

# WAVE-DIGITAL POLARITY AND CURRENT INVERTERS AND THEIR APPLICATION TO VIRTUAL ANALOG AUDIO PROCESSING

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## ABSTRACT

Wave digital filters (WDFs) allow for efficient real-time simulation of classic analog circuitry by DSP. This paper introduces two new nonenergetic two-port WDF adaptors that allow mixing wave digital subnetworks adopting different polarity and sign conventions and extends the definitions of absorbed instantaneous and steady-state pseudopower to the case in which the active sign convention is used. This new knowledge is applied to a WDF triode tube amplifier model and it is shown to result in a more faithful reproduction of the simulated system than the previous model.

**Index Terms**—Acoustic signal processing, amplifiers, circuit simulation, music, wave digital filters

## 1. INTRODUCTION

Wave digital filters (WDFs) are an extensively used technique for simulating analog electronics by DSP in music and audio technology. They were first introduced by Fettweis in 1971 [1], while [2, 3, 4, 5] are more exhaustive resources on the topic. WDFs allow one to accurately and efficiently digitize a wide class of lumped physical systems into networks of interconnected and reusable DSP filters that operate on physically meaningful quantities and whose coefficients depend on observable parameters of such physical systems.

The preservation of energy and hence the numerical stability of WDFs are often considered. However, this technique requires that all circuit elements are modeled assuming a given polarity and sign convention, thus leading to a situation in which elements and subcircuits that exhibit asymmetrical behavior may need to be modeled more than once.

This paper solves this problem by introducing two two-port adaptors to interface wave digital subnetworks adopting different conventions. It then extends the definitions of absorbed instantaneous and steady-state pseudopower to the case in which the active sign convention is used and develops further considerations regarding the nature and use of the new adaptors. It also shows that these adaptors they are

pseudopassive, pseudolossless and nonenergetic. In the end the results of this investigation are applied to the WDF-based vacuum-tube amplifier simulator described in [6].

## 2. WDF POLARITY AND CURRENT INVERTERS

This section describes the wave decomposition which the WDF theory is based on, then it includes a couple of considerations regarding polarity and sign conventions in WDFs that justify the need for the specific adaptors introduced in its last part.

### 2.1. WDF wave decomposition

WDF elements in a wave digital network are interconnected with each other via *ports*, each having one ingoing and one outgoing *terminal* and for which a port current  $I$  and a port voltage  $V$  can be defined.

The WDF elements do not operate on the Kirchoff pair, but instead they use the wave variables  $a$  and  $b$  as defined by the travelling-wave formulation of lumped electrical elements:

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 & R_0 \\ 1 & -R_0 \end{bmatrix} \begin{bmatrix} V \\ I \end{bmatrix}, \quad (1)$$

where  $a$  is the incoming wave component, or *incident wave*,  $b$  is the outgoing wave component, or *reflected wave*, and  $R_0$  is a port resistance parameter that relates Kirchoff variables via Ohm's law. Since the port resistance does not need to correspond to any physically meaningful quantity, it is used as an additional degree of freedom to simplify calculations. The original K-variables can be, then, obtained from the wave variables as follows:

$$V = \frac{a + b}{2}, \quad I = \frac{a - b}{2R_0}. \quad (2)$$

### 2.2. Reconsidering polarity and sign conventions

Note that (1), by itself, does not impose restrictions on the directions of  $V$  and  $I$ , and thus there would be four different

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valid combinations for each wave variable couple when modeling a WDF element ( $++$ ,  $+-$ ,  $-+$ ,  $--$ ). This ambiguity is solved by always adopting the passive sign convention and by choosing voltages to point to the positive direction, i.e., from the minus to the plus pole, as explicitly indicated in [2].

Such a rigid arrangement, however, implies that circuit elements exhibiting asymmetrical properties (e.g., polarization) have to be modeled once per each possible use case. This means up to  $2^n$  models for an  $n$ -port WDF element. Furthermore, it might be desirable, in some nontrivial cases, to use the active sign convention for some ports, thus leading to up to  $2^{2n}$  different models.

### 2.3. Gaining flexibility through new specific adaptors

Given the considerations above, it is natural to think that a general solution might consist of defining new specific two-port adaptor elements that act as translators between wave digital subnetworks using different conventions.

A generic two-port WDF adaptor is shown in Figure 1, in which  $a_1$  and  $a_2$ , as well as  $b_1$  and  $b_2$ , have opposite directions. This implies that, given (2), the direction of voltages is the same on the two ends while the direction of currents is opposite (this can be easily verified by considering a “null” adaptor by which  $b_1 = a_2$ ,  $b_2 = a_1$  and  $R_{02} = R_{01}$ ).

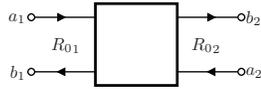


Fig. 1: Generic two-port WDF adaptor.

First we consider polarity inversion: in this case we have  $V_1 = -V_2$  and  $I_1 = I_2$ , hence:

$$\begin{cases} \frac{a_1 + b_1}{2} = -\frac{a_2 + b_2}{2} \\ \frac{a_1 - b_1}{2R_{01}} = \frac{a_2 - b_2}{2R_{02}} \end{cases},$$

thus solving for  $b_1$  and  $b_2$ :

$$\begin{cases} b_1 = -\frac{a_1(R_{01} - R_{02}) + 2a_2R_{01}}{R_{01} + R_{02}} \\ b_2 = -\frac{a_2(R_{02} - R_{01}) + 2a_1R_{02}}{R_{01} + R_{02}} \end{cases}.$$

In order to preserve computability properties of traditional WDFs, i.e., to avoid introducing instantaneous reflections, we have to choose  $R_{02} = R_{01}$ , thus getting:

$$b_1 = -a_2, \quad b_2 = -a_1, \quad R_{02} = R_{01}. \quad (3)$$

Following a completely analogous reasoning, we study sign convention change by examining the case in which  $V_1 = V_2$  and  $I_1 = I_2$ , leading to:

$$b_1 = a_2, \quad b_2 = a_1, \quad R_{02} = -R_{01}, \quad (4)$$

and similarly for the other case ( $V_1 = -V_2$ ,  $I_1 = I_2$ ):

$$b_1 = -a_2, \quad b_2 = -a_1, \quad R_{02} = -R_{01}. \quad (5)$$

It is possible to notice that (5) is, functionally, the combination of (3) and (4), thus we can just add two new two-port WDF adaptors to the current WDF formalism: a *polarity inverter*, corresponding to (3) and shown in Figure 2, and a *current inverter*, corresponding to (4) and shown in Figure 3.

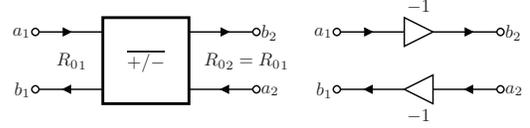


Fig. 2: WDF polarity inverter and its signal-flow diagram.

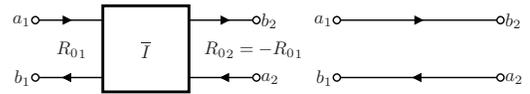


Fig. 3: WDF current inverter and its signal-flow diagram.

## 3. FURTHER CONSIDERATIONS

This section discusses the definitions of absorbed instantaneous and steady-state pseudopower of WDF elements when using different sign conventions, showing that the polarity and current inverters are pseudopassive, pseudolossless and nonenergetic. Moreover, it carries out some important considerations on negative port resistances and on the usage of the newly introduced adaptors in wave digital networks.

### 3.1. Passivity, losslessness and nonenergeticity

Fettweis [2] defines the *instantaneous pseudopower* absorbed by an  $n$ -port WDF element at time instant  $t$  as:

$$p(t) = \sum_{\nu=1}^n [a_{\nu}^2(t) - b_{\nu}^2(t)] G_{0\nu} \quad (6)$$

where  $G_{0\nu}$  is the port conductance at port  $\nu$ , and the *steady-state pseudopower* absorbed by an  $n$ -port WDF element as:

$$P = \sum_{\nu=1}^n (|A_{\nu}|^2 - |B_{\nu}|^2) G_{0\nu} \quad (7)$$

where  $A_{\nu}$  and  $B_{\nu}$  are, respectively, the incident and reflected wave at port  $\nu$  in an appropriate *reference frequency domain* whose complex frequency variable is referred to as  $\psi$ . The most common and appropriate choice for  $\psi$  is the bilinear transform of the  $z$ -variable.

A WDF element is, then, said to be: *pseudopassive* (or, simply, *passive*) if  $P \geq 0$  for  $Re(\psi) \geq 0$ , *pseudolossless* (or, simply, *lossless*) if  $P = 0$  for  $Re(\psi) = 0$ , *nonenergetic* if  $P = 0$  for all  $\psi$ . If the WDF element is delay-free, and hence

stateless, the requirements for pseudopassivity and nonenergeticity can be replaced by the corresponding requirements that for all  $t$  we have  $p(t) \geq 0$  and  $p(t) = 0$ , respectively.

In the case of our polarity and current inverters, we have that  $a_\nu^2(t) = b_\nu^2(t)$  for all  $t$ , hence  $p(t) = 0$ , that means that they are pseudopassive, pseudolossless and nonenergetic.

### 3.2. New absorbed pseudopower definitions

In general, it is clear that absorbed pseudopowers are intrinsic to WDF elements, but their definitions assume the use of the passive sign convention. Indeed, the signs of  $a_\nu$  and  $b_\nu$  have no influence on (6) and (7), contrary to the sign of  $G_{0\nu}$ . We, therefore, hereby extend such definitions so that they are irrespective of the chosen sign convention:

$$\hat{p}(t) = \sum_{\nu=1}^n k_\nu [a_\nu^2(t) - b_\nu^2(t)] G_{0\nu} \quad (8)$$

$$\hat{P} = \sum_{\nu=1}^n k_\nu (|A_\nu|^2 - |B_\nu|^2) G_{0\nu} \quad (9)$$

where  $k_\nu = 1$  if the passive sign convention is used at port  $\nu$ ,  $-1$  otherwise.

### 3.3. On negative port resistances

While it is obviously required that  $R_0 \neq 0$  for port currents to be defined, the WDF theory generally also assumes that  $R_0 > 0$ , which at first seems to be in sharp contrast with the port resistance sign change carried out by the current inverter adaptor element. The rationale behind such requirement, as explained in [2], is that this condition has to be met for basic WDF elements to be passive. Yet, we have just seen how the definitions of instantaneous and steady-state absorbed pseudopower can be extended to allow the usage of the active sign convention by adding a sign coefficient (i.e.,  $k_\nu$ ). It is immediate to check that, when using the active sign convention, for basic WDF elements to stay passive it is required that  $R_0 < 0$ , instead. This means that the sign of  $R_0$  is closely related to the sign convention used, and it can be safely assumed to be strictly positive for ports that use the passive sign convention and likewise to be strictly negative for ports that use the active sign convention.

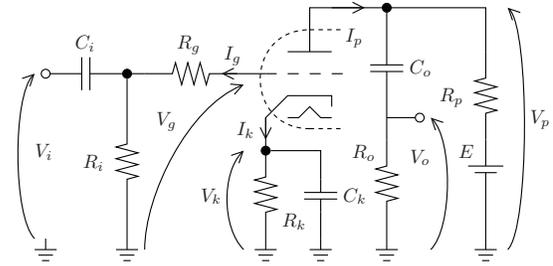
### 3.4. Usage within wave digital networks

Note that (3) and (4) are not dependent on any assumption w.r.t. polarity or sign conventions on any side, but they do only ensure a certain relationship between voltages and currents on the two sides. This consideration, together with the nonenergeticity of our polarity and current inverters, allows them to be inserted between any couple of WDF ports, any number of times and in any place of a wave digital subnetwork, as long as consistency is kept.

It should be clear that the two subnetworks at each end of such adaptors operate as if the subnetwork at the other end used the same conventions that are used locally, but, since the evaluation of voltages and currents is always to be performed locally, the local conventions should be kept well in mind when extracting physically observable quantities from the wave variables.

## 4. CASE STUDY: TRIODE TUBE AMPLIFIER MODEL

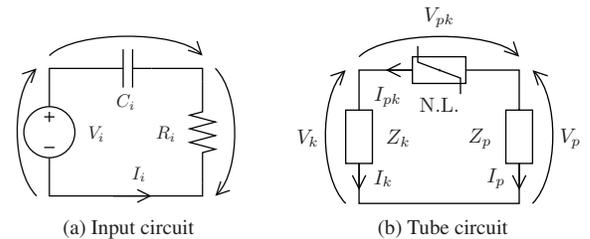
This section describes a nontrivial scenario in which the polarity inverter adaptor is used to properly take into account polarity issues without remodeling part of the circuit. In particular, it examines the triode tube amplifier shown as a schematic diagram in Figure 4 and more thoroughly discussed in [6].



**Fig. 4:** Triode amplifier stage with voltage and current annotations.

We assume that the tube model employed does not consider the grid current (i.e.,  $I_g = 0$ ) and that  $V_k$  changes slowly compared to  $V_g$ , so that  $V_{gk}(n+1) \approx V_g(n+1) - V_k(n)$ . Both these approximations are also enforced in the paper by Karjalainen and Pakarinen [6].

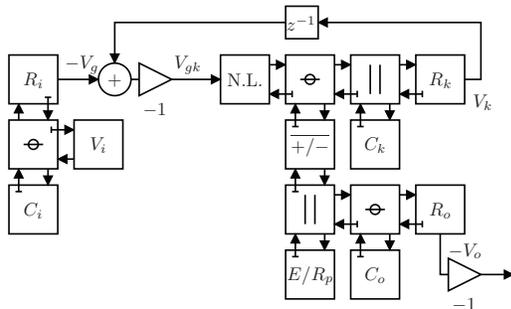
This allows us to model the whole circuit as the two separate circuits shown in Figure 5: an *input circuit* depicted in Figure 5a and a *tube circuit* in Figure 5b. The triode element is then considered as a nonlinear resistor placed between the plate and the cathode terminals (N.L. in Figure 5b) and is controlled by  $V_{gk}$ , where  $V_g$  is computed from the input circuit and  $V_k$  is extracted from the tube circuit itself.



**Fig. 5:** Resulting circuits with voltage and current annotations by assuming  $I_g = 0$ .

Traditional WDF series adaptors are derived by taking the

current and the voltages in the same direction for all involved elements, but this is not the case for the tube circuit. Hence, we will use the polarity inverter adaptor to interface the WDF model of the  $Z_p$  impedance to the WDF series adaptor that connects it with N.L. and  $Z_k$  in the resulting WDF-based implementation shown in Figure 6.



**Fig. 6:** Resulting WDF-based implementation.

Remembering that the computation of voltages from wave variables using (2) is affected by the local conventions used, it is clear that the voltage across  $R_i$  is  $-V_g$ , while the voltage across  $R_k$  corresponds to  $V_k$ . The voltage across  $R_o$  is actually  $-V_o$  since we have the same situation as that in Figure 5b for the neighboring series adaptor. We could, otherwise, use two more polarity inverters for  $R_i$  and  $R_o$ .

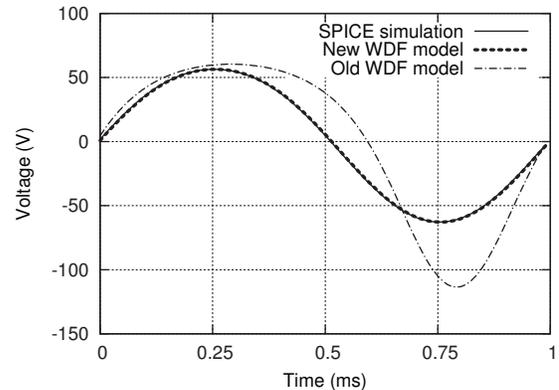
The “traditional alternative” to the discussed approach would have been to remodel one part of the tube circuit to match the conventions used in the other.

Figure 7 shows how the new WDF model produces more accurate results than the one described in the original paper by confronting both with a SPICE simulation output. The introduced polarity inverter element corrects the matching of polarities between  $Z_p$  and the rest of the WDF tree. While we can show the improvement with a sine wave input in this case, in general it is advisable to also use test input signals that have a strongly asymmetrical behavior (e.g., pulse waves with nonzero offset and duty cycle other than 50%), since multiple polarity-related mistakes can sometimes compensate each other with symmetrically behaving inputs.

## 5. SUMMARY

This paper has introduced two new nonenergetic two-port WDF adaptor elements to the current WDF formalism that allow interfacing wave digital subnetworks using different polarity and sign conventions. Such addition does also remove the need for developing specific models of circuit elements exhibiting asymmetrical behavior for each polarity and sign convention couple considered at each port.

Then, it extended the definitions of instantaneous and steady-state pseudopower of WDF elements to the case in which the active sign convention is used and correlates negative port resistances to the active sign convention, thus solving



**Fig. 7:** Output voltage when using a sine wave input (1 V amplitude, 1 kHz frequency). Note that the output of the new WDF model and that of the SPICE simulation are on top of each other.

an apparent contradiction that may have arisen w.r.t. earlier works.

Finally, the newly introduced polarity inverter element was used in a concrete scenario to correct the matching of polarities in the WDF model of the triode tube amplifier stage discussed in [6].

## 6. ACKNOWLEDGMENTS

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