A PATCH-BASED APPROACH FOR RANDOM-VALUED IMPULSE NOISE REMOVAL

Julie Delon *

LTCI (CNRS UMR 5141) Télécom ParisTech 46, rue Barrault, 75013 Paris, FRANCE

ABSTRACT

In this paper, we show that a patch-based approach can successfully be applied for impulse noise removal. This requires careful choices for both the distance between patches and for the statistical estimator of the original patch. This method proves to be particularly powerful, especially for the restoration of textured areas, and compares favorably to recent restoration methods.

Index Terms— restoration, impulse noise, non local methods, image patch, order statistics.

1. INTRODUCTION

Impulse noise is generally caused by errors appearing during the acquisition or the transmission of images. Removing this kind of noise while preserving image details and textures is of great importance before most image analysis tasks (edge detection, segmentation, etc). Two models of impulse noise are generally used in the literature. In the first one, called saltand-pepper noise, each grey level is replaced with a given probability by 0 or M, where [0, M] designs the range of the original image. In this paper, we focus on the second model, called random-valued impulse noise, where each grey level value is replaced with probability p, called noise ratio, by a random value in the interval $\{0, \ldots, M\}$. More precisely, if the discrete damaged image is denoted by u and the original image is denoted by u^0 , both defined on a discrete domain Ω , u is a realization of a random image U such that $\{U(x), x \in \Omega\}$ are independent random variables and such that the law of U(x) is a mixture between a uniform distribution on $\{0, \ldots, M\}$ (with weight p) and a Dirac mass centered at $u_0(x)$ (with weight 1-p). Observe that detecting and removing random-valued impulse noise is much more difficult in practice than removing salt-and-pepper noise.

The traditional approaches for impulse noise removal act locally and non linearly on images. Among them, let us mention the median and its extensions [1, 2]. These approaches modify all pixels indifferently, while impulse noise affects Agnès Desolneux

MAP5 (UMR CNRS 8145) Université Paris Descartes 45 rue des St-Pères, 75006 Paris, FRANCE

only a portion of the pixels. In order to avoid this shortcoming, the trend for nearly twenty years has been to propose different impulse noise detectors and to restrict the restoration to pixels detected as corrupted. For instance, this idea underlies the switching median filter [3], the adaptive center weighted median filter (ACWMF) [4] or the pixel-wise median absolute deviation [5]. Unfortunately, median-based methods tend to destroy details and textures in images when the noise ratio is large. A successful alternative consists in combining a well chosen impulse detector, generally relying on local order statistics of grey level differences, with a global or at least semi-local restoration approach. This is the case of [6], which combines the Rank Order Absolute Differences (ROAD) for detection with a trilateral filter for restoration. This scheme is also followed by the authors of [7, 8], who rely on ACWMF, or ROLD (Rank-Ordered Logarithmic Difference) for detection, before applying a variational approach to restore corrupted pixels.

The approach presented in this paper for impulse noise removal relies on the patch redundancy inside images. In the last fifteen years, a great deal of restoration methods have been developed in order to take advantage of this property [9, 10, 11]. Let us mention in particular the Non Local Means, which have been first introduced by Buades et al [9] to tackle Gaussian noise in images, and are now declined or improved for different kind of noises [12, 10, 13, 14, 15]. The mathematical framework adapted to deal with this redundancy is the one of statistical estimation: we aim at estimating the real underlying patch behind different degraded versions. The goal of this paper is to model properly this estimation scheme in the case of images suffering from impulse noise. Let us make clear the different steps of this scheme. At each point x in Ω , we want to estimate $u^0(x)$ from all the values u(y) when y spans a region \mathcal{V}_x around x (in this paper \mathcal{V}_x is a square of size $(2t+1) \times (2t+1)$ centered at x). The first step of the estimation consists in deciding which pixels y can be trusted in the estimation of $u^0(x)$. For this task, we introduce in Section 2 a measure of similarity D between patches, designed to be robust to impulse noise. This permits to order the pixels y in \mathcal{V}_x in function of $D(\mathbf{P}_x, \mathbf{P}_y)$, where \mathbf{P}_x and \mathbf{P}_y are $(2f+1) \times (2f+1)$ patches centered at x and y. We call \mathcal{V}_r^n

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the subset of \mathcal{V}_x corresponding to the *n* smallest distances, and only rely on the n-tuple $(u(y), y \in \mathcal{V}_x^n)$ for the estimation of $u^0(x)$. The second question, tackled in Section 3, concerns the choice of a judicious estimator \mathcal{E} of $u^0(x)$. Experiments and comparisons with recent approaches are displayed in Section 4.

2. ROBUST DISTANCE BETWEEN PATCHES

The success of any patch-based denoising procedure relies greatly on the ability to find similar patches in the noisy image u. More precisely, for a given patch **P** in u, we aim at discovering all patches Q such that the unknown original patches \mathbf{P}^0 and \mathbf{Q}^0 in u^0 are equal or at least similar. Since **P** and **Q** are affected by impulse noise, the euclidean (L^2) distance between P and Q relies on some outliers and cannot be trusted. In this work, we propose to rely on a robust dissimilarity measure, inspired by order statistics [16]. Let $r_1 \leq r_2 \leq \cdots \leq r_{(2f+1)^2}$ be the values obtained by ordering the $(2f + 1)^2$ values of the differences $|\mathbf{P}(y) - \mathbf{Q}(y)|$. If ${\bf P}$ and ${\bf Q}$ are two independent random patches such that $\mathbf{P}^0 = \mathbf{Q}^0$, the probability that the k^{th} difference r_k stems from two untouched pixels is $\mathcal{B}((2f+1)^2, k, (1-p)^2)$ (with the approximation that the smallest distances correspond to untouched pixels), where \mathcal{B} denotes the tail of the binomial distribution¹. We take these probabilities into account and propose to use the robust distance

$$D(\mathbf{P}, \mathbf{Q}) = \sum_{k=1}^{(2f+1)^2} \mathcal{B}((2f+1)^2, k, (1-p)^2) r_k^2.$$
 (1)

3. CHOICE OF THE ESTIMATOR \mathcal{E}

In this section, we aim at defining a good estimator \mathcal{E} of $u^0(x)$. The estimator \mathcal{E} should only rely on the values u(y) for y in \mathcal{V}_x^n (this set can be computed once D is defined). Let us start with a much simpler problem. Instead of estimating $u^0(x)$ from other values u(y) in the degraded image u, assume a case where several independent realizations of the random value U(x) are observed. In this case, it can be easily shown that the maximum likelihood estimator (MLE) of $u^0(x)$ corresponds to the most represented value among these samples. If we compute the discrete histogram of these n samples (on M + 1 bins), this value is the place where the histogram attains its maximum ². Synthetic experiments prove that the bias and variance of the median estimator are larger than those of the MLE for this model, especially when the noise ratio p is high.

In practice, values u(y), $y \in \mathcal{V}_x^n$, cannot really be considered as independent realizations of U(x), since the underlying patches are only similar, not perfectly equal. In

order to take this uncertainty into account, we assume that these values are n independent realizations of a mixture between a uniform distribution on $\{0, \ldots, M\}$ and a Gaussian distribution of mean $u^0(x)$ and unknown standard deviation s(x). We note $g_{u^0(x)}^{s(x)}$ this Gaussian distribution. Let us denote by v_1, \ldots, v_n the observations $u(y), y \in \mathcal{V}_n^x$ and by V_1, \ldots, V_n the corresponding random variables. The MLE $(\mathcal{E}_{u^0(x)}, \mathcal{E}_{s(x)})$ of the couple $(u^0(x), s(x))$ is obtained by looking for the value (θ, σ) which maximizes the quantity $\log \mathbb{P}[V_1 = v_1, \ldots, V_n = v_n | u^0(x) = \theta, s(x) = \sigma]$. This can be rewritten by using the histogram h of the values $\{v_1, \ldots, v_n\}$ on $\{0, \ldots, M\}$,

$$(\mathcal{E}_{u^0(x)}, \mathcal{E}_{s(x)}) = \arg\max_{\theta, \sigma} \sum_{m=0}^{M} h(m) \log\left(\frac{p}{M+1} + (1-p)g_{\theta}^{\sigma}(m)\right).$$

As a consequence, the MLE is the value where the discrete convolution $h * f^{\sigma}(\theta)$ attains its maximum, with $f^{\sigma} : m \mapsto \log\left(\frac{p}{M+1} + (1-p)g_0^{\sigma}(m)\right)$.

Observe that the estimator \mathcal{E} depends on a good estimation of p and on a number n of trusted patches. We will see in Section 4 how to estimate p globally on the degraded image. As for the number of patches, it is computed empirically as the smallest n such that the probability $\mathbb{P}[|\mathcal{E}_{u^0(x)}(V_1, \ldots, V_n) - u_0(x)| \ge \sigma]$ is smaller than 0.01, for $\sigma = 5$ (see Table 1). Let us remark at this point that this number varies slowly with values of σ between 5 and 20. In other words, the choice of σ is not crucial.

p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
n	8	10	14	18	22	32	50	92

Table 1. Number n of patches used in \mathcal{E} for each value of p.

4. EXPERIMENTS AND DISCUSSION

Implementation details. The first step of our denoising procedure consists in estimating the noise ratio p. For this task, we rely on the detector ROAD (for "Rank Ordered Absolute Differences"), proposed in [6]. This detector can be described as follows: for each pixel x, the absolute differences between u(x) and u(y) are computed for all $y \neq x$ in a centered 3×3 patch around x. These differences are ordered. The value ROAD(x) is obtained by computing the sum of the 4 smallest differences. This value measures how close u(x) is from its neighbors. When ROAD(x) is above a given threshold τ , set as 70 in our experiments, x is considered as noisy. Other detection procedures could be used for estimating p [4, 5, 8, 17]. We choose to rely on ROAD mainly because the corresponding estimation of p is generally good and fast to compute. Figure 1 shows the quality of this estimation for different values of p on the 512×512 image Lena.

 $^{{}^{1}\}mathcal{B}(n,k,q) = \sum_{i=k}^{n} {n \choose i} p^{i} (1-p)^{n-i}.$

²If the histogram attains its maximum at more than one bin, one of them can be chosen randomly, with equal probabilities.



Fig. 1. Estimation of the impulse noise parameter p on the 512×512 version of *Lena*. The results \overline{p} are shown as boxplot graphics, where the horizontal axis represents the tested values of p, from 0.1 to 0.9. Boxes are the statistics obtained from 100 samples for each value of p.

Once p is computed, we estimate n empirically, as explained at the end of Section 3. Values of n can also be tabulated once for all for quantized values of p (see Table 1). The algorithm continues as follows. For each point x in Ω , we seek the n nearest of neighbors \mathbf{P}_y of \mathbf{P}_x when y spans \mathcal{V}_x . This permits to define \mathcal{V}_x^n , the subset of \mathcal{V}_x corresponding to these n nearest neighbors. The MLE $(\mathcal{E}_{u_0(x)}, \mathcal{E}_{s(x)})$ is then computed at each x from the n-tuple $(u(y), y \in \mathcal{V}_x^n)$. The image $u_1(x) = \mathcal{E}_{u_0(x)}$ constitutes a first denoised version of u(x). However, this restored image is sometimes a little bit too smooth. In order to recover the grain of the original image, we take into account the estimated standard deviation s(x) at each point for defining a map of noisy pixels:

$$\mathcal{M} = \{ x \in \Omega; \ |\mathcal{E}_{u_0(x)} - u(x)| > \mathcal{E}_{s(x)} \}.$$

The new restored image is then defined as a mixture of u_1 and u: $\tilde{u}_1(x) = u_1(x).\mathbf{1}_{x \in \mathcal{M}} + u(x).(1 - \mathbf{1}_{x \in \mathcal{M}})$. In practice, we repeat these steps twice and the output of the algorithm is given by \tilde{u}_2 . In all our experiments, the half-size of the research neighborhood and the half-size of the patches are set as t = 7 and f = 3.

A refined version of the algorithm can be obtained by following the approach introduced in [18]. Notice that a point xbelongs to all patches $\mathbf{P}_{x+\delta}$, $\delta \in [-f, f] \times [-f, f]$. The idea of the refinement is to take into account in the estimation all the information from these patches. In this version, the MLE is computed at each x from

$$(u(y-\delta); \ \delta \in [-f,f] \times [-f,f] \text{ and } y \in \mathcal{V}_{x+\delta}^n).$$

This refined version is the one used in the experiments.

Experiments. In this paragraph, we compare our scheme with the recent state of the art approaches [6, 8]. One common way to measure the quality of the restored image v is to use the PSNR, given by the formula

$$PSNR(u_0, v) = 10 \log_{10} \frac{255^2 \# \Omega}{\sum_{x \in \Omega} (u_0(x) - v(x))^2},$$

where $\#\Omega$ is the size of the support of u_0 . Table 2 describes the PSNR obtained with the three methods on the 512×512 classical images *Lena*, *Bridge* and *Baboon* (the images used here are those available on the website www.math. cuhk.edu.hk/~rchan/paper/dcx). The PSNR results for [6, 8] are taken from Table 1 in [8]. Our result is thus obtained with a different noise sample. These tables show very similar performances between our method and ROLD-EPR, and prove that a patch-based approach is well founded for random-valued impulse noise removal. Let us also insist on the fact that the parameters of our approach (patch size, research neighborhood size, number of iterations, ROAD threshold) were fixed for all these experiments. Better results can be obtained by optimizing these values for each image.

Figure 2 provides a visual comparison of the results of different algorithms on the 512×512 versions of Lena and *Barbara*, with respective noise ratios p = 40% and p = 50%. Enlarged portions of the images are shown for better visualization. The code used for the trilateral filter is the one provided by its authors ³ and its parameters are set as advised in [6]. The code of ROLD-TVL1 was kindly provided by V. Duval [19] and its parameters are optimized empirically to obtain the best visual result. This algorithm is used here instead of ROLD-EPR, whose code was not available. If the regularization term in TVL1 is slightly different from the one of EPR, the results of ROLD-EPR and ROLD-TVL1 for impulse noise removal are in practice quite similar (see for instance [19]). On Lena, the result of our non local scheme seems smoother than the ones of ROLD-TVL1 and ROADtrilateral, in which some clues of impulse noise remain. On Barbara, while all schemes yield a quite reasonable regularization on constant regions, our scheme is the only one to handle the texture of the clothes properly.

Lena								
Method	p = 20%	p = 40%	p = 60%					
ROAD-Trilateral [6]	37.70	31.12	26.08					
ROLD-EPR [8]	37.45	32.76	29.03					
Our method	38.33	34.18	29.96					
Bridge								
Method	p = 20%	p = 40%	p = 60%					
ROAD-Trilateral [6]	27.60	24.01	20.84					
ROLD-EPR [8]	27.86	24.79	22.59					
Our method	27.68	27.68 24.80						
Baboon								
Method	p = 20%	p = 40%	p = 60%					
ROAD-Trilateral [6]	24.18	21.60	19.52					
ROLD-EPR [8]	24.49	21.92	20.38					
Our method	24.17	22.02	20.13					

Table 2. PSNR results of different restoration filters for the 512×512 images *Lena*, *Bridge* and *Baboon*.

³www.ssc.wisc.edu/~thuegeri/RTtrilateral.m



Fig. 2. Comparative results on *Lena* and *Barbara*. First line: noisy image (p = 40%), trilateral filter, ROLD+TVL1, our result. Second line: noisy image (p = 50%), trilateral filter, ROLD+TVL1, our result.

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