NON-RIGID POINT SET REGISTRATION: A BIDIRECTIONAL APPROACH

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ABSTRACT

In this paper we present a novel point set registration algorithm based on the robust Gaussian Mixture Model(GMM). We take advantage of a robust estimation to weigh the noise component in GMM. Moreover, a bidirectional EM process is introduced to model outliers in both point sets in contrast to traditional methods. The performance of the method is demonstrated and validated in carefully designed synthetic data and point sets extracted from medical images. Results show that the proposed method can improve the robustness and accuracy as compared to the traditional registration techniques.

***Index Terms*—** GMM, EXPECTATION MAXIMIZATION(EM), ROBUST ESTIMATION, NON-RIGID REGISTRATION

1. INTRODUCTION

The registration of point set is a fundamental problem in computer vision, medical image analysis, pattern recognition areas and has been extensively studied in the motion tracking, shape matching and image interpolation. The goal of registration is to find meaningful correspondence between two different point sets and determine the transformation that maps one set to the other. These point sets are extracted from input images representing feature points in these image.

The point set registration has attracted much attention in the last two decades. Many algorithms have been proposed. One of the commonly used methods is the Iterative Closest Point(ICP) algorithm[1], which iteratively assigns correspondence based on the Euclidean Distance and finds the least squares based transformation related to these point sets. Because ICP assigns the definite point-to-point correspondence in each iteration, it is easy to get stuck in local minima. So ICP is usually applied to rigid registration.

Several algorithms have been introduced to treat the non-rigid registration problem. The Robust Point Matching(RPM) improves the performance in contrast to the ICP by adopting soft assignment and deterministic annealing techniques. However, the RPM is not really based on the probability. Although it uses the similar Expectation Maximization(EM), it does not strictly compute the posterior probability in the "E" step.

Recently some algorithms based on the probability, especially the Gaussian Mixture Model(GMM), were proposed. Coherence Point Drift(CPD)[3] defines a velocity function for the template point set, namely the centroid of the GMM, and iteratively calculates the unknown parameter in the GMM by EM. An Expectation Conditional Maximization(ECM) for point registration algorithm called ECMPR is proposed in [4], which adopts the anisotropic covariance model and ECM to resolve the rigid and articulated point registration. The registration based on GMM has good performance. But if the data contain a large amount of noise, the results would be significant degraded. In order to enhance robustness, an extra uniform component can be included in the mixture model, as in [9]. The CPD and ECMPR also adopt this kind of method. So it would be better if we can estimate the probability that a point belongs to the uniform component. At the same time, in [3][4] one point set is taken as the observed data and the other is taken as the centroid of GMM. We notice that the uniform component just fit outliers in the observed data. The points in the other point set represent the Gaussian components with the same weight. When outliers exist in both point sets, some wrong Gaussian components will be used to fit the observed data, which will make the algorithm easing get stuck in local minima.

In order to make the registration model more robust, specially to outliers, we present a new approach to tackling the non-rigid registration between two point sets, with the following original contributions: we weigh the noise component in GMM by a robust estimation; the estimation of GMM density function is resolved by use of a bidirectional EM process instead of fixing one point set and moving the other.

The paper is organized as follows. The non-rigid point set registration method is proposed in Section 2. In Section 3, the proposed method is applied to artificial point sets and ultrasound medical image and the experimental results are presented. Conclusions are drawn in Section 4.

*This work was conducted when the author studied as a visiting student at University of Wisconsin-Milwaukee.

2. METHOD

2.1. Non-rigid point set registration based on GMM

Here we quickly review the non-rigid registration method based on the GMM. More detailed can be found in [2][3][4]. Given two point sets $X = \{x_1, x_2, ..., x_i, ..., x_n\}$ and $Y = \{y_1, y_2, ..., y_j, ..., y_m\}$, where x_i and y_j are d dimension vector, n and m are the number of the points. The goal of registration is to find the mapping between X and Y. We denote the mapping by $f : \Re^d \to \Re^d$, $y_j = f(x_i, \theta)$, where θ denotes the set of unknown transformation parameters.

In the framework of point set registration based on GMM, we choose any one of the two data sets, such as X, as the observed data and the other Y denotes the centroid of the GMM. Assuming X is the $i.i.d$ observed data, we have the probability [11]

$$P(X) = \prod_{i=1}^{n} \sum_{j=1}^{m} \pi_j P(x_i|\theta_j) \tag{1}$$

where the unknown parameter is the (π_j, θ_j) and $\theta_j = (\Sigma_j, \mu_j)$, where μ_j is the centroid, Σ_j is the covariance of GMM and π_j is set as constants. Here we adopt the non-rigid transformation in [3]: $\mu_j = f(y_j) = Y + GW$, where G is the Gaussian Basis Function and W is the transformation parameter. With the regularization constraint, the log likelihood function of (1) is

$$E(W) = -\sum_{i=1}^{N} log \sum_{j=1}^{M} exp^{-\frac{1}{2}\|\frac{x_i-(Y+GW_j))}{\Sigma_j}\|^2} + \frac{\lambda}{2} tr(W^T GW) \tag{2}$$

The registration problem can be resolved by estimating the unknown coefficients of the likelihood function. According to the EM algorithm, in the "E" step we can obtain the posterior probability[11]:

$$P_j(x_i; \Psi^k) = p_j(x_i; \theta^k) / (\sum_{h=1}^{m} p_h(x_i; \theta_h^{(k)}) + U) \tag{3}$$

where the U is a uniform distribution used to fit the noise component in GMM, and the transformation coefficient W can be obtained by taking the derivative of (2) in the "M" step. The isotropic covariance is computed as in [3].

2.2. Robust Estimation on the noise component

Actually there exists two classes of outliers in point set registration. The first one are noise point, which do not represent any feature of data; the second are the data points in one point set that can not match any points in the other set. For the first one, in [3][4] a uniform distribution is used to suppress its influence. In this subsection, we devise a robust estimator based on the M-estimation[10] to weigh the noise component in GMM. Here the average squared distance of the neighborhood from x_i can be seen as the squared residual error of the estimator on x_i. Hence the total squared error E is given by

$$e_i = \frac{1}{|\Omega_i|} \sum_{x_j \in \Omega_i} \|x_i - x_j\|^2 \tag{4}$$

Where Ω_i denotes x_i's neighborhood point. The size of the neighborhood window may not be a constant, but the number of neighborhood points of the x_i would be the same.

Then the weighting function is based on the average distance between the point and its neighborhood points and is monotonically increasing. $e'_i = e_i - \bar{e}$. where $\bar{e}$ is the mean of all e_i in observed point set. Using the Welsch function $\rho(e_i) = \frac{c^2}{2}[1 - exp(-(e_i/c)^2)]$. We can get

$$w_i(e'_i) = \frac{c^2}{2}[1 - exp(-(e'_i/c)^2)] \tag{5}$$

The weight means that a point belongs to the noise component if its point density among its vicinity deviate the point density mean very much. And we normalize each weight as $w_i = (w'_i - minw)/(maxw - minw)$ into the range $[0, 1]$.

By weighting the noise component, we revise the posterior probability function (3) in subsection. 2.1 as:

$$P_j(x_i; \Psi^k) = p_j(x_i; \theta^k) / (\sum_{h=1}^{m} p_h(x_i; \theta_h^{(k)}) + w_i U) \tag{6}$$

2.3. Bidirectional EM Process

For the second class outlier, we adopt a bidirectional EM process which is easy to implement. Unlike the method in [2][3], we do not fix the roles of two point sets. Firstly we arbitrarily choose one point set as observed data and the other as the centroid. For example, set X is chosen as the fixed set(observed data) and set Y is chosen as the moving set(centroid). Then we use the algorithm mentioned above to estimate the updated value Y' of the moving set Y. Secondly we take the Y' as fixed and X as the moving point set. Then the new value X' of X is estimated. We iteratively update two point sets until the parameter of the registration converge. Generally speaking, in the previous methods one point set is updated to its new position based on the other point set. In our approach two point sets interactively deform to their next positions. In each iterative procedure, there are two EM processes. Because each point set is modeled as the observed data, the weighted uniform component can fit the outliers in both point sets, which works well on the second class outliers mentioned in subsection 2.2. In order to match one data set with another, we will reset one of point set with initial positions after convergence and restart the bidirectional EM process. When the distance between two point sets reaches a threshold measured by covariance, a unidirectional EM will be performed to complete the registration. Table.1 shows the flow chart of our algorithm.

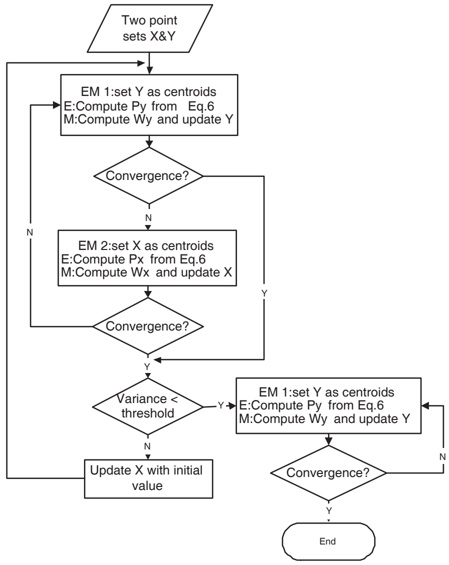
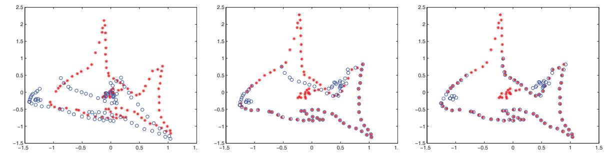


Table. 1. Flow chart of our algorithm.

3. EXPERIMENTAL RESULT

This section shows the performance of our algorithm on artificial data and medical images in heavy noise. The algorithm is implemented in Matlab, and tested on a Pentium4 CPU with 4GB RAM. The stopping criterion is: the change of registration parameter is above a threshold of 10^{-5}; the number of iteration is more than 200. On average the algorithm converges in several seconds. The 2D point set and two medical images used for comparison are taken from the CPD Matlab package[3].

First, two incomplete fish models are used. The fish head in the reference point set is removed and in the template point set the fin is removed. For the first experiment(Fig. 1), we just test the weighted noise component without the bidirectional EM processing(see Eq.6). The result shows that weighted heterogeneous outliers are excluded from the Gaussian component. For the second experiment(Fig. 2), 5%, 15% and 25% random outliers are added. RPM and CPD work well with a few outliers. Our method have good performance even with heavy outliers. For the third experiment(Fig. 3), we use 20 images with random outliers at different levels. RMP, CPD and our method are used for registration. After that, we remove these outliers and points that can not been matched. Then we compute the average hausdorff distance between two point sets. The smaller the distance, the better the results. Our method shows higher robustness. At last we apply our algorithm to point sets extracted from two medical images used in [3]. However, We use the Canny's algorithm to automatically extract point sets instead of manual extraction as in [3].



a b c

Figure. 1. Application of the weighted noise component to 2D fish point sets. (b) Registration without weighted noise component. (c) Registration with weighted noise component[Eq.6].

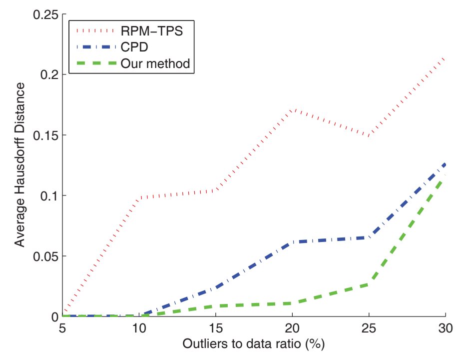


Figure. 3. A comparison of TPS-RPM, CPD and our method on the 2D fish point sets with respect to outliers(x-axis is the ratio of the number of outliers to the number of clean data points, y-axis is the average hausdorff distance of two point sets after registration and removing the outliers). Our algorithm shows more accurate registration performance compared to TPS-RPM and CPD.

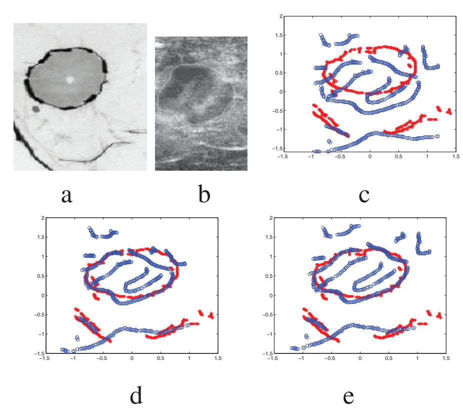
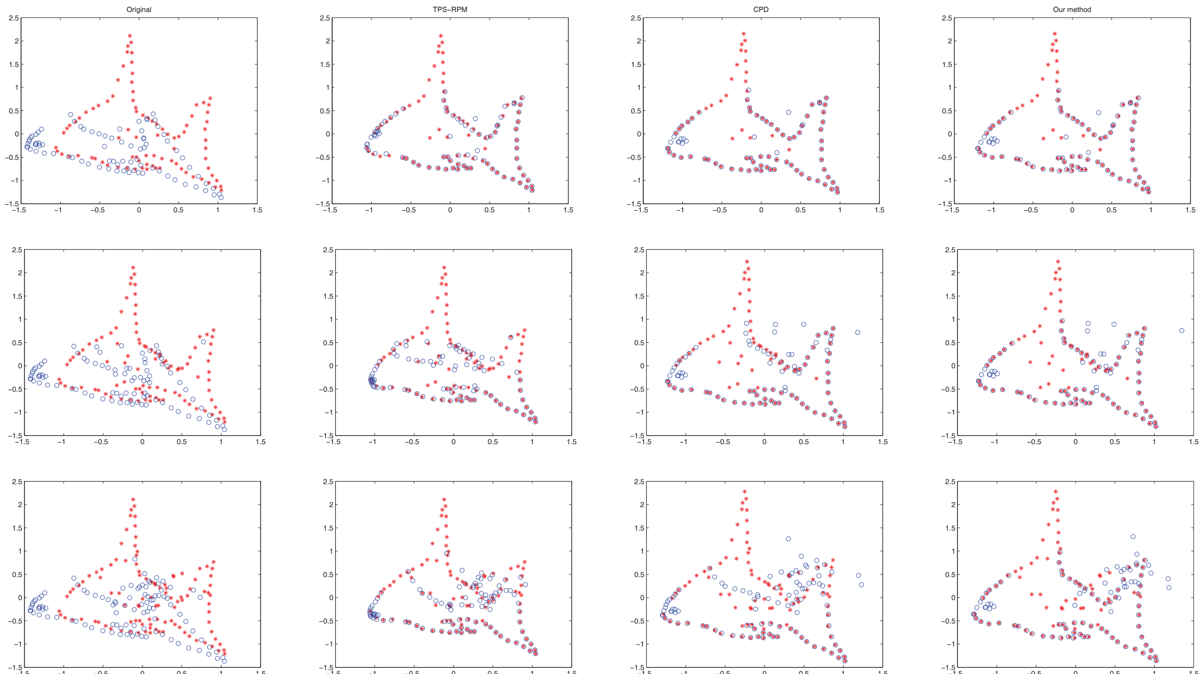


Figure. 4. Application of our algorithm to image registration of histopathology (a) and ultrasound elastography (b) images[3]. (c) Two point set were extracted by the Canny's algorithm. (d)The aligned point sets using CPD. (e) The aligned point sets using our method.



5%

15%

25%

Figure. 2: Registration results of RPM, CPD and our method with respect to different outlier ratios. The first column shows template(∘) and reference(∗) point sets with 5%, 15% and 25% outliers. The col 2-4 show the registered positions of template set superimposed over the reference set after applying TPS-RPM, CPD and our algorithms.

4. CONCLUSION

This paper introduces an improved probabilistic method for non-rigid point set registration. The registration is considered as a parameter estimation question of Gaussian Mixture Model with missing data. Unlike the previous method, we do not fix the roles of point set: one is taken as the centroid of GMM and the other is taken as the observed data, but exchange the roles alternatively. That is more reasonable for fitting the outliers in two point sets. Extensive experiments were presented to show the robustness and accuracy. Compared with the other two well-known algorithms, our approach performs better under non-rigid deformation with heavy outliers.

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