A TIME-VARYING GAUSSIAN MODEL FOR THE COMPLEX-VALUED EEG SPECTRUM DURING MENTAL IMAGERY TASKS

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ABSTRACT

Recent findings in neuroscience have shown that the spectral components of electroencephalogram (EEG) signals convey information regarding the mental task not only in their power but also in their phase. This calls for the utilization of complex-valued spectrum, instead of the commonly used power spectral density, in designing the brain computer interfaces. This paper studies the complex-valued spectrum of the EEG signal recorded during mental imagery tasks, and provides a statistical model for the EEG spectral components. Motivated by the results of a recent work by the authors, this paper proposes a time-varying noncircularly-symmetric Gaussian model for complex-valued EEG spectrum during a mental imagery trial. It will be shown that the mean of this Gaussian model is constant over time, whereas its variance and pseudo-variance follow an autoregressive conditional heteroscedastic (ARCH) model. The validity of this model is then verified using statistical tests.

Index Terms— Brain computer interface, improper complex Gaussian, complex-valued spectrum, autoregressive conditional heteroscedasticity

1. INTRODUCTION

Electroencephalogram (EEG) signals recorded from brain's activity during mental imagery tasks are widely used for spontaneous brain-computer interface (BCI) systems. A spontaneous BCI aims to provide a channel for the brain to communicate with the outside world by means of imagining certain motor tasks, e.g. hand/foot movement. A great portion of spontaneous BCIs utilize spectral components of the EEG data as discriminative features for classification of imagery tasks. Although complete representation of the EEG signal in the frequency domain results in a complex-valued representation, most methods only consider the *power spectral density* and ignore the *phase* of EEG spectrum.[1] However, recent studies have revealed that there exist relevant information carried in the phase of electrical activities of the brain, both in microscopic level (the phase of neural firings) and in macroscopic level (the phase of EEG) [1–5], which is ignored in the *power spectral density* (psd) representation.

While there are numerous studies in the literature on statistical characterization of EEG's power spectrum during mental imagery tasks, there exist no study on characterization of the complex-valued spectrum. To the best of the authors' knowledge, the only study is a recent work by the authors in [6] which has examined the complex-valued spectral features obtained from short-time Fourier transformation (STFT) of the EEG data. It has been shown in [6] that these spectral components can be modeled by a quasi-stationary Gaussian distribution. This model is shown to be *noncircularly-symmetric* or *improper* [7, 8], which confirms that there exist relevant information in the phase of EEG spectrum. A brief review of this model will be presented in Section 3.

The current paper utilizes the results of [6] to further study the time-varying nature of the complex-valued EEG spectral components. In particular, the time-varying properties of the mean and variance of the real and imaginary parts of the spectrum will be examined in Section 4. It will be shown that during a motor imagery trial the means of the real and imaginary parts of each spectral component are highly stationary, whereas their variances slowly change over time. This motivates us to propose an *autoregressive conditional heteroscedastic* (ARCH) model for the spectral components in Section 5. An ARCH model assumes that the variance at any time instance is a function of the samples at previous time instances. The validity of this model will be verified using the Engle's test for residual heteroscedasticity. Finally, these results will be incorporated into the the quasi-stationary model of [6] to provide a general time-varying complex-valued Gaussian model for the EEG spectral components.

2. DATA STRUCTURE AND EXPERIMENT SETUP

This paper uses data set V of the BCI competition III [9], which consists of EEG signals of three normal subjects (persons) recorded during four non-feedback sessions. During

[1] Note that the phase of EEG spectrum is different from the phase coupling of oscillatory activities from different parts of the brain, which is usually measured by *phase locking value*.

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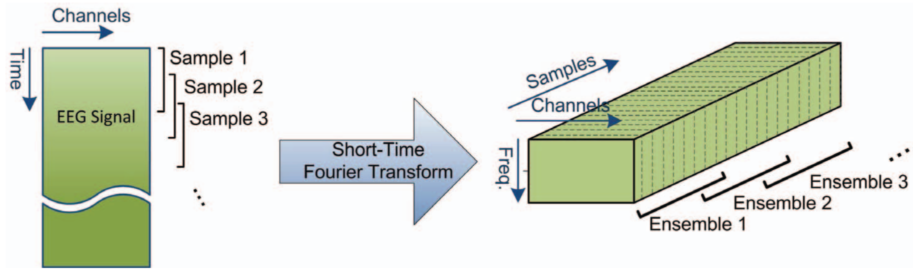


Fig. 1. Complex-valued EEG spectral components obtained using short-time Fourier transformation of multichannel EEG data.

each session, the subject sequentially imagines three different tasks: repetitive self-paced *left hand* movements (Task 1), repetitive self-paced *right hand* movements (Task 2), and *generation of words* beginning with the same random letter (Task 3). Each task lasts 15 seconds and is continuously followed by another randomly selected task requested by the operator. The EEG signals are recorded at 512Hz sampling rate using a Biosemi system with 32 electrodes located according to the International 10-20 system. Our analysis is performed on 8 centro-parietal channels: C3, Cz, C4, CP1, CP2, P3, Pz, and P4, which are recommended by the data set providers.

Consider a multichannel EEG signal recorded during a trial as shown in Figure 1. An STFT is applied on each channel of the data to obtain the frequency domain representation of the EEG at each time instant. The applied STFT uses Tukey windows of length 1 second with overlapping factor of $15/16$ and $\alpha = 1/8$. Then, the spectral components in the range of $8-30$Hz with a resolution of 2Hz are retained. This frequency band corresponds to the α rhythm ($8-12$ Hz) and β rhythm ($12-30$ Hz) of the brain which are known to be associated with mental imagery tasks. In this paper, $z(f,c,t) = x(f,c,t) + jy(f,c,t)$ denotes the fth spectral component of channel c obtained during the time interval $[t-1,t]$, where $x(f,c,t)$ and $y(f,c,t)$ are its real and imaginary parts[2].

3. GAUSSIAN MODEL FOR COMPLEX-VALUED EEG SPECTRAL COMPONENTS

Assume that a subject is performing a specific mental imagery task during the time interval $t \in [t_1, t_2]$. The EEG spectral component $z(f,c,t)$ is called stationary, if the probability density function (pdf) of $z(f,c,t)$ only depends on the variables f and c and is constant over time $t \in [t_1, t_2]$. Similarly, the variable $z(f,c,t)$ will be called quasistationary if its pdf changes very slowly with time and can be modeled to be constant if $z(f,c,t)$ is observed over a short period of time, i.e., $t_2 - t_1$ is small enough. In such a case, we consider the $z(f,c,t)$ components that are observed during this short period of time to form a set of samples with the same pdf. For simplicity we call this set of samples an *ensemble*. Figure 1 shows how a small set of samples can form an ensemble.

The work in [6] has shown that during a mental imagery task, the spectral components $z(f,c,t)$ can be modeled with a quasistationary *noncircularly-symmetric complex-valued* Gaussian distribution, whose first and second order statistics can be defined with the following parameters:

$$\begin{aligned}
\mu_z(f,c,t) &= E\left\{z(f,c,t)\right\} = \mu_x(f,c,t) + j\mu_y(f,c,t)\\
\sigma_z^2(f,c,t) &= E\left\{|z(f,c,t) - \mu_z(f,c,t)|^2\right\}\\
&= \sigma_x^2(f,c,t) + \sigma_y^2(f,c,t)\\
\gamma_z^2(f,c,t) &= E\left\{(z(f,c,t) - \mu_z(f,c,t))^2\right\}\\
&= (\sigma_x^2(f,c,t) - \sigma_y^2(f,c,t)) + j2\sigma_{xy}(f,c,t)
\end{aligned}$$

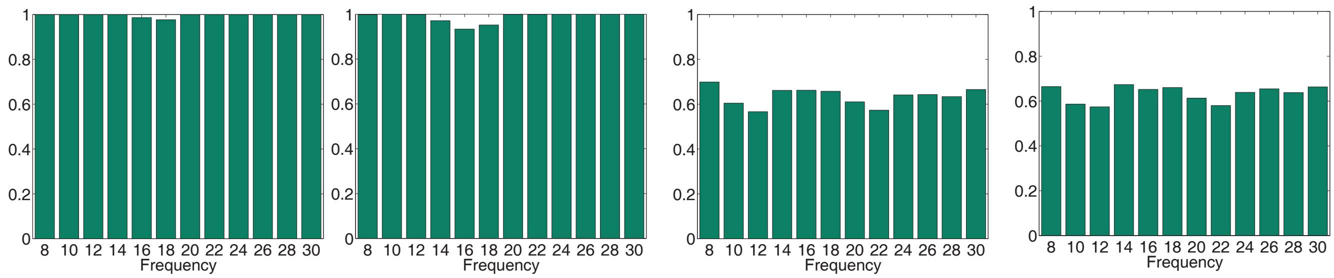
The parameters $\mu_z(f,c,t)$, $\sigma_z^2(f,c,t)$, and $\gamma_z^2(f,c,t)$ represent the time-varying mean, variance, and pseudo-variance of $z(f,c,t)$. It should be noted that second order characterization of $z(f,c,t)$ requires knowledge of both its variance and pseudo-variance, or alternatively knowledge of $\sigma_x^2(f,c,t)$, $\sigma_y^2(f,c,t)$, and $\sigma_{xy}(f,c,t)$. It is shown in [6] that $\sigma_x^2(f,c,t) \neq \sigma_y^2(f,c,t)$; hence, $z(f,c,t)$ is called *noncircularly-symmetric* or *improper*[3]. The next section examines the time-varying properties of these parameters.

4. TIME-VARYING CHARACTERISTICS OF THE EEG SPECTRUM

The results of [6] indicate that $\mu_x(f,c,t)$, $\mu_y(f,c,t)$, $\sigma_x^2(f,c,t)$, and $\sigma_y^2(f,c,t)$ parameters change very slowly during a trial and can be considered to be constant over observation intervals of length three seconds or less. In other words, for $t \in [t_1, t_1+3]$ we have $x(f,c,t) \sim \mathcal{N}\big(\mu_x(f,c,t_1), \sigma_x^2(f,c,t_1)\big)$ and $y(f,c,t) \sim \mathcal{N}\big(\mu_y(f,c,t_1), \sigma_y^2(f,c,t_1)\big)$. In this section, we divide the STFT samples obtained during a trial into overlapping intervals of length three seconds, and consider the samples within each interval to form an ensemble, as illustrated in Figure 1. Then, we perform the well

[2] In this paper, scalars are shown in lowercase (e.g., a). Also, E$\{.\}$ denotes expectation, and σ_x^2 and σ_{xy} respectively denote the variance of x and covariance of x and y

[3] For simplicity, in this paper we assume $\sigma_{xy}(f,c,t) = 0$ and only focus on the effect of $\sigma_x^2(f,c,t)$ and $\sigma_y^2(f,c,t)$.



(a) T-test for $\mu_x(f, c, t)$ (b) T-test for $\mu_y(f, c, t)$ (c) Chi-square test for $\sigma_x^2(f, c, t)$ (d) Chi-square test for $\sigma_y^2(f, c, t)$

Fig. 2. Percentage of ensembles verified to have a mean (a-b) or a variance (c-d) equal to the overall empirical mean or variance calculated using all the samples in a trial. (For brevity only the results of Subject 1 are presented.)

known T-test and Chi-square variance test to determine if the mean/variance of the samples within each ensemble is equal to the overall trial mean/variance, denoted by $\mu(f, c)$ and $\sigma^2(f, c)$, which is empirically calculated from all the samples in the trial. Each of these tests is separately performed on the real part and imaginary part of spectral components.

In order to study the ensemble means, we use the T-test which examines the null hypothesis that the $x(f, c, t)$ (or $y(f, c, t)$) samples within an ensemble have a Gaussian distribution with mean $\mu_x(f, c)$ (or $\mu_y(f, c)$) and unknown variance. This test is repeated over all the trials in the database, for each specific frequency and each channel. A significance level of 0.05 is used for all the statistical tests performed in this paper. Figure 2.a shows the results of this test for the real part of the spectrum ($x(f, c, t)$). In this figure, we have reported the average percentage of ensembles for which the null hypothesis of T-test is not rejected. In other words, the percentage of ensembles which are verified to have the same mean as the overall trial mean are presented in this figure. Figure 2.b shows similar results for the imaginary part of the spectrum ($y(f, c, t)$). Since all the subjects exhibited similar trends, only the results of Subject 1 are reported. These results are averaged over all channels and tasks. These figures reveal that the mean of spectral components are highly stationary over each mental imagery trial. Therefore, we can assume that the $\mu_x(f, c, t)$ and $\mu_y(f, c, t)$ parameters are constant over each trial and do not change with time index t.

In order to study the ensemble variances, we use the Chi-square variance test which examines the null hypothesis that the $x(f, c, t)$ (or $y(f, c, t)$) samples within an ensemble have a Gaussian distribution with $\sigma_x^2(f, c)$ (or $\sigma_y^2(f, c)$). Figures 2.c and 2.d show the results of this test for the real and imaginary parts of the spectrum ($x(f, c, t)$ and $y(f, c, t)$). Similar to the Figures a-b, the percentage of ensembles which are verified to have the same variance as the overall trial variance are presented in these figures. It can be seen that for more than $\%30$ of the ensembles the null hypothesis is rejected, which shows that unlike the means, the variances are time-varying and cannot be assumed to be constant over the entire trial.

The above results together with the results of [6] suggest that during a mental imagery trial, the complex-valued spectral components can be modeled with a time-varying noncircularly-symmetric Gaussian model with a constant mean and a time-varying variance and pseudo-variance. In this model, the variations of the variance and the pseudo-variance are slow enough such that a Gaussian distribution with fixed parameters accurately models the spectral components observed during a short interval (of length 3 seconds or less). This motivates us to examine the possibility of using an *autoregressive conditional heteroscedastic (ARCH)* model for the time-varying variance of the spectral components.

5. ARCH MODEL FOR SPECTRAL COMPONENTS

The main challenge in dealing with the time-varying Gaussian model proposed in the previous section is to model the variations of $\sigma_x^2(f, c, t)$ and $\sigma_y^2(f, c, t)$ parameters over time. This section examines if an ARCH model [10] can be used for this purpose. The ARCH model assumes that: (a) The variance of the signal is not constant and changes over time; hence the term heteroscedastic. (b) The variance at each time instance is a linear function of the previous samples; hence the term conditional autoregressive. Let $\varepsilon_x(f, c, t) = x(f, c, t) - \mu_x(f, c)$, then the ARCH model of order q implies that

$$\sigma_x^2(f, c, t) = \alpha_0(f, c) + \Sigma_{i=1}^{q} \alpha_i(f, c)\varepsilon_x^2(f, c, t - i\Delta t)$$

where $i\Delta t$ is the time lag between the current sample and the ith previous sample (As it was explained in Section 2, $\Delta t = 1/16$ second). A similar model can also be used for $\sigma_y^2(f, c, t)$ in terms of $\varepsilon_y(f, c, t) = y(f, c, t) - \mu_y(f, c)$. In order to test the validity of this model we have performed the following steps for $x(f, c, t)$ and $y(f, c, t)$ over a trial.

1. Start with $q = 1$;
2. Assuming that an ARCH(q) model describes the variations of the variance of the spectral components over time, estimate the parameters $\{\alpha_0, ..., \alpha_q\}$ and find the log-likelihood objective function value (LLF_q) associated with the parameter estimates;
3. Assuming that an ARCH(q+1) model describes the variations of the variance of the spectral components over time, estimate the parameters $\{\alpha_0, ..., \alpha_{q+1}\}$ and find the value of LLF_{q+1};

4. Perform the likelihood ratio test to determine if there is enough statistical evidence to increase the ARCH order from q to $q+1$.
5. If the likelihood ratio test confirms the order increase, then increase the ARCH order from q to $q+1$ and go to step 2. Otherwise, the ARCH(q) model suffices for modeling the variations of the signals' variance.

Let $q_x(f,c)$ and $q_y(f,c)$ denote the resulting order for $x(f,c,t)$ and $y(f,c,t)$. The validity of each of these model orders has been further confirmed using the Engle's test for residual heteroscedasticity [10] with a significance level of 0.05. Our analysis shows that (a) the value of $q_x(f,c)$ is equal to $q_y(f,c)$ in most of the cases (with only a few exceptions which will be mentioned later). Therefore, we will simply show the model order by $q(f,c)$. (b) The value of $q(f,c)$ does not change over different trials and tasks (with a few exceptions). Table 6 presents the results of $q(f,c)$ for subject 1 (The other two subjects show similar trends). The values marked by an asterisk are the ones for which the value of $q(f,c)$ was changing between 1 and 2 for the real/imaginary parts and/or for different tasks or trials.

The results of this table show that for all the frequency components the variations of $\sigma_x^2(f,c,t)$ and $\sigma_y^2(f,c,t)$, and hence the variations of $\sigma_z^2(f,c,t)$ and $\gamma_z^2(f,c,t)$, can be easily modeled using an ARCH model of order one or two. Therefore, we can conclude that the complex-valued spectral components $z(f,c,t)$ of the EEG signal recorded during a mental imagery trial can be modeled by a noncircularly-symmetric Gaussian distribution with constant mean and time-varying variance and pseudo-variance that follow an ARCH(1) or ARCH(2) model (depending on the values of f and c).

6. CONCLUSIONS AND REMARKS

This paper provided a time-varying Gaussian model for the complex-valued spectral components of the EEG signal, which are obtained using STFT during mental imagery tasks. It was shown that although the mean of each spectral component can be considered to be constant over a trial, its variance and pseudo-variance are time-varying. We examined the possibility of using an autoregressive conditional heteroscedastic (ARCH) model for analyzing the time-variations of the variance and pseudo-variance, and showed that an ARCH model of order 1 or 2 completely characterizes these time-variations.

The results of this paper can be used in modeling the non-stationarity of the spectral components of the EEG data recorded for brain computer interfacing (BCI) systems, which in turn can be utilized in the design of adaptive feature extractors and classifiers. Moreover, the proposed model paves the way for further analysis of the information conveyed in the complex-valued spectrum. Although recent findings in neuroscience have strongly suggested that the phase information conveyed in the complex-valued spectrum is relevant to the brain's mental activities, there exists no statistical framework for analysis of this complex-valued spectrum. The results of this paper can be used in the future as a framework for further statistical analysis of the complex-valued EEG spectrum.

	Channels							
Freq	C3	Cz	C4	CP1	CP2	P3	Pz	P4
8 Hz	2*	1	2*	1	1	2*	2*	2*
10 Hz	1	1	1	1	1	1	2*	1
12 Hz	2	2	2*	2	2	2*	2*	2
14 Hz	1	1	1	1	1	1	1	1
16 Hz	1	1	2*	1	1	1	1	1
18 Hz	1	1	1	1	1	1	1	1
20 Hz	2	2	2	2	2	2	2	2
22 Hz	1	1	1	1	1	1	1	1
24 Hz	1	1	1	1	1	1	1	1
26 Hz	1	1	1	1	1	1	1	1
28 Hz	2	2	2	2	2	2	2	2
30 Hz	1	1	1	1	1	1	1	1

Table 1. ARCH model order for different spectral components of each channel.

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