

A UNIFIED APPROACH FOR OPTIMIZATION OF SNAKUSCULES AND OVUSCULES

Adithya Kumar Pediredla and Chandra Sekhar Seelamantula

Department of Electrical Engineering

Indian Institute of Science

Bangalore-560012, India

Emails: adithya.ssa.iisc@gmail.com, chandra.sekhar@ieee.org

ABSTRACT

Automated image segmentation techniques are useful tools in biological image analysis and are an essential step in tracking applications. Typically, snakes or active contours are used for segmentation and they evolve under the influence of certain internal and external forces. Recently, a new class of shape-specific active contours have been introduced, which are known as *Snakuscules* and *Ovuscules*. These contours are based on a pair of concentric circles and ellipses as the shape templates, and the optimization is carried out by maximizing a contrast function between the outer and inner templates. In this paper, we present a unified approach to the formulation and optimization of Snakuscules and Ovuscules by considering a specific form of affine transformations acting on a pair of concentric circles. We show how the parameters of the affine transformation may be optimized for, to generate either Snakuscules or Ovuscules. Our approach allows for a unified formulation and relies only on generic regularization terms and not shape-specific regularization functions. We show how the calculations of the partial derivatives may be made efficient thanks to the Green's theorem. Results on synthesized as well as real data are presented.

Index Terms— Active contours, Snakes, Affine transformation, Snakuscule, Ovuscule.

1. INTRODUCTION

Active contours or snakes are efficient tools for image segmentation [1] and tracking [2]. Snakes evolve from a specified initialization towards the boundary of a desired object so as to minimize some suitably defined energy function. There are a large class of snake varieties depending on the type of curve representation: (i) point snakes, where the snake is represented as a collection of pixels [3], (ii) parametric snakes, where the curve is described in terms of chosen basis functions such as B-splines [4] or Fourier exponentials, and (iii) geometric snakes, which are level sets of appropriately defined surfaces. Of these snake types, the parametric variety offers computational advantages and allows for straightforward incorporation of smoothness and shape constraints.

Depending on the choice of the basis function, some specific shapes may also be preferred. For example, using exponential splines with appropriately chosen parameters, one may be able to reproduce ellipses [5].

Recently, in a couple of interesting publications, Thévenaz et al. introduced a new snake formalism based on specific shapes. Based on circles and ellipses as the contour definition, they proposed *Snakuscules* [6] and *Ovuscules* [7] as a pair of concentric contours, which are specified by points that lie on them. The points are then optimized for, to determine the best-fit circle or ellipse. The optimization cost function is a locally defined contrast function (contrast between the outer and inner contours). Thus, the snakuscule and ovuscule models differ significantly from traditional snake approaches. Snakuscules are parametrized by two points, and Ovuscules are specified by three points. In either parametrization, there is an extra degree of freedom, which gives rise to non-unique solutions. In order to limit the degrees of freedom and arrive at a unique solution, regularization terms are introduced in addition to the snake energies. Upon convergence of the augmented cost function, Snakuscules and Ovuscules lock on to the nearest circular and elliptical objects, respectively.

In this paper, we show that both Snakuscules and Ovuscules can be obtained by considering a restricted affine transformation of a *mother snake*, which we specify as a pair of concentric circles centered at the origin. By choosing the parameters of the affine transformation appropriately, either Snakuscules or Ovuscules can be obtained. As a result, a regularization function is not needed to enforce unicity of the solution. The snake energy is a normalized contrast function, which is a measure of the contrast between the inner and outer curves of the affine-transformed mother template. This is the same energy proposed by Thévenaz et al.

Organization of the paper: In Section 2, we provide the mathematical formulation first for generating Ovuscules from a mother snake. Section 3 contains the specification of the snake energy and its partial derivatives with respect to the affine transformation parameters. In Section 4, we show how Snakuscules may be obtained by imposing a simple constraint on the affine transformation parameters.

2. OVUSCULES

The mother snake is shown in Figure 1. It comprises two concentric circles centered at the origin. Both inner and outer circles are continuously parameterized as $(x_0(t), y_0(t)) = \left(\frac{\cos t}{\sqrt{2}}, \frac{\sin t}{\sqrt{2}}\right)$ and $(x_1(t), y_1(t)) = (\cos t, \sin t), t \in (0, 2\pi]$. The factor $\sqrt{2}$ ensures that the area of the inner circle is as much as the area of the annulus between the two circles. The reason is discussed in Section 3.1. The affine-transformed versions of the mother snake are given as

$$\begin{pmatrix} X_i \\ Y_i \end{pmatrix} = \begin{pmatrix} A \cos \theta & B \sin \theta \\ -A \sin \theta & B \cos \theta \end{pmatrix} \begin{pmatrix} x_i \\ y_i \end{pmatrix} + \begin{pmatrix} x_c \\ y_c \end{pmatrix}, \quad (1)$$

for $i = 0, 1$. The parameters x_c and y_c determine the center of the transformed snake; A and B are the semi-major and semi-minor axes of the ellipse; θ is the angle of rotation. Note that the entries in the affine transformation matrix are constrained such that the transformed snakes are Ovuscules. This particular version of the transformation is referred to as the restricted affine transform. The total number of degrees of freedom is five (the free parameters being $\{A, B, \theta, x_c, y_c\}$). The transformation is illustrated in Figure 1. From these parameters, one can derive other useful parameters of the ellipse such as the eccentricity and foci. Among all the Ovuscules in the family, we would like to obtain the one that best fits an object of interest. This is achieved by optimizing with respect to the free parameters and is discussed in the subsequent section.

3. SNAKE OPTIMIZATION

3.1. Snake energy

The snake energy is specified as the normalized contrast function:

$$\begin{aligned} E &= \frac{1}{AB} \left(\int \int_{\tau_1 \setminus \tau_0} f \, dx \, dy - \int \int_{\tau_0} f \, dx \, dy \right) \\ &= \frac{1}{AB} \left(\int \int_{\tau_1} f \, dx \, dy - 2 \int \int_{\tau_0} f \, dx \, dy \right), \quad (2) \end{aligned}$$

where τ_0 and τ_1 are the circular regions enclosed by the contours $(X_0(t), Y_0(t))$ and $(X_1(t), Y_1(t))$, respectively. The contrast is computed between the inner and outer curves. Maximizing the contrast enables the Ovuscules to lock on to bright objects in a dark neighborhood. The $\sqrt{2}$ factor in the definition of the mother snake ensures that in regions of constant intensity, the energy E is zero. Therefore, the Ovuscules remain stationary in such regions. The normalization term AB ensures that the area occupied by the inner contour is minimum. This also removes the ambiguity suffered by Ovuscules. Without the normalization, Ovuscules do capture bright objects, but may not always provide the expected outline. This aspect is illustrated in Figure 2, where all three

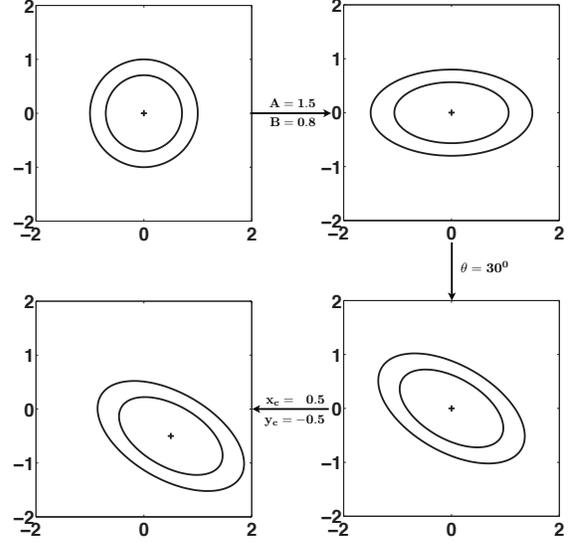


Fig. 1. Illustration of generating Ovuscules from the mother snake.

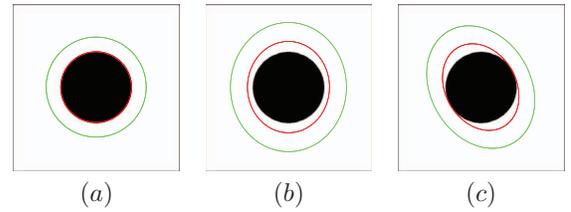


Fig. 2. (Inverted grayscale) Illustration of the need for the normalization term AB in equation (2). Best-fit Ovuscules (a) with normalization; (b) and (c): without normalization.

Ovuscules have the same contrast if there were no normalization. However, with the normalization factor, the Ovuscule in Figure 2(a) gives the best fit.

3.2. Partial derivatives

In order to perform optimization using gradient descent techniques, we require the partial derivatives of the cost function E with respect to the parameters $\{A, B, x_c, y_c, \theta\}$. The final expressions are given below (the derivations are given in the Appendix). The derivatives of the energy with respect to A and B are given by

$$\begin{aligned} \frac{\partial E}{\partial A} &= \frac{1}{A} \left(\int_{t=0}^{2\pi} (f(X_1, Y_1) - f(X_0, Y_0)) \cos^2 t \, dt - E \right), \text{ and} \\ \frac{\partial E}{\partial B} &= \frac{1}{B} \left(\int_{t=0}^{2\pi} (f(X_1, Y_1) - f(X_0, Y_0)) \sin^2 t \, dt - E \right). \end{aligned}$$

The partial derivative of E with respect to the orientation parameter θ is given by

$$\frac{\partial E}{\partial \theta} = \frac{1}{AB} \left(\int_{t=0}^{2\pi} (f(X_1, Y_1) - f(X_0, Y_0)) \times (B^2 - A^2) \sin t \cos t dt \right),$$

and those with respect to the coordinates of the center of the ovusculer are given by

$$\begin{aligned} \frac{\partial E}{\partial x_c} &= \frac{1}{AB} \left(\int_{t=0}^{2\pi} (\sqrt{2}f(X_1, Y_1) - 2f(X_0, Y_0)) \cos t dt \right), \\ \frac{\partial E}{\partial y_c} &= \frac{1}{AB} \left(\int_{t=0}^{2\pi} (\sqrt{2}f(X_1, Y_1) - 2f(X_0, Y_0)) \sin t dt \right). \end{aligned} \quad (3)$$

If f is a constant, then the partial derivatives are equal to zero, which means that the Ovusculer do not move in such regions. The optimization is carried out using a gradient-descent technique. To make the initialization interactive, we allow the user to specify two points, which are taken as the end-points of the mother snake.

The results of optimizing Ovusculer on Drosophila images and Shepp-Logan phantom [8] are shown in Figure 3. The Drosophila image is taken from the Image Archive of the Genetics Society of America (<http://www.drosophila-images.org/2007.shtml>). The image is the 2007 winner of the Drosophila Image Award [9]. Color images are converted to gray scale before optimization. The final results are displayed on the original color image. We observe that the Ovusculer evolve and lock on to the boundaries of interest quite effectively.

4. SNAKUSCULES

To optimize for Snakusculer instead of Ovusculer, we need to enforce the constraint: $A = B = r$ (r is the radius of the circle) in the restricted affine transformation of the mother snake. Corresponding to $A = B = r$, we have that $\frac{\partial E}{\partial \theta} = 0$, that is, the snake is not optimized for rotation — this is reasonable because the circle is isotropic and does not exhibit any directional preference. The partial derivative with respect to the parameter r in this case is given by

$$\frac{\partial E}{\partial r} = \frac{1}{r} \left(\int_{t=0}^{2\pi} f(X_1, Y_1) dt - \int_{t=0}^{2\pi} f(X_0, Y_0) dt - 2E \right).$$

The derivation for the above equation is given in Appendix A.2. The partial derivatives $\frac{\partial E}{\partial x_c}$ and $\frac{\partial E}{\partial y_c}$ are the same as in (3) with $A = B = r$. We see that circle fitting requires optimization of three parameters only, which is one less than the number considered in [6]. The constrained affine transform becomes a similarity transform for

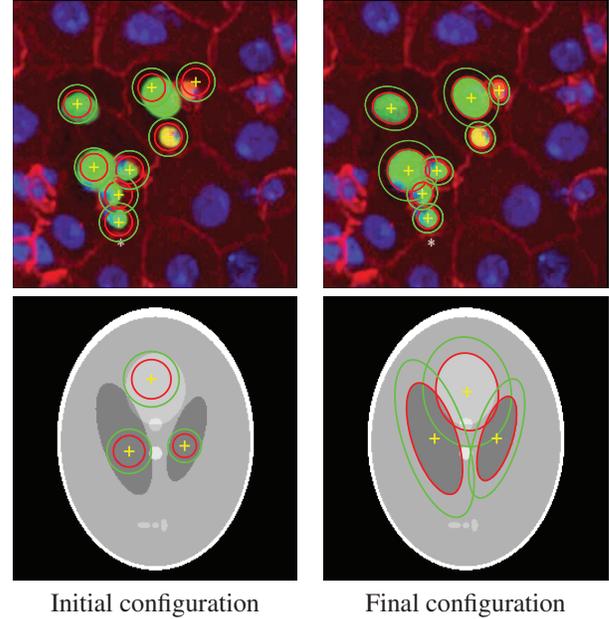


Fig. 3. Performance of Ovusculer on Drosophila Images and Shepp-Logan phantom [8]; to segment darker objects in bright background, the gray scale of the image is inverted.

Snakusculer. To illustrate the performance of the Snakusculer obtained using the proposed formulation, we show segmentation results on the optic disc in a retinal image. The data is taken from the online repository of the STARE project (<http://www.parl.clemson.edu/stare/images2.htm>). The results of the Snakusculer optimization are presented in Figure 4. We observe that the Snakusculer provides satisfactory segmentation of the optic disc.

5. CONCLUSIONS

We proposed a unified formulation of recently proposed Snakusculer and Ovusculer. The advantage of the new formulation is that explicit additional regularization terms are not required to ensure unique solutions. The cost function to generate Ovusculer can be modified slightly to result in Snakusculer. Thanks to the Green's theorem, the computation of the partial derivatives is made efficient. We presented examples on real data to demonstrate the efficiency of Snakusculer and Ovusculer fitting. The advantage of the proposed formalism is that other types of shape templates can also be accommodated within the same framework.

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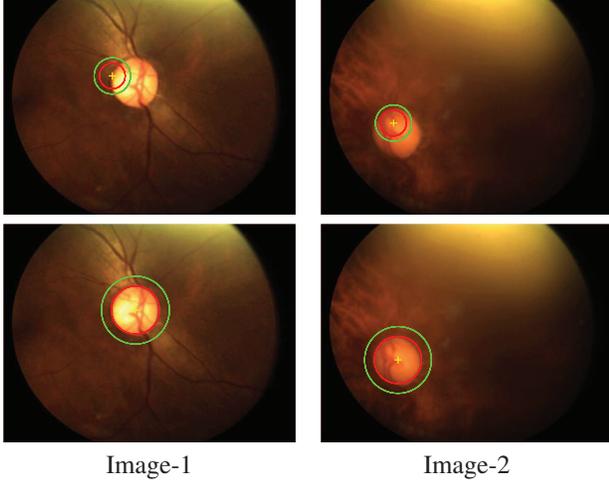


Fig. 4. Performance of Snakuscles on retinal images. The region of interest is the optical disc (the bright spot corresponding to the optic nerve head); the first row shows the initializations and the second row shows the converged result.

A. APPENDIX

A.1. Partial derivatives for Ovuscules

Since both integrals in (2) have the same form, it suffices to analyze one of them. Let $E_0 = \int \int_{\mathfrak{S}_0} f dx dy$, and $E_1 = \int \int_{\mathfrak{S}_1} f dx dy$. For ease of calculation, we transform the coordinate axes from (x, y) to (X, Y) , such that

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} A \cos \theta & B \sin \theta \\ -A \sin \theta & B \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} x_c \\ y_c \end{pmatrix}.$$

The image gets mapped from $f(x, y)$ to $F(X, Y)$. As a result,

$$E_0 = \int \int_{\mathfrak{S}_0} F(X, Y) dx dy, \text{ and } E_1 = \int \int_{\mathfrak{S}_1} F(X, Y) dx dy,$$

where $\mathfrak{S}_1 = \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] \times \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ and $\mathfrak{S}_0 = \left[-\frac{1}{2}, \frac{1}{2}\right] \times \left[-\frac{1}{2}, \frac{1}{2}\right]$.

For brevity of notation, we have dropped the parameter t to represent $(X(t), Y(t)), (x(t), y(t))$ as $(X, Y), (x, y)$, respectively. We next invoke the Green's theorem to convert the surface integrals to line integrals, which are relatively easy to compute and to perform differentiation. Specifically,

$$E_0 = - \oint_{\mathfrak{S}_0} F^Y dx = \oint_{\mathfrak{S}_0} F^X dy, \quad (4)$$

where $F^Y(X, Y) = \int_{-\infty}^Y F(X, \zeta) d\zeta$ and $F^X(X, Y) = \int_{-\infty}^X F(\zeta, Y) d\zeta$. E_0 is a function of (X, Y) , which are in turn functions of the parameters of the snake. Hence, the partial derivative of E_0 with respect to A is given by

$$\frac{\partial E_0}{\partial A} = \frac{\partial E_0}{\partial X} \frac{\partial X}{\partial A} + \frac{\partial E_0}{\partial Y} \frac{\partial Y}{\partial A}. \quad (5)$$

Applying the Green's theorem in (5), we get that

$$\begin{aligned} \frac{\partial E_0}{\partial A} &= \oint_{\mathfrak{S}_0} \frac{\partial F^X}{\partial X} \frac{\partial X}{\partial A} dy - \oint_{\mathfrak{S}_0} \frac{\partial F^Y}{\partial Y} \frac{\partial Y}{\partial A} dx \\ &= \oint_{t \in (0, 2\pi)} F(X, Y) x \cos \theta \{-A \sin \theta dx + B \cos \theta dy\} \\ &\quad + \oint_{t \in (0, 2\pi)} F(X, Y) x \sin \theta \{A \cos \theta dx + B \sin \theta dy\}, \\ \frac{\partial E_0}{\partial A} &= \frac{B}{2} \int_{t=0}^{2\pi} f(X_0, Y_0) \cos^2(t) dt. \end{aligned}$$

Similarly, one can derive partial derivatives of E_0 and E_1 with respect to the other parameters.

A.2. Partial derivatives for Snakuscles

$$\begin{aligned} \frac{\partial E_0}{\partial r} &= \oint_{\mathfrak{S}_0} \frac{\partial F^X}{\partial X} \frac{\partial X}{\partial r} dy - \oint_{\mathfrak{S}_0} \frac{\partial F^Y}{\partial Y} \frac{\partial Y}{\partial r} dx \\ &= \oint_{t \in (0, 2\pi)} F(X, Y) x r dy + \oint_{t \in (0, 2\pi)} F(X, Y) y r dx, \\ &= \frac{r}{2} \int_{t=0}^{2\pi} f(X_0, Y_0) dt. \end{aligned}$$

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