EFFICIENT ANISOTROPIC WAVELET PACKET BASIS SELECTION IN JPEG2000

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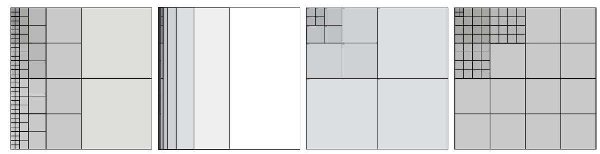
ABSTRACT

JPEG2000 Part 2 allows the application of arbitrary wavelet decomposition structures (wavelet packet bases). Efficient anisotropic wavelet packet basis selection for the coding framework of JPEG2000 has been developed and evaluated. Previous work focused on isotropic wavelet packet basis selection algorithms for JPEG2000, which serves as basis for the performance of the assessment of anisotropic wavelet basis selection. Several cost functions are applied in a top-down anisotropic wavelet packet selection scheme. Our evaluations employ state-of-the-art quality assessment tools supplementary to PSNR evaluations.

Index Terms— Image coding, rate-distortion optimization, wavelet packet bases, JPEG2000

1. INTRODUCTION

Wavelet packet bases (WPBs) [1] offer to adapt the wavelet transform to the source signal (image) characteristics and thus improve the compression performance (anisotropic are even more flexible). WPBs are an alternative to the classical dyadic wavelet decomposition (also referred to as pyramidal) and allow to further decompose all subbands and not just the LL subband, which leads to an enormous number of possible WPBs. The application of an adapted wavelet packet basis (WPB) for image compression purposes has been subject to investigation since the introduction of the first feasible selection technique called "best basis algorithm" (BBA)[1]. A brute-force search for the best WPB is computationally infeasible even for moderate maximum decomposition depths; for 2-D signals and isotropic wavelet decomposition depth d_{iso} of 4 (corresponds to an anisotropic depth d of 8) there are 4.9×10^{19} possible isotropic WPBs and even more anisotropic WPBs (AWPBs), namely 8.4×10^{94}. In Figure 1 examples of WPBs are shown, in Fig 1(b) the best anisotropic WPB for the artificial image is illustrated. The approach of [1] employs a rate-independent basis selection scheme, which is based on various additive cost functions which only estimate the actual coding cost and thus this scheme is not optimal in a RD (rate-distortion) sense. The employment of rate-distortion optimization criteria for WPB selection has been first demonstrated for classical wavelet-based compression schemes [2].



(a) Artificial (b) Aniso Artificial (c) Lena (d) WSQ

Fig. 1. Best WPBs for specific images and the WSQ-WPB

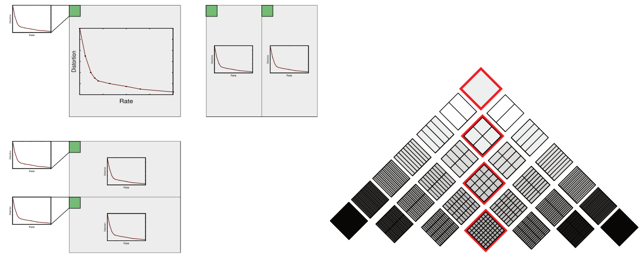
For certain compression schemes, a certain source image, and a specific target bitrate, the optimal WPB can be computed in feasible time. In previous work [3], a proprietary wavelet block-based compression scheme has been introduced incorporating the principle of [2] for WPB selection. Subsequent works of the authors [4] propose fast and efficient basis selection methods for their proprietary compression system with a lower computational complexity connected with a little loss of rate-distortion performance in comparison with the original work.

Interestingly, only very recent works discuss RD optimal (RDO) and efficient isotropic WPB selection in JPEG2000 [5, 6], while anisotropic WPB (AWPB) selection has not yet been discussed. Thus this works closes this gap and gives an in-depth analysis and experimental results for AWPB in JPEG2000.

The most relevant aspects of JPEG2000 for this work are briefly reviewed in Sect. 2. Sect. 3 presents the approach of rate-distortion optimal AWPB selection and in-depth discusses its complexity. More efficient AWPB selection schemes are discussed in Sect. 4, several more efficient cost functions are presented and and an analytical model for computational complexity on the basis of the average decomposition depth is given. Experimental results are presented in Sect. 5. Sect. 6 concludes the paper.

2. OVERVIEW OF JPEG2000

JPEG2000 Part 1 employs a pyramidal wavelet transform and uses the EBCOT-algorithm (embedded block coding with optimized truncation) to encode the wavelet coefficients. JPEG2000 Part 2 [7] allows the application of anisotropic



(a) Rate-distortion statistics of codeblocks a subband and its potential children: the anisotropic case (b) Necessary subbands of isotropic (red) vs. anisotropic best basis selection

WPBs. The wavelet coefficients of a subband are grouped in rectangular blocks (codeblocks), which are coded independently to separate bitstreams. A JPEG2000 file (codestream) consists of a main header followed by several packets. Each packet increases the decoded image quality. Each packet belongs to a certain quality layer and resolution. The embedded bitstream of a single codeblock has several potential truncation points, i.e., each codeblock has a separate RD function. The goal of an encoder is to arrange the bitstream data of all codeblocks in an RD optimal manner, i.e. to find the truncation points that minimize the distortion for a given rate. An optimally coded JPEG2000 codestream can be obtained by selecting an individual rate for each codeblock.

3. BEST ANISOTROPIC WAVELET PACKET BASES

Anisotropic wavelet packet bases (AWPB) allow to decompose a subband horizontally or vertically, while isotropic wavelet packet bases only allow a decomposition in both directions, i.e., horizontally and vertically simultaneously. Figure 2(a) illustrates a subband and its two possible decompositions (vertical and horizontal) and the RD statistics of the subbands. The RD statistics of a subband are derived from the RD statistics of its codeblocks. The root of an anisotropic wavelet packet decomposition tree represents the image, i.e., the root subband. A subband has either no or two horizontal children or two vertical children, which are derived by horizontal or vertical wavelet decomposition. For RDO AWPB, a parameter λ is selected and the AWP decomposition tree is visited in bottom-up fashion, the cost of a subband is determined according to the parameter λ (basically the RD point with a slope smaller than λ is selected) and it is compared to the cost of its children. The decomposition with the smallest cost is selected, which overall selects the best AWPB (minimum distortion) for a given image and a certain rate.

There are substantially more anisotropic decomposition trees than isotropic wavelet packet bases. The number of anisotropic wavelet packet bases is given by the following recursion ($A_{-2} = A_{-1} = 0$):

$$A_d = 1 + 2A_{d-1}^2 - A_{d-2}^4$$

At an anisotropic depth of 1 there are 3 possible anisotropic WPB (no decomposition, vertical horizontal), at an anisotropic depth of 2 there are already 18 bases (for the comparable isotropic depth of 1 we have only 2 possible isotropic WPB), at an anisotropic depth of 4 there are 540273 anisotropic WPB (compared to 17 isotropic WPB for the comparable isotropic depth of 2), at an anisotropic depth of 6 there are $\approx 3.75 \times 10^{23}$ bases (isotropic: 83522), and at an anisotropic depth of 8 there are $\approx 8.4 \times 10^{94}$ (isotropic: $\approx 4.9 \times 10^{19}$).

3.1. Complexity of Best AWPB Selection

RDO-AWPB selection is substantially more complex compared to isotropic WPB selection. This is reflected in its computational complexity estimate which is in $\mathcal{O}(d^2 \times n)$ (even the most efficient implementation), while isotropic WPB selection is in $\mathcal{O}(d_{iso} \times n)$. Note that $d_{iso} = \frac{d}{2}$. Figure 2(b) illustrates the necessary subbands for best basis selection for a isotropic decomposition depth of 3 which corresponds to an anisotropic decomposition depth of 6.

The asymptotic complexity of isotropic RDO WPB selection for a maximum decomposition depth d and an n-element signal is in $\mathcal{O}(d_{iso} \times n)$ and also in $\mathcal{O}(n\log n)$, because d_{iso} is bounded by $\log n$, which is the maximal decomposition depth. The asymptotic complexity of anisotropic RDO WPB selection for a maximum decomposition depth d and an n-element signal is in $\mathcal{O}(d^2 \times n)$ and also in $\mathcal{O}(n\log^2 n)$. Figure 2(b) illustrates the WPBs that have to be fully JPEG2000 compressed in order to obtain all subbands, e.g., the complexity of RDO WPB selection with $d_{iso} = 5$ is roughly the same as the complexity of anisotropic RDO WPB selection with $d = 2$. The the complexity of anisotropic RDO WPB selection with $d = 3$ already costs 10 full JPEG2000 compressions, and 1.66 times the complexity of isotropic RDO WPB selection with $d_{iso} = 5$.

4. MORE EFFICIENT AWPB SELECTION

For practical application efficient wavelet packet basis selection is fundamental. An approach towards cutting the cost of basis selection is to refrain from using the actual coding costs (as done in RDO WPB selection) and employ computationally more efficient cost functions. If these cost functions are additive then the determined basis is optimal in the sense of the cost function. However, this optimality in terms of a cost function does not necessarily go hand in hand with optimality in terms of RD performance, our main goal in the scope of this work. The BBA with cost functions still is computationally complex, especially for anisotropic WPBs, as all the subbands have to be computed, i.e., the full decomposition tree has to be computed, which is then visited in a bottom up fashion. The computational complexity of the anisotropic BBA is in $\mathcal{O}(d^2 \times n)$, while the isotropic BBA is in $\mathcal{O}(d_{iso} \times n)$. For the BBA a cost for every possible subband has to be computed, the number of subbands is far more in the anisotropic case than in the isotropic case (see Fig. 2(b)).

Computational complexity is significantly reduced if the decomposition tree is visited in a top-down fashion and if the subbands are only computed if necessary. This algorithm is briefly sketched:

- Decompose a subband (in case of anisotropic WPBs horizontally and vertically).
- Compute costs of the subband and its children.
- If subbands cost is minimal, stop, else evaluate each child of the minimum branch.

This top-down algorithm does not guarantee finding an optimal basis for an additive cost function, however, the optimality in terms of a cost function does not imply optimality in a rate-distortion sense anyways. Alternatively to the optimal wavelet packet basis in a rate-distortion sense with the actual coding bitrate as cost function it has often been proposed to employ simpler cost functions for best basis selection (although these may not result in best bases in a rate-distortion sense). In this section we will present common cost functions. Let c_i represent the value of the coefficients of a subband. The following additive cost functions are calculated, L1-norm: $\sum_i |c_i|$, L2-norm: $\sum_i c_i^2$, LogE - log energy metric: $\sum_i \ln(c_i^2)$, and EIC - entropy information cost or Shannon metric [1]): $\sum_i c_i^2 \ln(c_i^2)$. Furthermore, we employ an entropy based cost function, which basically computes an entropy estimate for the quantized coefficients of a subband. The coefficients are quantized, i.e., divided by q and rounded to the next integer, their distribution statistics are calculated and these data is used to compute an entropy estimate. This entropy estimate is weighted with / multiplied by the number of coefficients in the subband. We refer to this cost function as weighed entropy estimate, Eq, where q indicates the divisor in the quantization process. We have extended the basic entropy cost by a penalty for subbands that become smaller then the codeblock size (a constant $k = 2$ is added for every codeblock contained in the subband, which can be interpreted as two bit extra coding cost). Additionally we consider the JPEG2000 specific cost of signalling the quantization information for a subband, i.e., the signalling cost is added to the subband cost. For quantization type expounded the cost is 16 (2 bytes in the main header) and for quantization type reversible the extra cost is 8 (1 byte in the main header) [8].

4.1. Complexity

The main impact on the complexity of all top-down approaches is the average decomposition depth, $\bar{d}$, of the selected AWPB, which is highly source image dependent (see figure 1), e.g., $\bar{d} \approx 2.66$ ($\bar{d}_{iso} \approx 1.33$) for the Lena image, the isotropic WPB of the Artificial image has a $\bar{d}$ of 3.88 and the anisotropic of 1.94. JPEG2000 Part 2 imposes an upper bound for $\bar{d}$ of $6.\dot{6}$. Listing 1 gives an algorithm to determine the average decomposition depth of an encoding of an AWPB, a 0 indicates no decomposition, a 1 indicates a decomposition and is followed by a bit indicating horizontal or vertical decomposition. An upper bound of the complexity of a top-down approach for an image with a WPB with $\bar{d}$ is $C_T(\bar{d}) \leq (\bar{d}+1) \times (\mathcal{W} + \mathcal{F}) + \mathcal{B} + \mathcal{D}$, where $\mathcal{C}$ is the cost of JPEG2000 coding the image with one level wavelet decomposition and $\mathcal{D}$ represents the fixed time, e.g., Java start-up time and file IO (timing measurements are given in Sec. 3.1). $\mathcal{C}$ is mainly comprised of the wavelet transform cost, $\mathcal{W}$, the coefficient coding, $\mathcal{B}$, and $\mathcal{D}$. $\mathcal{F}$ denotes the cost of evaluating the cost function on all coefficients. The performance improvements of a top-down approach rely on $\bar{d} + 1 << d$ and $\mathcal{F} << \mathcal{B}$.

Algorithm 1 Average decomposition depth of an anisotropic wavelet packet tree

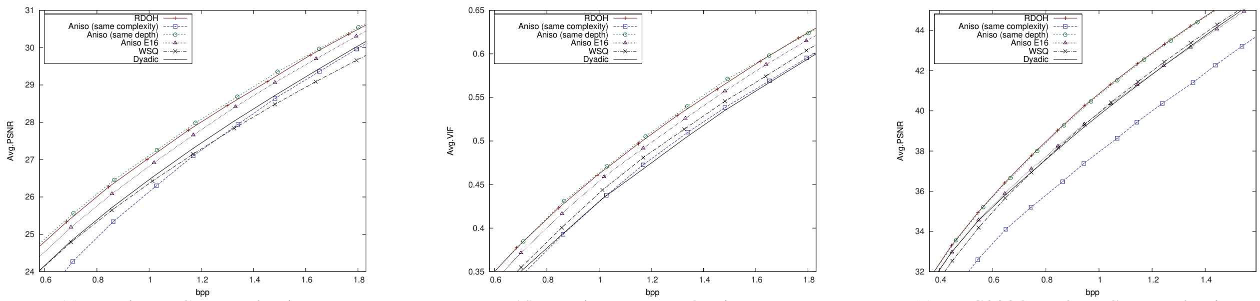
```
function A(s)
    b = s.pop_front()
    if b == '0' then
      return 0
    else if b == '1' then
        s.pop_front()
      return 1 + (A(s) + A(s))/2
    end if
end function
```

In our implementation top-down best basis search complexity is almost the same as that of a single full JPEG2000 compression (efficient).

5. EXPERIMENTAL RESULTS

The results have been produced with a custom implementation, which is based on the JJ2000 reference implementation. For lossy compression the 9-7 irreversible filter with quantization type expounded has been employed. A maximum decomposition depth of 5 (if not explicitly stated otherwise), one quality layer, and 64x64 codeblocks have been employed. Additionally to the well-known PSNR we have assessed the image quality with state-of-the-art metrics, such as the VIF, MSSIM, and SSIM [9]. The Matlab package metrix_mux has been employed.

We present results for highly textured data (Brodatz database) and for fingerprint data (FVC2004 database). There are two ways to compare anisotropic WPB to isotropic WPB, based on the complexity to determine the basis or based on the complexity to compress the image with the given basis. If we follow the first approach (same selection complexity), we have to compare isotropic RDO WPB selection (decomposition depth 5) to anisotropic wavelet packet selection with a joint decomposition depth of 2, as in both cases the complexity is roughly equal (about six full JPEG2000 compressions).



(c) Brodatz: PSNR evaluation (d) Brodatz: VIF evaluation (e) FVC2004 DB4B: PSNR evaluation

Fig. 2. Rate-quality performance

If we choose the second approach, i.e., equal complexity in terms of compression with the computed optimal WPB, the equivalent anisotropic WPB has a maximum joint decomposition depth of 10 (for the isotropic decomposition depth 5). The complexity of anisotropic RDO WPB selection would roughly correspond to 66 full JPEG2000 compressions. Thus the complexity is too high for practical application and even straight forward computation. In order to estimate the performance of such deep decompositions we give the best performance of all results, i.e., from cost functions in the anisotropic case and the isotropic RDO. Given the results of the best performance of cost functions in the isotropic case this approach is well justified [6].

At the same complexity anisotropic WPB selection does not lead to competitive results at all (see Fig. 2(c) and Fig. 2(e)). At the same depth the good results of isotropic RDO WPB (labelled RDOH) can even be improved. However, the increased complexity may not justify the necessary computational effort in some application scenarios. Similar results are achieved with state-of-the-art quality metrics, Fig. 2(d) shows the result for the VIF. The top-down approach with our entropy based cost functions (with the JPEG2000 specific cost) performed best (with a q of 16).

6. CONCLUSION

Our algorithms enable the efficient selection of anisotropic WPB in the JPEG2000 coding framework. In terms of compression performance our results show that for highly textured data, anisotropic wavelet packet bases perform significantly better than the dyadic decomposition. Anisotropic WPB slightly outperform isotropic WPB. For RDO selection the slight performance gain has to be traded off for a significantly increased computational complexity. However, our proposed top-down approach with the improved cost functions induces a very small additional complexity and improves compression performance. The improved PSNR performance also leads to improvements in terms of state-of-the-art objective image quality assessment.

7. REFERENCES

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