ANALYSIS OF 2D SOUND REPRODUCTION WITH FIXED-DIRECTIVITY LOUDSPEAKERS

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ABSTRACT

The implementation of 3D sound reproduction is well founded theoretically, but the requirements for the number of loudspeakers and array geometry make it impractical using conventional technology. In practice, a 2D array of loudspeakers is commonly used which restricts the reproduction of sources to those in the horizontal plane and requires a restricted form for the free-field loudspeaker driving signals based on the sectorial spherical harmonics. However, reflections can impair reproduction quality when using the free-field solution. Since first-order loudspeakers are commercially available which increase the direct to reflected sound ratio, we extend the sectorial solution for the loudspeaker excitation signals to the first-order case. We also investigate the reproduction accuracy for both zeroth and first-order loudspeakers using numerical simulations.

Index Terms— Sound reproduction, Ambisonics, spherical harmonics

1. INTRODUCTION

Sound reproduction systems aim to use sufficient information from a sound field to allow reproduction over a volume of space larger than a human head, so that a listener hears the original sound field. The Helmholtz integral equation shows that this is possible in theory, and wave-field synthesis (WFS) is an approach based on this integral [1,2]. Alternatively, the spherical harmonic description of sound fields shows that the field may be recorded by calculating the spherical harmonic mode responses and reproducing these using a 3D array of loudspeakers. Higher order Ambisonics (HOA) is based on this approach [2–7]. Ambisonics may be viewed as a method for collecting sound from a given direction and assigning that sound to a loudspeaker in the same direction to reproduce the field.

The number of loudspeakers required for reproduction over a radius r and wavenumber k is approximately $(kr+1)^2$ [6,7]. For example reproduction up to 8 kHz over a sphere of radius 100 mm (kr ~ 15) would require 250 loudspeakers. Clearly, 3D reproduction in the home is impractical using conventional technology.

The theory of 2D reproduction is based on cylindrical coordinate solutions to the wave equation, and assumes a height-invariant sound field [4,7,8]. The number of sources required is approximately $2kr+1$ which for large kr is $2/[kr]$ of the number for the 3D case (31 loudspeakers using the example above). While challenging, this is more achievable than 3D reproduction. However the cylindrical coordinate approach is not realistic since loudspeakers typically approximate monopoles and produce a $1/r$ attenuation with distance r.

One solution to the problem for WFS is known as 2.5 D reproduction, and is obtained by applying the stationary phase approximation to the line source amplitudes to allow implementation using monopole sources [4,9]. A spherical harmonic solution to the 2D reproduction problem may also be determined for a circular array of monopoles using only the sectorial sound field components [4,10].

Sound reproduction in rooms produces reflections from the room surfaces which can compromise perceived localisation. These can be reduced using active compensation [6]. A simpler alternative is to reduce reflections by using fixed directivity loudspeakers, which are commercially available [11]. This paper therefore considers 2D sound reproduction using a circular array of loudspeakers with fixed first-order directivities. We generalise the sectorial mode match solution in [10] to the first order case, and investigate the accuracy of sound field reproduction for zeroth and first-order loudspeakers by numerical simulation, and by comparison with the cylindrical 2D case and with a pressure matching solution.

2. SOUND FIELD REPRODUCTION USING FIRST ORDER SOURCES

The sound field in a source-free region of space can be expressed in spherical coordinates as [12]

$$p(r,\theta,\phi,k)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}A_n^m(k)\,j_n(kr)\,Y_n^m(\theta,\phi) \qquad (1)$$

where [13]

$$Y_n^m(\theta,\phi)=\sqrt{\frac{(2n+1)}{4\pi}\frac{(n-|m|)!}{(n+|m|)!}}P_n^{|m|}(\cos\theta)e^{im\phi} \quad (2)$$

is the *(n,m)*th spherical harmonic, $P_n^m(.)$ is the associated Legendre function, and $A_n^m(k)$ are the frequency dependent coefficients which are the Fourier transforms of the complex signals representing the sound field. Since $j_n(kr)$ is small for $n > kr$ a maximum integer order $n = N = \lceil ekr_{\max}/2 \rceil$ is sufficient to represent the field [14]. The spherical harmonics $Y_{|m|}^m(\theta,\phi)$ are termed the sectorial harmonics, and these produce lobes only in the (*x*,*y*) plane. To examine the sectorial approximation, we first rewrite Eq. (1) as

$$p(r,\theta,\phi,k)=\sum_{m=-M}^{M}\left[\sum_{n=|m|}^{M}A_n^m(k)\,j_n(kr)Y_n^m(\theta,0)\right]e^{im\phi} \quad (3)$$

where the azimuthal part of the spherical harmonic is explicitly written and $M = N$ is the expansion order, producing a total of $(M+1)^2$ terms. The field may thus be viewed as consisting of a sum of $2M+1$ 'phase modes' $\exp(im\phi)$, with each term consisting of a sum of terms over *n*. For $\theta=\pi/2$ the $(M+1)M/2$ spherical harmonics for $n=|m|+2q+1,\ q=0,1,...$ are zero (Fig. 1). Furthermore, for sources near $\theta_s=\pi/2$ the spherical harmonic spectrum tends to have the largest magnitudes for $n = |m|$. In this case the sectorial approximation

$$\breve{p}(r,\theta,\phi,k)=\sum_{m=-M}^{M}A_{|m|}^m j_{|m|}(kr)Y_{|m|}^m(\theta,0)e^{im\phi} \quad (4)$$

is – as will be verified below – a reasonably close approximation to the sound field, which can be represented by a sum of $2M + 1$ terms, a considerable reduction from the $(M + 1)^2$ terms required for the full expansion.

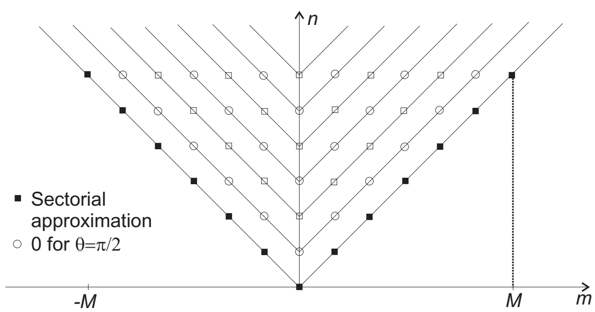

Figure 1: Sectorial harmonic approximation

Consider a single source at $\vec{r}_s$ which has a first order response consisting of a monopole term and a radial dipole term [11]

$$p_a(\vec{r},\vec{r}_s,k)=\frac{e^{ik|\vec{r}-\vec{r}_s|}}{4\pi|\vec{r}-\vec{r}_s|}\left\{a-(1-a)\left[1+\frac{i}{k|\vec{r}-\vec{r}_s|}\right]\cos\gamma\right\} \quad (5)$$

where $a\in[0,1]$ is the first-order weighting parameter and γ is the angle from the loudspeaker axis. For $a = 0.25$ the response is hyper-cardioid which produces the maximum direct to reverberant ratio [11]. The spherical harmonic expansion of the first order source is [11]

$$p_a(\vec{r},\vec{r}_s,k)= k\sum_{n=0}^{\infty}\sum_{m=-n}^{n}j_n(kr_<)b_n(kr_>,a)Y_n^m(\theta,\phi)Y_n^m(\theta_s,\phi_s)^* \quad (6)$$

where $b_n(kr_>,a)=iah_n(kr_>)+(1-a)h_n'(kr_>)$, $r_<$ denotes the smaller of r and r_s and $r_>$ the larger.

The sound field produced by a continuous circular distribution of first-order sources at a radius r_L with weighting $w(\phi_s)$ is

$$\hat{p}(r,\theta,\phi)=k\times \sum_{n=0}^{\infty}\sum_{m=-n}^{n}j_n(kr)b_n(kr_L,a)Y_n^m(\theta,\phi)Y_n^m(\theta_s,0)\int_0^{2\pi}w(\phi_s)e^{-im\phi_s}d\phi_s \quad (7)$$

Expressing the weight function as

$$w(\phi_s)=\sum_{m=-\infty}^{\infty}\beta_m e^{im\phi_s} \quad (8)$$

and equating to Eq. (3) yields, for each $m\in[-M,M]$

$$\beta_m=\frac{\sum_{n=|m|}^{M}j_n(kr)A_n^m(k)Y_n^m(\theta,0)}{2\pi k\sum_{n=|m|}^{M}j_n(kr)b_n(kr_L,a)Y_n^m(\theta,0)Y_n^m\left(\frac{\pi}{2},0\right)} \quad (9)$$

This equation depends on *kr*. However, letting *kr* tend to zero [10], or equivalently by matching only the $n=|m|$ sectorial modes, the coefficients can be simplified and

$$w(\phi_s)=\frac{1}{2\pi k}\sum_{m=-M}^{M}\frac{A_{|m|}^m(k)e^{im\phi_s}}{\left[iah_{|m|}(kr_L)+(1-a)h_{|m|}'(kr_L)\right]Y_{|m|}^m\left(\frac{\pi}{2},0\right)} \quad (10)$$

The weights for a discrete array of *L* loudspeakers are obtained by sampling the continuous weight function at *L* regularly spaced angles $\phi_l=2\pi l/L$, which is possible for $L > 2M+1$. The weights for *L* loudspeakers are then

$$w(\phi_l) = \frac{1}{kL}\sum_{m=-M}^{M}\frac{A_{|m|}^{m}(k)\,e^{im\phi_l}}{\left[iah_{|m|}(kr_L)+(1-a)\,h'_{|m|}(kr_L)\right]Y_{|m|}^{m}\left(\frac{\pi}{2},0\right)} \tag{11}$$

This describes a 'decoder' for deriving loudspeaker signals for a given sectorial HOA approximation to a sound field.

3. COMPARISON WITH 2D REPRODUCTION AND PRESSURE MATCHING

For comparison, we first consider the simple source solution for creating a 2D sound field $p(R,\phi,z) = p(R,\phi)$ which is independent of *z*, using a regularly spaced array of *L* first order line sources. Following the same procedure as above yields the general cylindrical solution [8]

$$w_C(\phi_l) = \frac{1}{L}\sum_{m=-M}^{M}\frac{A_m(kR)}{\left[aH_m(kR_L)-i(1-a)\,H'_m(kR_L)\right]}e^{im\phi_l} \tag{12}$$

For a source at (R_s,ϕ_s), $A_m(k) = H_m(kR_s)\exp(-im\phi_s)$.

A second comparison may be obtained by minimizing the squared error between the desired sound pressure and the field produced by the loudspeaker array at a number of positions $\vec{r}_v, v\in[1,V]$. The sum of squared errors can be written in matrix form as $\varepsilon_P = \left\|\mathbf{P}w_p - d\right\|^2$, where **P** is a *V* by *L* matrix of sound pressures produced at the *V* matching points by the *L* sources (Eq. 5), w_p is an *L* by 1 vector of weights and *d* is an *V* by 1 vector of desired sound pressures at the matching points. Assuming that $V > L$ and including a constraint on the maximum weight power with parameter λ produces the well known least squares error solution [5]

$$w_p = \left[\mathbf{P}^H\mathbf{P}+\lambda\mathbf{I}\right]^{-1}\mathbf{P}^H d \tag{13}$$

where *H* denotes the conjugate transpose and **I** is the *L* by *L* identity matrix. In practice we used a grid of 576 points in the (*x*,*y*) plane obtained from a spiral function out to a radius

$$r_{\max} = (L-1)/(2k) \tag{14}$$

which is the maximum radius at which reproduction is possible. In addition, each grid point had a *z* coordinate uniformly distributed over $[-0.1r_{\max}, 0.1r_{\max}]$.

4. SIMULATION RESULTS

We consider numerical simulations of a surround system consisting of $L = 31$ loudspeakers at a radius of 2.5 m. The desired field is that due to a single monopole source at a radius of 4 m and at an angle π/L, radiating a frequency of 500 Hz. (For the 2D comparison we use a line source.) Fig. 1 shows the field produced using the sectorial mode match solution, Eq. 11, assuming monopole loudspeakers ($a = 1$). The field is accurate out to r_{max}= 1.62 m (dotted circle). However a large exterior field is also generated, particularly in directions close to the source angle where the speaker amplitudes are largest.

Fig. 2 shows the field using $a = 0.25$ hypercardioid speakers. The hypercardioid produces a small rearward lobe, and this produces a reduced and more phase coherent outward traveling wave. The exterior field amplitude is reduced, and there is no discernable effect on the interior reproduction accuracy.

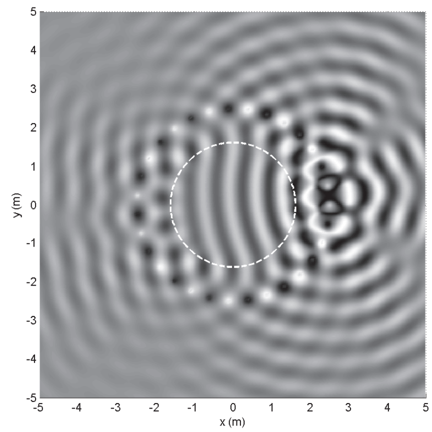

Figure 2: Sound field for *L* = 31 speakers at 2.5 m radius, *a* = 1.0

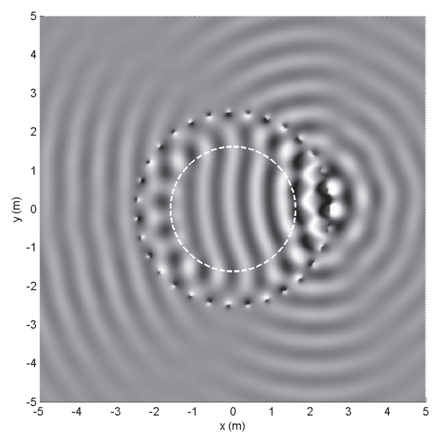

Figure 3: Sound field for *L* = 31 speakers at 2.5 m radius, *a* = 0.25

To assess the error in a compact form, we consider the normalized radial error in the (*x*,*y*) plane

$$\overline{\varepsilon}_C(kr) = \frac{\int_0^{2\pi}\left|p(r,\pi/2,\phi,k)-\hat{p}(r,\pi/2,\phi,k)\right|^2 d\phi}{\int_0^{2\pi}\left|p(r,\pi/2,\phi,k)\right|^2 d\phi} \tag{15}$$

and to assess the out-of plane behaviour we also consider the normalised error over a sphere

$$\overline{\varepsilon}_s(kr) = \frac{\int_{\Omega} \left| p(r,\theta,\phi,k) - \hat{p}(r,\theta,\phi,k) \right|^2 d\Omega}{\int_{\Omega} \left| p(r,\theta,\phi,k) \right|^2 d\Omega} \quad (16)$$

where Ω is solid angle. The circular and spherical radial errors for the sectorial mode and pressure matching solutions are shown in Fig. 4a, together with the error for the 2D case. The mode solution error is smaller at small *kr* than the pressure matching solution. While the in-plane pressure matching performance is better at larger *kr*, the spherical errors are the same. The pressure matching error can be varied by altering the matching radius. For example, reducing the radius of the matching grid produces lower error at small *kr* but increases it at larger *kr*. Generally, the sectorial mode match approximation appears to be similar in performance to pressure matching. However, as expected the reproduction error of both approaches is higher than the 2D case [4]. In general the circular loudspeaker array is able to maintain lower errors in the (*x*,*y*) plane but the error is larger for non-zero *z*, since the loudspeaker array has no vertical beamforming capability.

Fig. 4b shows the radial errors for the hypercardioid case. The modal solution error is unaffected at small *kr* and is reduced by around 1 dB at large *kr*. Thus the directional speakers produce no penalty in reproduction accuracy.

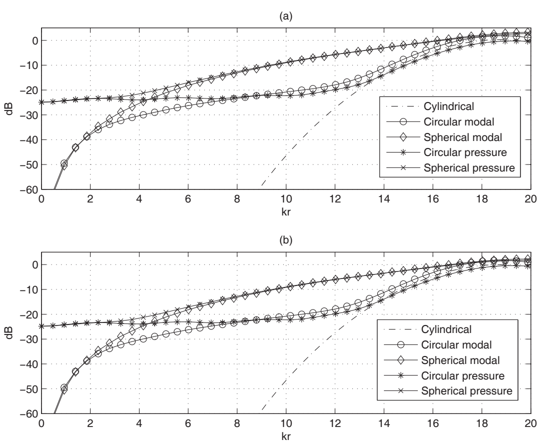

Figure 4: Radial errors: (a) *a* = 1, (b): *a* = 0.25

5. CONCLUSIONS

We have presented a derivation of the general decoder function for a circular array of loudspeakers with fixed first-order directivities. For comparison, we have included the 2D solution for first-order directivity line sources and a pressure matching solution. The sectorial mode matching solution extends the monopole result in [10] and allows for a reduction of the reverberant field by reducing the radiation of sound outside the array. It is similar in performance to the pressure matching approach, although the pressure matching approach can provide trade-offs in the error at small and large *kr*. Generally we found that full mode matching at a single radius (Eq. 9) reduced reproduction error at that radius, but the error was similar or larger to the sectorial solution error at other radii.

A circular array of loudspeakers cannot achieve the reproduction accuracy of an idealized 2D array of line sources or a 3D array of loudspeakers. However, it uses approximately 2/[*kr*] of the number required for the 3D case and allows a similarly reduced set of Ambisonics signals. The sectorial approximation is reasonably accurate for lateral sources and in this case the use of a circular array is more practical. A more detailed quantification of the sectorial approximation remains a future goal.

6. REFERENCES

[1] A. J. Berkhout, D. de Vries and P. Vogel, "Acoustic control by wave field synthesis," *J. Acous. Soc. Am.*, 93 (5), pp 2764-2778, 1993.

[2] J. Ahrens and S. Spors, "Sound field reproduction using planar and linear arrays of loudspeakers," IEEE Trans. Audio, Speech, Lang. Proc., 18 (8), pp 2038–2050, 2010.

[3] J. Daniel, "Spatial sound encoding including near field effect: Introducing distance coding filters and a viable new ambisonics format," AES 23rd Int. Convention, 2003

[4] J. Ahrens and S. Spors, "An analytical approach to sound field reproduction using circular and spherical loudspeaker distributions," Acustica/Acta Acustica, 94, pp 988-999, 2008

[5] H. Y. Sun and P. Svensson, "Optimal Higher Order Ambisonics Encoding with Predefined Constraints, IEEE Trans. Audio, Speech, Lang. Proc., to appear 2011

[6] M. A. Poletti, "Three dimensional surround sound systems based on spherical harmonics," J. Audio Eng. Soc., 53 (11), pp 1004-1025, November 2005

[7] Y. J. Wu and T. D. Abhayapala, "Theory and design of soundfield reproduction using continuous loudspeaker concept," IEEE Trans. Audio, Speech, Lang. Proc., 17 (1), pp 107–116, 2009

[8] M. A. Poletti and T. D. Abhayapala, "Interior and exterior sound field control using general two dimensional first-order sources," J Acoust. Soc. Am., 129 (1), pp 234–244, January 2011

[9] R. Nicol and M. Emerit, "3D-sound reproduction over an extensive listening area: A hybrid method derived from Holophony and Ambisonic," AES 16th Intl. Conf., Finland 1999

[10] J. Ahrens and S. Spors, "Analytical driving functions for higher order ambisonics," ICASSP 2008, pp 373 – 376, 2008

[11] M. A. Poletti, F. M. Fazi and P. A. Nelson, "Sound-field reproduction systems using fixed-directivity loudspeakers," J. Acoustical Society of America, 127 (6), pp 3590–3601, June 2010

[12] E. G. Williams, Fourier Acoustics, Academic Press, San Deigo, 1999

[13] D. Colton and R. Kress, Inverse Acoustic and Electromagnetic Scattering Theory, Applied Mathematical Sciences Vol. 93, 2nd Edition, Springer, 1998

[14]] R. Kennedy, P. Sadhegi, T. D. Abhayapala and H. M. Jones, "Intrinsic limits of dimensionality and richness in random multipath fields," IEEE Trans. Sig. Proc., 55 (6), pp 2542–2556, 2007