(f) and 1(g) that more details are included in the final output image especially the shadow areas by using multi-light input images. The proposed vector field is then compared with the maximum gradient field. Clearly, there are artifacts due to shadow edges in the final image as in Figure. 1(i) by using the maximum gradient field while these are removed by using the proposed vector field. Finally, the proposed details enhancement method is compared with the scheme presented in [4]. In [4], the base layer is calculated as a weighted sum of the input images. Even though users can control the strength and location of shadows through adjusting the weight factors of input images, the shadows of the final images are still a little messy as demonstrated in Figure 1(j), while the proposed details enhancement method can provide natural shading information in the final output images.

The proposed method is implemented in non-optimized C code. The running time of the proposed bilateral filter for the sequence in Figures 1(a)-1(e) is 0.28s. The run time of existing bilateral filter with a fast implementation in [4] is 1.4s for an image with 3M pixels. Clearly, the computational complexity of the proposed and existing bilateral filters is almost the same when there is only one input image. When there are multiple input images, the proposed bilateral filter can be used to reduce the time on extraction of fine details. For example, the time can be reduced by 80% for the image sequence in Figures 1(a)-1(e).



(a) The first input image (b) The second input image (c) The third input image (d) The fourth input image (e) The fifth input image

(f) An enhanced image with only one input image (g) An enhanced image with multi-light input images via the proposed filter (h) The extracted fine details of multi-light images (i) An enhanced image via the maximum gradient field (j) An enhanced image via the scheme in [4]

Fig. 1. Comparison of different detail enhancement schemes

4. DE-NOISING OF DIFFERENTLY EXPOSED IMAGES

In this section, the proposed filter is used to study de-nosing of differently exposed LDR images. The objective is to show that the proposed filter can be applied to remove noise from selected areas of an image.

A set of differently exposed DR images are usually taken from a HDR scene and they are used to synthesize an image with desired details. To capture a HDR scene under low lighting condition, cameras are usually set to high ISO settings so as to reduce motion blur of the differently exposed LDR images. However, high ISO setting causes noise on LDR images and this eventually degrades the quality of the final HDR image.

A weighted frame averaging method was proposed in [13] to reduce noise from differently exposed LDR images. Same as method in [14], the method in [13] is based on an observation that an image with a longer exposure time, i.e., a brighter image, normally includes less noise. All LDR images are firstly arranged according to their exposure times. Since the knowledge of exposures might not be available in some applications, the average intensity values of an image are used in the scheme [13]. Each LDR image is corrected using several successive LDR images with longer exposures in the same sequence. Instead of mapping all LDR images to the HDR domain as in [14], all LDR images with longer exposures are first calibrated according to the image to be denoised by using the IMFs among them in [13]. All mapped LDR images and the image to be denoised are then averaged with a predefined weighting function to generate the corrected LDR image. With the observation that LDR images with longer exposures include less noise, the weighting function is designed such that higher weights are given to pixels captured with longer exposure times. The noise in the LDR image with a shorter exposure can thus be reduced and the noise in the corresponding areas of synthesized image are also effectively removed. Both methods in [13] and [14] can be applied to reduce noise from images with smaller exposure times well. *However, neither [13] nor [14] provides any method to remove noise from the brightest image which could also include noise, especially in dark areas.* In this section, the proposed bilateral filter is applied to remove noise only from dark areas of the brightest image.

Defining a weighting function $w_6(z)$ as

$$w_6(z) = \frac{1}{1+\exp((z-128)/8)},$$

the vector field $\vec{V}(i,j)$ is computed as

$$V_x(i,j) = w_6(Z_1(i,j+1))w_6(Z_1(i,j))\nabla Z_{1,x}(i,j);$$
$$V_y(i,j) = w_6(Z_1(i+1,j))w_6(Z_1(i,j))\nabla Z_{1,y}(i,j).$$

The noise is extracted from the vector field $\vec{V}(i,j)$ by using the proposed bilateral filter. The values of r, σ_s and σ_r are chosen as 2, 4, and 26, respectively. The proposed de-noising scheme is compared with a de-noising scheme based on the bilateral filter in [6]. In the latter, the whole image is de-noised, i.e., the value of $w_6(z)$ is always 1. The brightest LDR image from a set of differently exposed LDR images is shown in Figure 2(a). It is shown in Figures 2(b) and 2(c) that both the proposed bilateral filter and the bilateral filter in [6] can be adopted to remove noise from dark areas of the input image while only the proposed filter

(a) An input noisy image (b) The de-noised image by the proposed bilateral filter (c) The de-noised image by the existing bilateral filter

Fig. 2. Comparison of different de-noising algorithms.

can be applied to keep sharpness in bright regions of the input image.

5. CONCLUSION

In this paper, existing bilateral filter is first extended to the gradient domain. The extended bilateral filter is then applied to study detail enhancement via multi-light images and noise reduction of differently exposed low dynamic range images. Similar to the joint bilateral filter in [15] and the switched bilateral filter in [7], the proposed filter can be further extended for different applications. Both types of extensions as well as applications of extended bilateral filters will be investigated in our future research.

References

[1] T. Mertens, J. Kautz, and F. V. Reeth, “ Exposure fusion: a simple and practical alternative to high dynamic range photography,” Computer Graphics Forum, vol. 28, pp.161-171, 2009.

[2] P. Charbonnier, L. Blanc-Feraud, G. Aubert, and M. Barlaud M, “Deterministic edge-preserving regularization in computed imaging,” IEEE Transactions on Image Processing, vol. 6, no. 2, pp. 298-311, Feb. 1997.

[3] Z. Farbman, R. Fattal, D. Lischinshi, and R. Szeliski, “Edge-preserving decompositions for multi-scale tone and details manipulation,” ACM Transactions on Graphics (Proc. SIGGRAPH), vol. 27, no. 3, pp. 249-256, Aug. 2008.

[4] R. Fattal, M. Agrawala, and S. Rusinkiewicz, “Multiscale shape and details enhancement for multi-light image collections,” *ACM Transactions on Graphics (Proc. SIGGRAPH)*, vol. 26, no.3, pp.51:1-51:10, Aug. 2007.

[5] L. Rudin L., S. Osher, and E. Fatemi, “Nonlinear total variation based noise removal algorithms,” Physica D., vol. 60, pp. 259-268, 1992.

[6] C. Tomasi and R. Manduchi, “Bilateral filtering for gray and color images,” In *Proc. IEEE Int. Conf. on Computer Vision*, pp. 836-846, 1998.

[7] C. H. Lin, J. S. Tsai, and C. T. Chiu, “Switching bilateral filter with a texture/noise detector for universal noise removal,” IEEE Trans. on image Processing, vol. 19, pp. 2307-2320, Sept. 2010.

[8] Z. G. Li, J. H. Zheng, C. H. Yeo, and S. Rahardja, “Quadratic optimization based small scale details extraction,” In *2011 IEEE International Conference on Acoustics, Speech and Signal Processing*, pp. 1309-1312, Czech Republic, 22-27 May 2011

[9] J. H. Zheng, Z. G. Li, S. Rahardja, S. S. Yao, and W. Yao, “Collaborative image processing algorithm for details refinement and enhancement via multi-light images,” In *IEEE ICASSP 2010*, pp. 1382-1385, Mar. 2010.

[10] P. Perez, M. Gangnet, and A. Blake, “Poisson image editing,” *ACM Transactions on Graphics (Proc. SIGGRAPH)*, vol. 22, no.3, pp. 313-318, Aug. 2003.

[11] P. E. Debevec and J. Malik, “Recovering high dynamic range radiance maps from photographs,” In *Proceedings of SIGGRAPH 1997*, pp.369-378, 1997.

[12] E. Reinhard, G. Ward, S. Pattanaik and P. E. Debevec, High dynamic range imaging: acquisition, display and image-based lighting, Morgan Kaufmann, 2005.

[13] W. Yao, Z. G. Li, and S. Rahardja, “Intensity mapping function based weighted frame averaging for high dynamic range imaging,” In *the 6th IEEE Conference on Industrial Electronics and Applications*, pp.1568-1571, Beijing, China, June 2011.

[14] A. O. Akyuz and E. Reinhard, “Noise reduction in high dynamic range imaging”, Journal of Visual Communication and Image Representation, vol. 16, pp.366-376, 2007.

[15] G. Petschnigg, M. Agrawala, and H. Hoppe, “Digital photography with flash and no-flash image pairs,” ACM Transactions on Graphics (Proc. SIGGRAPH), Vol. 23, No. 3, pp. 664-672, Aug. 2004.

[16] P. Choudhury and J. Tumblin, “The trilateral filter for high contrast images and meshes,” In *Eurographics Symposium on Rendering*, 2003.