A NOVEL WEIGHTING METHOD USING FUZZY SET THEORY FOR SPATIAL ADAPTIVE PATCH-BASED IMAGE DENOISING

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ABSTRACT

Based on fuzzy set theory, this paper proposes a novel weighting method for spatial adaptive patch-based image denoising algorithm which can be considered as an extension of nonlocal means filtering. First, a fuzzy clustering algorithm for weighting data points is applied to reduce the estimate bias which arises from the unrelated points. The weighting function is determined by optimal fuzzy partitions of the similarity of image patches. Then, the control parameter of the weighting function for each iterative step is modified to make a more rational weight distribution. Finally, a fuzzy control idea is introduced to deal with residual noisy pixels which fail in fuzzy clustering because of lacking similar patches in their neighborhoods. Experiment results show that the proposed weighting method improves the performance of the original algorithm and preserves more details in images.

Index Terms**—** Image denoising, fuzzy set theory, patch-based, fuzzy clustering, fuzzy control

1. INTRODUCTION

Exploiting nonlocal self-similarities in images, nonlocal means filtering introduced by Buades et al. [1] achieves a good denoising result. In [1], each pixel value is estimated by the weighted average of data points in its neighborhood. Each weight is proportional to the similarity between image patches corresponding to the point being processed and data points. Therefore, the nonlocal means filtering can be considered as a patch-based method which assumes that small patches in the neighborhood of an estimation point contain essential process required for denoising.

Combining the patch-based technique with the idea of pointwise adaptive estimation, Kervrann et al. proposed an optimal spatial adaptation for patch-based image denoising [2, 3] to extend the nonlocal means algorithm. In their method, an adaptive neighborhood is chosen iteratively at each spatial position to balance the accuracy of approximation and stochastic errors, and the weights are computed from restored patches obtained at previous iteration. This adaptive selection procedure copes well with spatial inhomogeneities across the image domain. However, there are still several drawbacks in this method. One is that a rough soft Gaussian threshold function is used to make a weight distribution. This Gaussian function would yield an additional estimate bias because the weights of the unrelated data points are not zero. Another problem is that the same control parameter is simply used at different iterative steps, which leads to a poor weight distribution because the patches used to compute weights have variable biases at iterations.

To overcome the drawbacks mentioned above, we propose a novel weighting strategy using fuzzy set theory. Fuzzy set theory was introduced first by Zadeh in [4], which can be seen as a generalization of classical set theory. In the proposed method, we apply fuzzy clustering to determine the weighting function of data points, and adjust the control parameter of the weighting function according to the biases of input patches. Actually, this fuzzy weighting function is a semi-soft threshold function, which implements optimal fuzzy partitions for the patch similarity and sets the weights of the unrelated points as zero. In addition, because some pixels having no similar data points in their neighborhoods would fail in fuzzy clustering, we utilize adjacent data points to correct such pixel values through fuzzy control.

The rest of the paper is organized as follows. In Section 2, we review the related work about the patch-based method and pointwise spatial adaptation. Section 3 describes the fuzzy weighting method for patch-based denoising. Section 4 presents experimental results and analysis. Finally, the paper is concluded in Section 5.

2. RELATED WORK

2.1. Patch-based method

Consider the following noisy image model:

$$y(i) = x(i) + n(i)\,, \quad i \in I\,, \tag{1}$$

where i represents the spatial coordinate of the discrete image domain $I \subset \mathbb{Z}^2$, $y(i)$ is the observed value, $x(i)$ is the true value and $n(i)$ is the noise perturbation assumed to be a signal-independent Gaussian zero-mean random variable with variance σ^2. For a pixel i, the estimated value $\hat{x}(i)$ is computed as a weighted average of data points in its neighborhood [1]:

$$\hat{x}(i) = \frac{\sum_{j \in \Delta_i} L_h(\mathbf{y}_i - \mathbf{y}_j)\, y(i)}{\sum_{j \in \Delta_i} L_h(\mathbf{y}_i - \mathbf{y}_j)}\,, \tag{2}$$

where Δ_i represents the neighborhood of point i, $\mathbf{y}_i$ is the observed image patch centered at point i, $L_h(\cdot)$ are rescaled versions of Gaussian kernel functions. The similarity between two points i and j is measured by the Euclidean distance $\| \mathbf{y}_i - \mathbf{y}_j \|_2$ between the corresponding patches. We can control the level of blurring by varying h.

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2.2. Pointwise spatial adaptation

In [2], the size of the neighborhood Δ_i in (2) is chosen iteratively for each point i to balance the accuracy of approximation and stochastic errors. For the sake of simplicity and computational efficiency, the set of admissible neighborhoods is finite and is chosen as a geometric grid of nested square windows

$$\Phi = \{\Delta_{i,n} : |\Delta_{i,n}| = (2^n + 1) \times (2^n + 1), n = 1, \ldots, N\}, \quad (3)$$

where $|\Delta_{i,n}|$ is the cardinality of $\Delta_{i,n}$ and N is the number of elements of Φ.

At point i, for each iteration n, the iterative estimator and its variance are defined as

$$\hat{x}_{i,n} = \sum_{j \in \Delta_{i,n}} w_{i\sim j,n} y(i), \; v^2(\hat{x}_{i,n}) = \sigma^2 \sum_{j \in \Delta_{i,n}} (w_{i\sim j,n})^2. \quad (4)$$

Notice that $\hat{x}_{i,0} = y_i$ and $v^2(\hat{x}_{i,0}) = \sigma^2$. The weights $w_{i\sim j,n}$ are computed from restored $p \times p$ patches $\hat{\mathbf{x}}_{i,n-1}$ and $\hat{\mathbf{x}}_{j,n-1}$ obtained at iteration n-1 as

$$w_{i\sim j,n} = \frac{L_{\lambda_\alpha}(\mathrm{dist}(\hat{\mathbf{x}}_{i,n-1}, \hat{\mathbf{x}}_{j,n-1}))}{\sum_{k \in \Delta_{i,n}} L_{\lambda_\alpha}(\mathrm{dist}(\hat{\mathbf{x}}_{i,n-1}, \hat{\mathbf{x}}_{k,n-1}))}, \quad (5)$$

where $L_{\lambda_\alpha}(z) = \exp(-z^2 / 2\lambda_\alpha)$, λ_α is a parameter used to control the decay rate of the function, which is chosen as a quantile of a $\chi^2_{p^2,1-\alpha}$ distribution, and $\mathrm{dist}(\hat{\mathbf{x}}_{i,n}, \hat{\mathbf{x}}_{j,n})$ represents a symmetric distance normalized by the variances:

$$\mathrm{dist}^2(\hat{\mathbf{x}}_{i,n}, \hat{\mathbf{x}}_{j,n}) = (\hat{\mathbf{x}}_{i,n} - \hat{\mathbf{x}}_{j,n})^T \mathbf{V}_{ij,n}^{-1} (\hat{\mathbf{x}}_{i,n} - \hat{\mathbf{x}}_{j,n}), \quad (6)$$

where $\mathbf{V}_{ij,n}^{-1} = \mathrm{diag}\left[v^{-2}(\hat{x}_{i,n}^{(k)})/2 + v^{-2}(\hat{x}_{j,n}^{(k)})/2 \right]$ is a $p \times p$ diagonal matrix where the index k is used to denote a spatial position in a patch.

The optimal neighborhood of point i is determined as

$$\Delta_{i,\hat{n}} = \arg \max_{\Delta_{i,n} \in \Phi} \left\{ |\Delta_{i,n}| : |\hat{x}_{i,n} - \hat{x}_{i,n'}| \le \tau v(\hat{x}_{i,n'}) \text{ for all } n' < n \right\} \quad (7)$$

where τ is a positive constant, and the final estimator $\hat{x}(i) = \hat{x}_{i,\hat{n}}$.

3. WEIGHTING METHOD USING FUZZY SET THEORY

3.1. Fuzzy weighting function

In the weighting function (5), unrelated data points have small but nonzero weights. When there are a lot of unrelated points in the neighborhood of the estimated pixel, the additional estimate bias resulting from unrelated points can not be ignored.

In this paper, fuzzy clustering is used to improve the weighting function. If we consider image patch $\mathbf{y}_j$ of $p \times p$ pixels as a vector belonging to $\mathbb{R}^{p^2}$, then these vectors $\mathbf{y}_j$, $j \in \Delta_i$ in a neighborhood of point i form a vector set $\mathbf{Y}_i$. Let fuzzy partition matrix $\mathbf{D} = (d_{t,j})_{c \times |\Delta_i|}$, and the criterion function of fuzzy c-means clustering can be defined as [5]

$$J(\mathbf{D}, \mathbf{U}) = \sum_{t=1}^{c} \sum_{j \in \Delta_i} (d_{t,j})^2 \mathrm{dist}^2(\mathbf{u}_t, \mathbf{y}_j), \quad (8)$$

where $\mathbf{u}_t$ is the prototype of the tth cluster, $\mathbf{U} = [\mathbf{u}_1, \ldots, \mathbf{u}_c]$ is the matrix of cluster prototypes, $\mathrm{dist}(\mathbf{u}_t, \mathbf{y}_j)$ represents a distance between the vector $\mathbf{u}_t$ and $\mathbf{y}_j$, and the element $d_{t,j}$ of the matrix $\mathbf{D}$ represents the degree to which the vector $\mathbf{y}_j$ belongs to the tth cluster and satisfies the following conditions:

$$d_{t,j} \in [0,1], \; \forall t, j, \; \sum_{t=1}^{c} d_{t,j} = 1, \; \forall j, \; \text{and} \sum_{j \in \Delta_i} d_{t,j} > 0, \; \forall t. \quad (9)$$

Given the matrix $\mathbf{U}$, for the vector set $\mathbf{Y}_i$, if there is a partition matrix $\mathbf{D}^*$ which minimizes the criterion function $J(\mathbf{D}, \mathbf{U})$, then $\mathbf{D}^*$ is known as optimal fuzzy c-partitions of the set $\mathbf{Y}_i$ [5]:

$$\mathbf{D}^* = \arg \min_{\mathbf{D}} J(\mathbf{D}, \mathbf{U}). \quad (10)$$

Here, we partition the vector set $\mathbf{Y}_i$ into two clusters, i.e., $c = 2$. One cluster consists of vectors similar to the vector $\mathbf{y}_i$ of point i, and the cluster prototype $\mathbf{u}_1 = \mathbf{y}_i$. The other consists of non-similar vectors, and the prototype $\mathbf{u}_2$ is the vector $\bar{\mathbf{y}}_i$ of the point unrelated to point i. In this way, the weighting function is determined by optimal fuzzy 2-partitions $\mathbf{D}^* = (d^*_{t,j})_{2 \times |\Delta_i|}$:

$$w_{i\sim j} = \frac{d^*_{1,j}}{\sum_{k=1}^{|\Delta_i|} d^*_{1,k}}, \; j \in \Delta_i, \quad (11)$$

where $d^*_{1,j}$ represents the similar degree between the vectors $\mathbf{y}_i$ and $\mathbf{y}_j$, $j \in \Delta_i$. Next, we detail the calculation of $d^*_{1,j}$.

According to (10), for any $j \in \Delta_i$,

1) if there is l ($l = 1$ or $l = 2$) to make $\mathbf{y}_j = \mathbf{u}_l$, then

$$d^*_{t,j} = \begin{cases} 1, & \mathrm{t} = l \\ 0, & \mathrm{t} \ne l \end{cases} \quad (12)$$

2) if there is $\mathbf{y}_j \ne \mathbf{u}_t$ for any t, then

$$d^*_{t,j} = \frac{1}{\sum_{k=1}^{2} \frac{\mathrm{dist}^2(\mathbf{u}_t, \mathbf{y}_j)}{\mathrm{dist}^2(\mathbf{u}_k, \mathbf{y}_j)}}, \; t = 1, 2 \quad (13)$$

Here we introduce a threshold T. If $\mathrm{dist}(\mathbf{y}_i, \mathbf{y}_j) \ge T$, point j is considered to be unrelated to point i. That is to say, in the set $\mathbf{Y}_i$, all the vectors outside an open ball centered at $\mathbf{y}_i$ with radius T are considered to be significantly different from $\mathbf{y}_i$. Therefore, let $\bar{\mathbf{y}}_i$ ($\mathbf{u}_2$) be the vector on the border of the ball, i.e. $\mathrm{dist}(y_i, \bar{y}_i) = T$, and the distance between $\mathbf{y}_j$ and $\bar{\mathbf{y}}_i$ can be defined as

$$\mathrm{dist}(\bar{\mathbf{y}}_i, \mathbf{y}_j) \triangleq T - \mathrm{dist}(\mathbf{y}_i, \mathbf{y}_j). \quad (14)$$

According to (12), (13) and (14), $d^*_{1,j}$ can be calculated as

$$d^*_{1,j} = \begin{cases} 0, & \mathrm{dist}(\mathbf{y}_i, \mathbf{y}_j) \ge T \\ 1, & \mathrm{dist}(\mathbf{y}_i, \mathbf{y}_j) = 0 \\ \dfrac{1}{1 + \dfrac{\mathrm{dist}^2(\mathbf{y}_i, \mathbf{y}_j)}{(T - \mathrm{dist}(\mathbf{y}_i, \mathbf{y}_j))^2}}, & 0 < \mathrm{dist}(\mathbf{y}_i, \mathbf{y}_j) < T \end{cases} \quad (15)$$

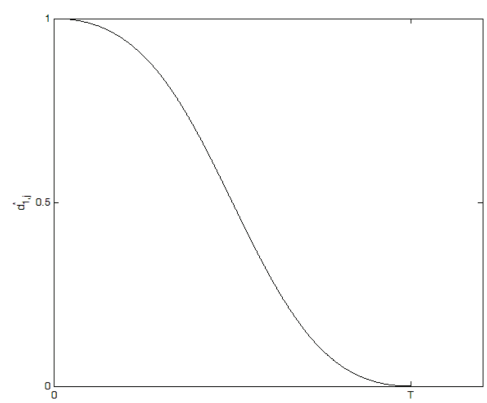


Fig. 1. Semi-soft threshold function.

The graph of the function (15) is shown in Fig. 1. We can see that it is actually a semi-soft threshold function. Compared with

NW_i	N_i	NE_i
W_i	i	E_i
SW_i	S_i	SE_i

Fig. 2. 3×3 neighborhood of point *i*.

the soft Gaussian threshold, it sets the weights of unrelated data points to zero and implements optimal fuzzy partitions of similar data points.

3.2. Adjustment of the control parameter

This section analyses the adjustment of the control parameter T in (15). Because the estimator $\hat{x}_{i,n}$ has the bias $b(\hat{x}_{i,n})$ at iteration n and $b(\hat{x}_{i,n})$ increases with $\Delta_{i,n}$ growing [2], T should have larger value when n grows.

Let $G_{i,n} = (\hat{x}_{i,n} - b(\hat{x}_{i,n}))$, which is normal distributed:

$$G_{i,n} \sim N(0, v^2(\hat{x}_{i,n})) \ . \tag{16}$$

With a multiplier 1/2, we rewrite the distance formula (6) as follows:

$$\begin{aligned}
\mathrm{dist}(\hat{\mathbf{x}}_{i,n}, \hat{\mathbf{x}}_{j,n}) &= \sqrt{\frac{1}{2}\left[\sum_{k=1}^{p^2}\frac{(\hat{x}_{i,n}^{(k)} - \hat{x}_{j,n}^{(k)})^2}{2v^2(\hat{x}_{i,n}^{(k)})} + \sum_{k=1}^{p^2}\frac{(\hat{x}_{j,n}^{(k)} - \hat{x}_{i,n}^{(k)})^2}{2v^2(\hat{x}_{j,n}^{(k)})}\right]} \\
&\qquad \left(\text{assume that } \sum_{k=1}^{p^2}\frac{(\hat{x}_{i,n}^{(k)} - \hat{x}_{j,n}^{(k)})^2}{2v^2(\hat{x}_{i,n}^{(k)})} \ge \sum_{k=1}^{p^2}\frac{(\hat{x}_{j,n}^{(k)} - \hat{x}_{i,n}^{(k)})^2}{2v^2(\hat{x}_{j,n}^{(k)})}\right) \\
&\le \sqrt{\sum_{k=1}^{p^2}\frac{(\hat{x}_{i,n}^{(k)} - \hat{x}_{j,n}^{(k)})^2}{2v^2(\hat{x}_{i,n}^{(k)})}} \\
&\le \sqrt{\sum_{k=1}^{p^2}\frac{(G_{i,n} - G_{j,n})^2}{2v^2(\hat{x}_{i,n}^{(k)})}} + \sqrt{\sum_{k=1}^{p^2}\frac{b^2(\hat{x}_{i,n}^{(k)})}{2v^2(\hat{x}_{i,n}^{(k)})}} + \sqrt{\sum_{k=1}^{p^2}\frac{b^2(\hat{x}_{j,n}^{(k)})}{2v^2(\hat{x}_{i,n}^{(k)})}} \\
&\qquad \left(\text{assume that } \sum_{k=1}^{p^2}\frac{b^2(\hat{x}_{i,n}^{(k)})}{2v^2(\hat{x}_{i,n}^{(k)})} \ge \sum_{k=1}^{p^2}\frac{b^2(\hat{x}_{j,n}^{(k)})}{2v^2(\hat{x}_{i,n}^{(k)})}\right) \\
&\le \sqrt{\sum_{k=1}^{p^2}\frac{(G_{i,n} - G_{j,n})^2}{2v^2(\hat{x}_{i,n}^{(k)})}} + \sqrt{2\sum_{k=1}^{p^2}\left(\frac{b(\hat{x}_{i,n}^{(k)})}{v(\hat{x}_{i,n}^{(k)})}\right)^2} \\
&\qquad \left(\left|b(\hat{x}_{i,n}^{(k)})\right| \le \frac{C_1}{\sqrt{2}}\left|\Delta_{i,n}\right|^{1/2},\ v(\hat{x}_{i,n}^{(k)}) \approx \sigma\left|\Delta_{i,n}\right|^{-\gamma^2/2} \ \text{(see [2])}\right) \\
&\le \underbrace{\sqrt{\sum_{k=1}^{p^2}\frac{(G_{i,n} - G_{j,n})^2}{2v^2(\hat{x}_{i,n}^{(k)})}}}_{\text{First term}} + \underbrace{\sqrt{2}p\left(\frac{C_1}{\sqrt{2}\sigma}\left|\Delta_{i,n}\right|^{\frac{1+\gamma^2}{2}}\right)}_{\text{Second term}}
\end{aligned} \tag{17}$$

where C_1 is a real constant, γ is a constant close to 1, the fist term follows approximately a chi-square distribution with p^2 degrees of freedom, and the second term is determined by the cardinality $\left|\Delta_{i,n}\right|$ of $\Delta_{i,n}$. In the meantime, we have trivially

$$\mathrm{dist}(\hat{\mathbf{x}}_{i,n}, \hat{\mathbf{x}}_{j,n}) \ge \sqrt{\sum_{k=1}^{p^2}\frac{(\hat{x}_{j,n}^{(k)} - \hat{x}_{i,n}^{(k)})^2}{2v^2(\hat{x}_{j,n}^{(k)})}} \ge \sqrt{\sum_{k=1}^{p^2}\frac{(G_{j,n} - G_{i,n})^2}{2v^2(\hat{x}_{j,n}^{(k)})}} \ . \tag{18}$$

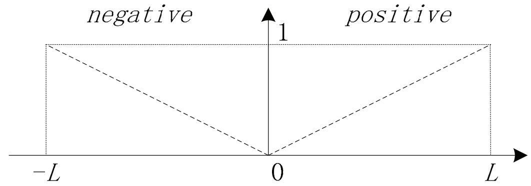


Fig. 3. Membership function *positive* and *negative*.

Therefore, consider the worst case, we set the control parameter

$$T = \sqrt{\lambda_\alpha} + \sqrt{2}p\left(C\left|\Delta_{i,n}\right|^{\frac{1+\gamma^2}{2}}\right), \tag{19}$$

where λ_α is a quantile of $\chi^2_{p^2,1-\alpha}$ and $C = C_1/\sqrt{2}\sigma$. The choice of C and γ will be discussed in Section 4.

3.3. Residual noisy pixel correction with fuzzy control

In the case that some pixels have no similar data point in their neighborhoods, such as corner points, the above fuzzy clustering will be failure and the noise at such points will be left. In order to remove the residual quasi-impulse noise and simultaneously preserve edges in the image, we weight adjacent points through fuzzy control to correct these noisy pixel values.

Consider a 3×3 neighborhood of point *i* as shown in Fig. 2, the gradient at the central point *i* in the direction D ($D\in dir = \{NW, W, SW, S, SE, E, NE, N\}$) is defined as the difference between point *i* and its adjacent point in the direction D, for example,

$$\nabla_{NW}(i) = x(NW_i) - x(i) \tag{20}$$

where $\nabla_{NW}(i)$ represents the gradient in the direction *NW*.

Consider the case of the adjacent point NW_i (similar in other adjacent points). If two of the gradients at point *i*, SW_i and NE_i in the direction *NW* are small, it is safe to assume that no edge is present in the direction *SW-NE*. Based on this idea, we calculate the fuzzy gradient value $\nabla^F_{NW}(i)$ by the following fuzzy control rule [6] to avoid the effect of the adjacent points in edge regions:

IF ($\nabla_{NW}(i)$ is *small* AND $\nabla_{NW}(SW_i)$ is *small*) OR

($\nabla_{NW}(i)$ is *small* AND $\nabla_{NW}(NE_i)$ is *small*) OR

($\nabla_{NW}(SW_i)$ is *small* AND $\nabla_{NW}(NE_i)$ is *small*)

THEN $\nabla^F_{NW}(i)$ is *small*. (21)

The rule is implemented using the minimum to represent the AND-operator and the maximum for the OR-operator. The membership function $m_K(\nabla_D(\cdot))$ for the property *small* is defined as:

$$m_K(\nabla_D(\cdot)) = \begin{cases} 1 - \dfrac{\left|\nabla_D(\cdot)\right|}{K}, & 0 \le \left|\nabla_D(\cdot)\right| \le K \\ 0, & \left|\nabla_D(\cdot)\right| > \mathrm{K} \end{cases} \tag{22}$$

where K is an adaptive parameter related to the noise variance σ^2.

To computer the weights of adjacent points in the correction term *c*, we apply the following two rules [6] to each direction by using the fuzzy gradient values obtained in (21):

C_D^+ : IF $\nabla^F_D(i)$ is *small* AND $\nabla_D(i)$ is *positive* THEN C_D^+ is *positive*; (23)

C_D^- : IF $\nabla^F_D(i)$ is *small* AND $\nabla_D(i)$ is *negative* THEN C_D^- is *negative*. (24)

Fig. 3 shows the linear membership functions for the properties *positive* and *negative*. L represents the number of gray levels.

Table 1. PSNR comparison for the denoising of the gray-scale *Barbara*, *Lena*, *Boats*, *House* and *Peppers* test images with different levels of Gaussian noise.

Image	σ	PSNR		
		SA-NLM [2]	BLS-GSM [7]	Proposed Method
Barbara 512^2	20	30.37	30.27	**31.16**
	30	28.13	28.10	**29.22**
Lena 512^2	20	32.64	32.68	**32.72**
	30	30.90	30.89	**31.05**
Boats 512^2	20	30.12	**30.36**	30.35
	30	28.30	28.51	**28.62**
House 256^2	20	32.90	32.30	**33.10**
	30	31.40	30.50	**31.49**
Peppers 256^2	20	30.59	30.27	**30.80**
	30	28.75	28.29	**28.88**

Finally, the correction term c and the estimated value of residual noisy point i is calculated as follows:

$$\hat{x}(i) = y_{\text{res}}(i) + c \text{ , and } c = \frac{L}{8}\sum_{D\in dir}(C_D^+ - C_D^-) \tag{25}$$

Note that the correction is only performed at a few pixels. Compared with the search of similar patches and the iterative procedure which mainly determine the computational complexity, the running time of this step can be ignored.

4. EXPERIMENTAL RESULTS

In our experiments, all the gray-scale test images are corrupted with zero-mean additive Gaussian white noise with variance σ^2 (σ = 20, 30). The image patch size is fixed at 9×9, i.e. p = 9, the quantile $\lambda_{0.001} = \chi^2_{81,0.999}$ =126.1 and K=3σ^2 [6]. According to [2], an idea choice of the neighborhood will be the largest neighborhood $\Delta_{i,n}$ such that $b(\hat{x}_{i,n})$ is still not larger than $\gamma v(\hat{x}_{i,n})$. Here, we assume appropriately that when n = 3, $b(\hat{x}_{i,n})$ is not larger than $\gamma v(\hat{x}_{i,n})$ exactly and γ = 0.7, which obtains the desired results in experiments. Accordingly,

$$b(\hat{x}_{i,n}) / v(\hat{x}_{i,n}) = C\left|\Delta_{i,3}\right|^{\frac{1+0.7^2}{2}} = 0.7 \text{ ,} \tag{26}$$

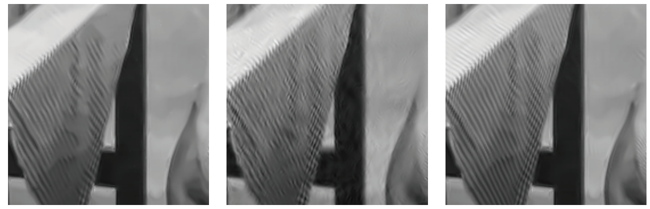
then we get C = 0.0265.

Table 1 shows the PSNR values of the proposed method, the original method, i.e. SA-NLM [2], and BLS-GSM [7]. It can be seen that the proposed method outperforms the other methods, except for *Boats* with σ = 20. In that case, the proposed method has a little less PSNR value than BLS-GSM.

The fragments of *Barbara* (σ = 20) denoised by these methods are shown in Fig. 4. It can be observed that more texture information is preserved in (c) (the proposed method). Fig. 5 shows the denoised *Boats* (σ = 20). We can see that although BLS-GSM gives higher PSNR value, the proposed method has fewer artifacts and better visual quality than BLS-GSM.

5. CONCLUSION

This paper proposed a novel fuzzy weighting method for patch-based image denoising. In this method, we use fuzzy clustering to determine the weighting function to reduce the interference of



(a) (b) (c)

Fig. 4. Fragments of *Barbara* (σ = 20): (a) SA-NLM estimate; (b) BLS-GSM estimate; (c) Proposed method estimate.

(a) (b) (c)

Fig. 5. Fragments of *Boats* (σ = 20): (a) SA-NLM estimate; (b) BLS-GSM estimate; (c) Proposed method estimate.

unrelated data points, and adjust the control parameter of the fuction based on the biases of restored patches. Finally, we use fuzzy control to remove the residual noise. By introducing fuzzy set theory, the proposed method optimizes the weighting distribution of data points. Experimental results show that compared with the original method and other state-of-the-art method, the proposed method has better objective and subjective performance. Future work will focus on the more accurate parameter adjustment.

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