

A LASSO BASED ENSEMBLE EMPIRICAL MODE DECOMPOSITION APPROACH TO DESIGNING ADAPTIVE CLUTTER SUPPRESSION FILTERS

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ABSTRACT

Accurate estimation of the blood flow velocity in ultrasound imaging is an important tool for medical diagnostics. In this paper, we adopt an improved empirical mode decomposition (EMD) framework called ensemble EMD (EEMD). To reduce the errors caused by the outliers in data when using a uniform weight in conventional EEMD, a regularized LASSO EEMD algorithm is proposed to solve for the multiple regression weights. An adaptive clutter rejection filter can then be designed to remove the clutter components. According to our simulation study, the proposed LASSO EEMD approach performs better than the state-of-the-art eigen-based and EMD method in estimating the blood flow velocity. Although the LASSO EEMD derived filter only achieves slightly better results than the cubic regression derived filters at most part of the simulated blood flow center frequencies, the proposed LASSO EEMD algorithm achieves much improved performance over cubic regression at extreme cases when the blood flow center frequency is close to or much higher than that of the clutter.

Index Terms— Blood flow velocity estimation, clutter rejection, empirical mode decomposition, LASSO, ridge regression

1. INTRODUCTION

As a non-invasive and high-level vascular visualization tool, ultrasound color flow imaging has been used for various types of cardiovascular flow studies over the past twenty years [1]. It is important for these images to provide accurate blood flow velocity estimation in medical diagnostics. As demonstrated in several evaluation studies, failure to provide accurate flow information may lead to an increased risk of misdiagnosis and cause assessment difficulties during long-term monitoring of patients [2]. However, there are several factors that tend to reduce the accuracy of blood flow velocity estimation, namely: (i) the clutter signal originating from scattering sources, such as tissues and vessel walls, makes the power ratio between the clutter and blood flow signals extremely large, often up to 40dB or more, and (ii) the available sample number of the Doppler signals is often very limited, usually less than 20 pulses.

To suppress the strong clutter, polynomial regression filter achieves better performance than the conventional digit filters because of its time-varying characteristics [3]. However, it is not adaptive because it is often sensitive to the selection of the order of the polynomials. Furthermore, when the clutter frequency shift is large, the regression curve fails to approximate the real clutter, even for clutters with light powers. To alleviate the above difficulties, an adaptive filter is needed. An eigen-based clutter filter generates the eigenvectors [4] or the orthogonal vectors of singular value decomposition [5] of the autocorrelation matrix. These

orthogonal vectors are used as the basic function of the clutter subspace. Then the estimated clutter from the linear combination of these basis functions is removed from the input. These methods are carefully-designed in theory and perform well when the clutter is non-stationary or consisted of multi-frequency components. It has been demonstrated though that the above eigen-based methods are sensitive to the choice of the clutter subspace dimension. So a data-driven, frequency-energy based decomposition method to suppress the clutter is required.

As a noise-assisted nonlinear and non-stationary data decomposition method, empirical mode decomposition (EMD) and ensemble EMD (EEMD) have been explored to solve the above problem [6]. Different applications have shown the effectiveness of this versatile time-frequency analysis tool. However, each intrinsic mode function (IMF) is often represented as the average of corresponding components at different experiment trials. This fixed uniform weight is not a reasonable choice since the existence of outliers in data. In this paper a regularized EEMD to reduce decomposition errors, called LASSO EEMD, is proposed. It can further be shown that the results obtained with LASSO EEMD and EEMD are asymptotically equivalent. Based on the proposed LASSO EEMD, a novel adaptive clutter rejection filter can be designed. Overall the LASSO EEMD derived filter achieves more accurate blood flow velocity estimation over the state-of-the-art eigen-based and conventional EMD filter. Although the LASSO EEMD derived filter only achieves slightly better results than the cubic regression derived filters at most part of the simulated blood flow center frequencies, the proposed LASSO EEMD algorithm achieves much improved performance over cubic regression at extreme cases when the blood flow center frequency is close to or much higher than that of the clutter.

2. LASSO EEMD

In the following we briefly review the main concept of EEMD and describe the proposed LASSO EEMD framework in detail. Finally, an adaptive clutter rejection filter is designed.

2.1. Ensemble Empirical Mode Decomposition (EEMD)

EMD is an adaptive time-scale choosing scheme on extreme points which is realized as a "sifting" process [7]. The basis function obtained in such a way is complete and almost orthogonal. In EMD, an intrinsic mode function (IMF) is obtained by subtracting from the input signal the average signal of the upper and lower envelopes of the input. The time-scale of each IMF is data-driven and increasing. Based on this property, the first IMF represents the fastest-changing component, while the last IMF and the residual are corresponding to the slowest-varying component often with the highest energy. Therefore EMD is regarded as a frequency-energy based signal decomposition method such that the input signal x with length L is expressed

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as a vector:

$$\mathbf{x} = \sum_{i=1}^n c_i + r_n, \quad (1)$$

where c_i denotes the i^{th} IMF and r_n is the residual. Much attention has been paid to EMD once it was proposed. However, the major drawback of EMD is the frequent appearance of mode mixing which is defined as either a single IMF consisting of oscillations of dramatically disparate time scales, or a component of a similar time-scale residing in different IMFs. It could not only cause serious aliasing in the time-frequency distribution, but also make physical meaning of the individual IMF unclear. Specifically, if the frequency of a distortion component is very close to that of an expected component, such mode mixing often drastically degrades the filter performance.

To solve this problem, an adaptive data analysis method, called ensemble empirical mode decomposition (EEMD), was introduced and the robustness of the original EMD algorithm was significantly improved. The EEMD procedure is realized in the following steps:

- i) Add a white noise sequence to the targeted data $\mathbf{x}_i = \mathbf{x} + e_i$;
- ii) Decompose the revised data into IMF components;
- iii) Repeat the previous two steps with different white noise series in each trial;
- iv) Obtain the result as the mean of the N corresponding IMFs:

$$c_j = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N c_{jk}. \quad (2)$$

When $N = 1$, the EEMD is regarded as EMD. Experimental results demonstrate that EEMD can ease the problem of mode mixing [8]. However, a challenging issue is that the fixed uniform weight for each element obtained by different trials is not as effective due to the presence of outliers. To simplify the presentation we use the following notations:

$$Y = (c_1 \ c_2 \ \cdots \ c_n)^T, \quad (3)$$

$$Z = \begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1N} \\ c_{21} & c_{22} & \cdots & c_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nN} \end{pmatrix}. \quad (4)$$

2.2. LASSO EEMD and Convex Programming

Next we propose a regularized framework reduce the estimation errors of multiple regression weights often caused by data outliers. It is called LASSO EEMD in short. For input signal $\mathbf{x}$ with length L , the relationship between the result of EMD and that of the general EEMD is modeled with a standard multiple linear regression model as:

$$Y = Z\beta + e, \quad (5)$$

where β is $N \times 1$ weight coefficient and unknown. If each element of β is unique and equal to $1/N$, the model is simply the conventional EEMD shown in Eq. (2).

Based on the unbiased and minimum variance properties, the solution to Eq. (5) can be simply expressed as:

$$\hat{\beta} = (Z^T Z)^{-1} Z^T Y, \quad (6)$$

where $\hat{\beta}$ is the well-known least squares estimate, which is a good one if the correlation matrix $Z^T Z$ is nearly a unity matrix with T

denoting the transpose of a matrix. However, if $Z^T Z$ is not nearly a unity one, the least squares solution in Eq. (6) is sensitive to outliers in data. To reduce the effect of outliers, a regularization term is usually added to the objective function as follow,

$$\begin{aligned} \hat{\beta} &= \arg \min_{\beta} \frac{1}{2} E(e^T e) + \lambda \sum_{i=1}^N |\beta_i|^p \\ &= \arg \min_{\beta} \frac{1}{2} (Y - Z\beta)^T (Y - Z\beta) + \lambda \sum_{j=1}^N |\beta_j|^p. \end{aligned} \quad (7)$$

When $p = 1$, it is known as LASSO [9]. Although a closed form solution is available based on ridge regression ($p = 2$) [10], LASSO achieves less error for the general convex optimization problem. However, there is no closed form solution to Eq. (7). For the j^{th} element of β , β_j is often updated in an iterative manner by solving

$$\begin{aligned} \hat{\beta}_j &= \arg \min_{\beta_j} \left[\frac{1}{2} \sum_{i=1}^n \left(y_i - \sum_{j=1}^N \beta_j z_{ij} \right)^T \left(y_i - \sum_{j=1}^N \beta_j z_{ij} \right) \right. \\ &\quad \left. + \lambda \sum_{j=1}^N |\beta_j| \right]. \end{aligned} \quad (8)$$

The solution can be computed explicitly in the first order case by taking the derivative of the bracket term in the right hand side of Eq. (8)

$$\begin{aligned} 0 &= - \sum_{i=1}^n z_{ij}^T (y_i - z_{ij} \beta_j) + \lambda \text{sign}(\beta_j) \\ &= -a + b\beta_j + \lambda \text{sign}(\beta_j), \end{aligned} \quad (9)$$

where

$$a = \sum_{i=1}^n z_{ij}^T y_i, b = \sum_{i=1}^n z_{ij}^T z_{ij}.$$

By Karush-Kuhn-Tucker Theorem [11], if $a \geq 0$,

$$\beta_j = \frac{a - \lambda \text{sign}(\beta_j)}{b} = \begin{cases} \left(\frac{a-\lambda}{b} \right) \text{sign} \left(\frac{a}{b} \right) & a \geq \lambda \\ 0 & \text{otherwise} \end{cases}, \quad (10)$$

if $a < 0$,

$$\beta_j = \frac{a - \lambda \text{sign}(\beta_j)}{b} = \begin{cases} \left(\frac{-a-\lambda}{b} \right) \text{sign} \left(\frac{a}{b} \right) & -a \geq \lambda \\ 0 & \text{otherwise} \end{cases}. \quad (11)$$

Combining Eqs. (10) and (11), the final solution is solved as:

$$\hat{\beta}_j = \left(\left| \frac{a}{b} \right| - \frac{\lambda}{b} \right)_+ \text{sign} \left(\frac{a}{b} \right), \quad (12)$$

$$\text{where } (x)_+ = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}.$$

Based on the obtained β , the input signal is decomposed as

$$\mathbf{x} = \sum_{i=1}^n \sum_{j=1}^N \hat{\beta}_j c_{ij}. \quad (13)$$

2.3. Clutter Suppression Filter Design

In this following section, an adaptive clutter suppression filter is designed based on the proposed LASSO EEMD solution obtained in Eq. (12). The input Doppler signal $\mathbf{x}$ is composed of

$$\mathbf{x} = \mathbf{b} + \mathbf{c} + \mathbf{n}, \quad (14)$$

where $\mathbf{b}$ is the blood flow component, $\mathbf{c}$ denotes the clutter and $\mathbf{n}$ is the noise [12]. The length of input signal is $L = 16$. So the number of IMFs is determined by the logarithm of signal length, which is 3 (excluding the residual) in the whole proceeding of the LASSO EEMD filter design.

$$\mathbf{x} = c_1 + c_2 + c_3 + r. \quad (15)$$

In this representation, the first IMF c_1 represents the high frequency blood flow component. While the last IMF c_3 and residual r are the low-frequency component with the strongest energy. c_3 is regarded as the clutter and to be removed from the input signal by the clutter suppression filter. The second IMF, c_2 , is regarded as the mixture of the blood flow and clutter, which is then decomposed by LASSO EEMD again. Another set of three IMFs are then obtained. Then the same procedure is repeated until enough components are obtained. At each layer of the decomposition, the first IMF is summed with the previous blood flow component. Finally the output $\mathbf{u}$ after filtering is formalized as

$$\mathbf{u} = 1c_1 + 2c_1 + \dots + kc_1, \quad (16)$$

where kc_i represents the i^{th} IMF at the k^{th} LASSO EEMD implementation stage and $1c_1 = c_1$.

3. SIMULATION AND EXPERIMENTAL RESULTS

As stated in previous studies [13], the distribution of the frequency and power for clutter, blood flow signal, and low level random noise can be summarized in Figure 1. It is found that the clutter is usually of low frequency and high power with a narrow bandwidth. On the contrary, the flow component is in high frequency and at low power. Furthermore the random noise is with a wide bandwidth but low power. So the simulation signals are carefully designed based on the characteristics of these three components. In the following set of experiments, the simulation model utilized here is originated from the above-mentioned Doppler mask. To keep the input parameters manageable, some parameters are given and fixed throughout the simulation, while others of a great interest are varied. Table 1 lists the parameters and their values. In the following, all frequency values are stated as fractions of the pulse repetition frequency (PRF). Of special interest is the row labeled "flow center frequency" which varies from 0.075PRF, a value close to the fixed clutter center frequency of 0.01PRF, to 0.5PRF, a value much larger than the fixed value of 0.01PRF. A total of 18 flow center frequencies were used.

The velocities estimated respectively by the regression filter of different orders, eigen-based filter of different clutter subspaces, EMD and LASSO EEMD filters are plotted in Figure 2 as a function of the varying blood flow center frequency as described in the simulation. For the eigen-based filters, two methods of different clutter subspace dimension are presented. In the simulation, the dimension of the clutter subspace is spanned by 3 and 4 basis functions, which are denoted as 'eigen-based3' and 'eigen-based4', respectively, throughout the remainder of the paper.

It was noted that the quadratic regression filter failed to suppress the clutter effectively. The cubic regression could approximate

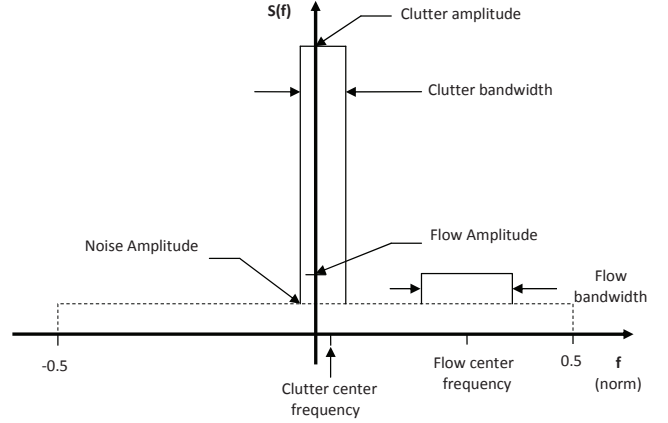


Fig. 1. Frequency distribution used for Doppler signal simulation

Table 1. Parameters Used for the Simulation

Parameter	Value(s)
clutter bandwidth	0.02 PRF
clutter center frequency	0.01 PRF
clutter-to-flow signal power ratio	40 dB
flow bandwidth	0.1 PRF
flow center frequency f_b	0.075 to 0.5 PRF in the steps of 0.025 PRF
flow-to-noise power ratio	10 dB
number of sample per Doppler ensemble	16

the clutter better than quadratic regression with less deviation. Although the filter with low-dimension clutter subspace achieved more accurate velocity estimation than the those with high-dimension clutter subspace, the eigen-based methods caused negative deviation for blood flow velocity estimation. Compared with the above quadratic regression and eigen-based methods, EMD filter showed a better performance. However, it caused large negative deviation when the center frequency of the flow component was close to that of the clutter. As a result, the filtered output signal contained some clutter information which degraded the filter effect. Compared with the EMD filter, the LASSO EEMD derived filter achieved more accurate velocity estimation. Furthermore, the LASSO EEMD reduced the effect of mode mixing which led to the presence of the clutter and blood components in different IMFs. On the other hand, the LASSO EEMD filter produced more IMFs which led to a filter that achieved a better separation. To clearly demonstrate these results, we calculated the mean absolute velocity estimation error $e_{overall}$ over the all the competing methods, and list them in Table 2.

$$e_{overall} = \frac{|f_b - \hat{f}_b|}{N_b}, \quad (17)$$

where $\hat{f}_b$ is the estimated flow center frequency and N_b is the number of simulated flow center frequencies, which was 18 in our case.

It was also found that the LASSO EEMD derived filter achieved some improvements in blood flow velocity estimation over all competing filters. Although LASSO EEMD only obtained a slightly better result than that in cubic regression filters for the overall mean absolute error, it outperformed cubic regression when the center frequencies of the blood flow and the clutter are close to each other.

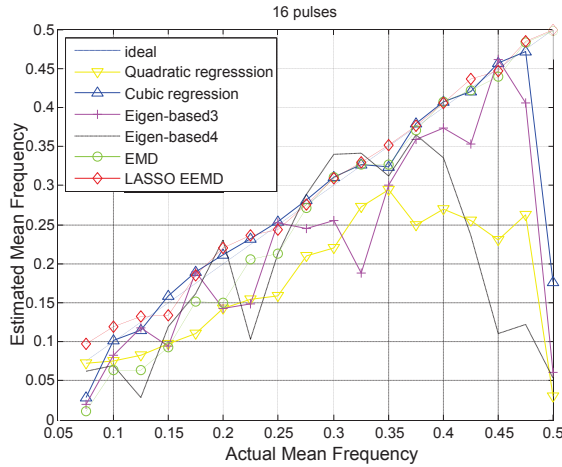


Fig. 2. Blood flow velocity estimation for narrowband clutters

Table 2. Overall Mean Error comparison

Methods	Overall Mean Error
Quadratic regression	1.3034
Eigen-based4	1.0828
Eigen-based3	0.6724
EMD	0.4154
Cubic regression	0.1730
LASSO EEMD	0.1488

This is shown in Table 3, where we pinpoint a few frequency components to show the effectiveness of the LASSO EEMD derived filters for the center frequency of the blood flow with values at 0.075, 0.275 and 0.5 PRF. Comparing the values listed in the rows labeled as "Cubic regression" and "LASSO EEMD", it clearly showed that LASSO EEMD gained a larger improvement than cubic regression when the center frequency of the blood flow was 0.075 and 0.5 PRF, two extreme situations for the flow center frequency either very close to or much larger than the clutter center frequency.

4. CONCLUSION

In this study, a multiple regression model for general EEMD is first established. Next a regularized LASSO EEMD is proposed to reduce the impact of outliers in the data. Finally a standard convex optimization procedure is formulated to obtain the multiple regression weights so that an adaptive clutter suppression filter can be designed.

Experimental results demonstrate that the proposed LASSO EEMD derived filter achieves less velocity estimation errors when

compared with the filters obtained with other competing methods across the entire range of the flow center frequencies simulated. For the overall mean error, the LASSO EEMD derived filters only performs slightly better than cubic regression. However for low and high flow center frequencies LASSO EEMD achieves much better results than cubic regression.

The proposed filter has a great potential to numerous fields of applications. It does not only suppress the clutter, but also removes other category of noise from Doppler imaging.

5. REFERENCES

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Table 3. Pinpoint Frequency Components Error Comparison

Methods	0.075(PRF)	0.275(PRF)	0.5(PRF)
Quadratic regression	0.0020	0.0647	0.4701
Eigen-based4	0.0130	0.0134	0.4467
Eigen-based3	0.0549	0.0301	0.4399
EMD	0.0639	0.0043	0.0020
Cubic regression	0.0470	0.0063	0.3229
LASSO EEMD	0.0221	0.0001	0.0001