A MODEL BASED EXCURSION PROTECTION ALGORITHM FOR LOUDSPEAKERS

Chenchi (Eric) Luo, James McClellan**

Center for Signal and Image Processing
Georgia Institute of Technology

Milind Borkar, Arthur Redfern

Systems and Applications R&D Center
Texas Instruments

ABSTRACT

Loudspeakers in portable consumer electronic devices are frequently small in size. Due to the low sensitivity of their drive units, they are pushed to their power handling and mechanical limits by powerful amplifiers in an attempt to reach high volumes. To protect against excessive diaphragm excursions, a model based algorithm is proposed which regulates the voltage input signal to the loudspeaker while minimizing unnecessary system interventions.

Index Terms— loudspeakers, excursion protection

1. INTRODUCTION

A problem for loudspeaker designers is inexpensively producing high sound volumes and high sound quality from small loudspeakers [1]. It is typically not practical to use professional drive units for portable loudspeakers. Instead, drive units with low sensitivities and power handling capacities are used. Therefore, in order to reach a high sound pressure levels, these units are pushed to their power handling and mechanical limits.

Figure 1 shows the cross section of a typical loudspeaker. The voice coil is attached to a diaphragm which is mounted on a fixed frame via a suspension. A magnetic field is generated by a permanent magnet that is conducted to the region of the coil via a magnetic circuit which generates a concentrated magnetic field in the region of the coil gap. Holes in the rear frame provide ventilation to the rear enclosure.

According to laws of electrodynamics, due to the presence of the magnetic field, an electrical current passing through the voice coil will generate an electromotive force (EMF) f_c which moves the diaphragm by an excursion x_d . The movement of the diaphragm produces the sound wave output.

Excessive diaphram excursion is one of the two main causes of loudspeaker failure (the other is voice coil overheating). In order to directly monitor diaphragm excursion, a sensor has be to installed which is typically not practical for portable devices with limited space and cost. As an alternative, a nonlinear loudspeaker model was developed in [3] to model excursions based on input voltage, and a highpass filter was triggered to filter out low frequency components when a large excursion was predicted.

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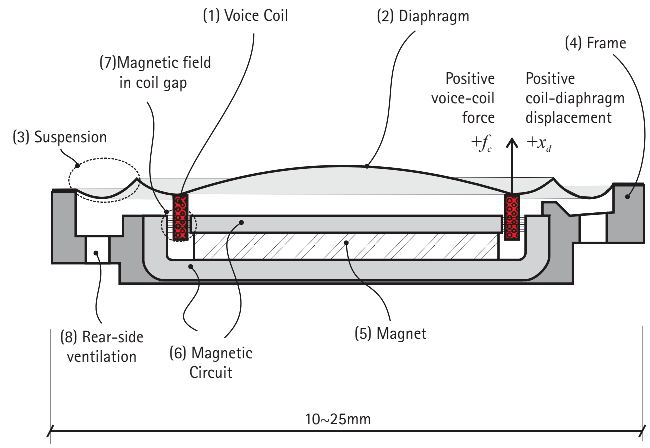

Fig. 1. Cross section of a typical micro-loudspeaker, from [2]

For high peak to average ratio signals, large excursion events are localized in time and introducing a high pass filter may unnecessarily remove low frequency content. As such, this paper proposes using a model based approach to predict excursions, but addresses the excursion events in the time domain to minimize the distortion caused by the algorithm intervention.

Figure 2 shows a block diagram of the proposed model based excursion control algorithm. Given the current input voltage $v_\mathrm{c}[n]$, the diaphragm excursion at next sampling interval $x_\mathrm{d}[n+1]$ is predicted according to a forward loudspeaker voltage to excursion model. An excursion compression algorithm is then applied to $x_\mathrm{d}[n+1]$ to obtain a compressed excursion $x_\mathrm{d}^\mathrm{c}[n+1]$. Finally, the resulting voltage $v_\mathrm{c}^*[n]$ required to reach the compressed excursion is calculated from an inverse loudspeaker excursion to voltage model.

This paper is organized as follows. A continuous-time loudspeaker model is developed in section 2 and discretized in section 3. The digital loudspeaker model is able to predict excursion changes given the input voltage and vice versa. Section 4 compares the distortion and protection effects between conventional high pass filtering and the proposed algorithm. Finally, conclusions are provided in Section 5.

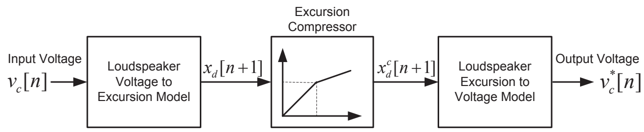

Fig. 2. A block diagram of the model based excursion control algorithm.

2. A CONTINUOUS-TIME NONLINEAR LOUDSPEAKER MODEL

A continuous-time model for the electrical behavior of a loudspeaker is [2]:

$$v_{\mathrm{c}}(t) = R_{\mathrm{eb}} i_{\mathrm{c}}(t) + \phi_0 \dot{x}_{\mathrm{d}}(t), \tag{1}$$

where $v_{\mathrm{c}}(t)$ is the voltage input across the terminals of the voice coil, $i_{\mathrm{c}}(t)$ is the voice coil current, R_{eb} is the blocked electrical resistance, $x_{\mathrm{d}}(t)$ and $\dot{x}_{\mathrm{d}}(t)$ are the diaphragm excursion and velocity, ϕ_0 is the transduction coefficient at the equilibrium state $x_{\mathrm{d}}(t) = 0$.

The mechanical dynamics of a loudspeaker can be modeled as a single-degree-of-freedom mechanical oscillator:

$$m_{\mathrm{d}} \ddot{x}_{\mathrm{d}}(t) + c_{\mathrm{d}} \dot{x}_{\mathrm{d}}(t) + k_{\mathrm{d}} x_{\mathrm{d}}(t) = f_{\mathrm{c}}(t), \tag{2}$$

where m_{d} is the mass of the diaphragm, c_{d} and k_{d} are the mechanical resistance and stiffness due to diaphragm suspension and $f_{\mathrm{c}}(t)$ is the EMF exerted on the voice coil. At the equilibrium state where $x_{\mathrm{d}}(t) = 0$,

$$f_{\mathrm{c}}(t) = \phi_0 i_{\mathrm{c}}(t). \tag{3}$$

Combining the electrical and mechanical loudspeaker models, we can write the s-domain transfer function of excursion versus voltage input at the equilibrium state as

$$\frac{x_{\mathrm{d}}(s)}{v_{\mathrm{c}}(s)} = \frac{(\phi_0 / R_{\mathrm{eb}})}{m_{\mathrm{d}} s^2 + (c_{\mathrm{d}} + \phi_0^2 / R_{\mathrm{eb}}) s + k_{\mathrm{d}}}. \tag{4}$$

A z-domain transfer function can be obtained by applying either a bilinear transformation or the impulse invariance method to (4).

However, in order to yield a more precise model several additional nonlinear factors need to be taken into consideration [4]. Mechanical nonlinearities are caused by the fact that the transduction coefficient ϕ and suspension stiffness k_{d} vary as quadratic functions of the excursion $x_{\mathrm{d}}(t)$. As such, a more precise expression for $f_{\mathrm{c}}(t)$ is given by

$$f_{\mathrm{c}}(t) = \phi(x_{\mathrm{d}}(t)) i_{\mathrm{c}}(t) - k_1(x_{\mathrm{d}}(t)) x_{\mathrm{d}}(t), \tag{5}$$

where $k_1(x_{\mathrm{d}}[n])$ is the variation of the suspension stiffness as a function of excursion

$$k_1(x_{\mathrm{d}}(t)) = k_{\mathrm{d}}(x_{\mathrm{d}}(t)) - k_{\mathrm{d}}(0). \tag{6}$$

Likewise, electrical nonlinearities are caused by R_{eb} depending on temperature T

$$R_{\mathrm{eb}}(T) = R_{\mathrm{eb}}(T_0) \left(1 + \alpha(T - T_0)\right), \tag{7}$$

where α is the temperature coefficient ($\alpha_{\text{copper}} = 0.004 K^{-1}$) and T_0 is the ambient temperature. Therefore, (1) should be rewritten as

$$v_{\mathrm{c}}(t) = R_{\mathrm{eb}}(t) i_{\mathrm{c}}(t) + \phi(x_{\mathrm{d}}(t)) \dot{x}_{\mathrm{d}}(t). \tag{8}$$

Equations (2,5,8) complete the continuous-time nonlinear loudspeaker model. Examples of the measurement and tracking of these parameters can be found in [5] and [6].

3. A DISCRETE-TIME NONLINEAR LOUDSPEAKER MODEL

To introduce digital processing into the loudspeaker protection system it is essential to develop a digital loudspeaker model. From (2), the transfer function of the mechanical receptance is

$$X_{\mathrm{m}}(s) = \frac{x_{\mathrm{d}}(s)}{f_{\mathrm{c}}(s)} = \frac{1}{m_{\mathrm{d}} s^2 + c_{\mathrm{d}} s + k_{\mathrm{d}}}. \tag{9}$$

A z-domain transfer function for $X_{\mathrm{m}}(s)$ can be derived using the impulse-invariance method:

$$H_{X_{\mathrm{m}}}(z) = \frac{x_{\mathrm{d}}(z)}{f_{\mathrm{c}}(z)} = \frac{b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}}, \tag{10}$$

where a_1, a_2, b_1 are a function of m_d, c_d, k_d and the sampling frequency F_{s}. This allows us to express the discrete-time diaphragm excursion as

$$x_{\mathrm{d}}[n] = b_1 f_{\mathrm{c}}[n-1] - a_1 x_{\mathrm{d}}[n-1] - a_2 x_{\mathrm{d}}[n-2] \tag{11}$$

where the discrete-time EMF exerted on the voice coil is found from (8) and (5):

$$\begin{aligned} f_{\mathrm{c}}[n] = \phi(x_{\mathrm{d}}[n]) \left(\frac{1}{R_{\mathrm{eb}}[n]} \{ v_{\mathrm{c}}[n] - \phi(x_{\mathrm{d}}[n]) \dot{x}_{\mathrm{d}}[n] \} \right) \\ - k_1(x_{\mathrm{d}}[n]) x_{\mathrm{d}}[n] \end{aligned} \tag{12}$$

The diaphragm velocity $\dot{x}_{\mathrm{d}}[n]$ is determined by differentiating $x_{\mathrm{d}}[n]$ according to a 1st order IIR filter

$$\dot{x}_{\mathrm{d}}[n] = 2F_s(x_{\mathrm{d}}[n] - x_{\mathrm{d}}[n-1]) - a_{\mathrm{dt}} \dot{x}_{\mathrm{d}}[n-1], \tag{13}$$

where F_s is the sampling frequency and $0 < a_{\mathrm{dt}} \leq 1$ is a differentiator coefficient used to ensure the stability.

Equations (11), (12) and (13) complete the discrete-time loudspeaker excursion prediction model. The diaphragm excursion $x_{\mathrm{d}}[n]$ depends on the previous voltage input $v_{\mathrm{c}}[n-1]$, previous diaphragm excursions $x_{\mathrm{d}}[n-1]$, $x_{\mathrm{d}}[n-2]$, and velocity $\dot{x}_{\mathrm{d}}[n-1]$.

Note that due to the point-wise nonlinearity of k_d and ϕ with regard to $x_{\mathrm{d}}(t)$, the excursion to voltage relationship can also be inverted. According to (11),

$$f_{\mathrm{c}}[n] = \frac{1}{b_1}(x_{\mathrm{d}}[n+1] + a_1 x_{\mathrm{d}}[n] + a_2 x_{\mathrm{d}}[n-1]) \tag{14}$$

According to (8) and (5),

$$v_{\mathrm{c}}[n] = R_{\mathrm{eb}}[n]\left\{\frac{1}{\phi(x_{\mathrm{d}}[n])}(f_{\mathrm{c}}[n] + k_1(x_{\mathrm{d}}[n])x_{\mathrm{d}}[n])\right\} + \phi(x_{\mathrm{d}}[n])\dot{x}_{\mathrm{d}}[n] \tag{15}$$

4. MODEL BASED EXCURSION PROTECTION

The easiest way to protect a loudspeaker from unsafe excursion is to lower the gain. A disadvantage of reducing the gain is the obvious reduction of perceived loudness, which is a key issue for small loudspeakers and drivers. To overcome this drawback, the characteristics of the excursion to voltage transfer function in (4) are taken advantage of.

Figure 3(a) shows an example of the frequency response of the transfer function in (4). The lowpass characteristic suggests that large excursions are caused by low frequency components in the input signal. Therefore, loudspeakers can be protected against excessive excursion by reducing the gain in the low frequency signal components, i.e., simply highpass filtering the input signal.

Figure 3(b) shows an example of the frequency response of an example highpass filter. The disadvantage associated with the highpass filtering technique is the loss of low frequency content, which is potentially more than necessary when there are only a few excessive excursions.

As shown in Figure 2, this paper proposes using the voltage to excursion model to predict excursions, and addresses excessive excursions directly using an excursion compression algorithm, then uses the excursion to voltage model to determine the required modification (if any) to the voltage signal. The benefit of this formulation is that a minimal magnitude and number of modifcations are made to the voltage signal to address excessive excursion.

Figure 4 shows a gain mapping characteristic of the excursion compression module. Instead of hard clipping the excursion at the safe limit, the compression is only triggered when the input excursion exceeds a pre-defined threshold. Different compression ratios can be applied with the ratio = ∞ corresponding to hard clipping. Logical modifications such as soft knees can be introduced to allow for smoother compression and less perceived distortion.

Figure 5 shows an example of the distortion and excursion protection effects when playing a clip of music with both the traditional highpass filtering protection method and the proposed excursion protection method. The frequency response of the highpass filter is the same as the one shown in Figure 3(b). The sampling frequency is F_s = 16 kHz and the safe excursion limit is $x_{\max} = \pm 0.5$ mm. The gain mappling curve applied in the proposed algorithm has a threshold of $0.9x_{\max}$ and a compression ratio of 2.

As shown in Figure 5(b), without protection the excursions occasionally exceed the safe limits as indicated by the green horizontal lines. Highpass filtering the input voltage signal is able to reduce the excursions at the cost of constantly suppressing the bass. As a result, the input voltage signal to distortion ratio (SDR), where the distortion is defined as $\sum_n (v_{\mathrm{c}}[n] - v_{\mathrm{c}}^*[n])^2$, is degraded to -0.93 dB. With the proposed protection algorithm, interventions to the input voltages are minimized and justified by the underlying loudspeaker model. The distortions introduced by the proposed protection algorithm are minimal when comparing the spectrograms of the original and protected voltage input signal. Consequently, the input voltage SDR increases to 18.97 dB with the proposed protection algorithm.

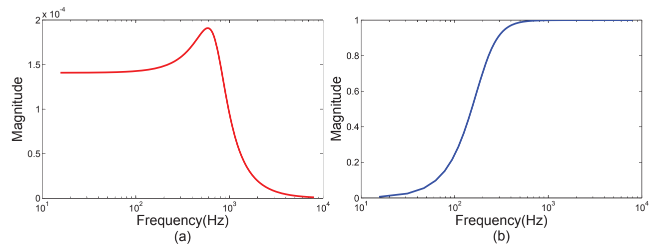

Fig. 3. (a) A linearized micro-loudspeaker excursion to voltage frequency response with $m_{\mathrm{d}} = 0.014$g, $c_{\mathrm{d}} = 0.039$ Ns/m, $k_{\mathrm{d}} = 284$ N/m, $\phi_0 = 0.3$ N/A and $R_{\mathrm{eb}} = 7.5\Omega$. (b) The frequency response of a corresponding digital Butterworth highpass filter with $f_{\mathrm{cutoff}} = 200$ Hz and $F_s = 16$ kHz.

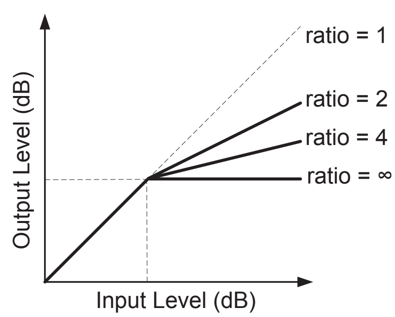

Fig. 4. The input-output relationship of the excursion compression module.

5. CONCLUSIONS

This paper proposed a model based excursion protection algorithm to protect loudspeakers from excessive diaphragm excursion, one of the key causes of loudspeaker failure. A

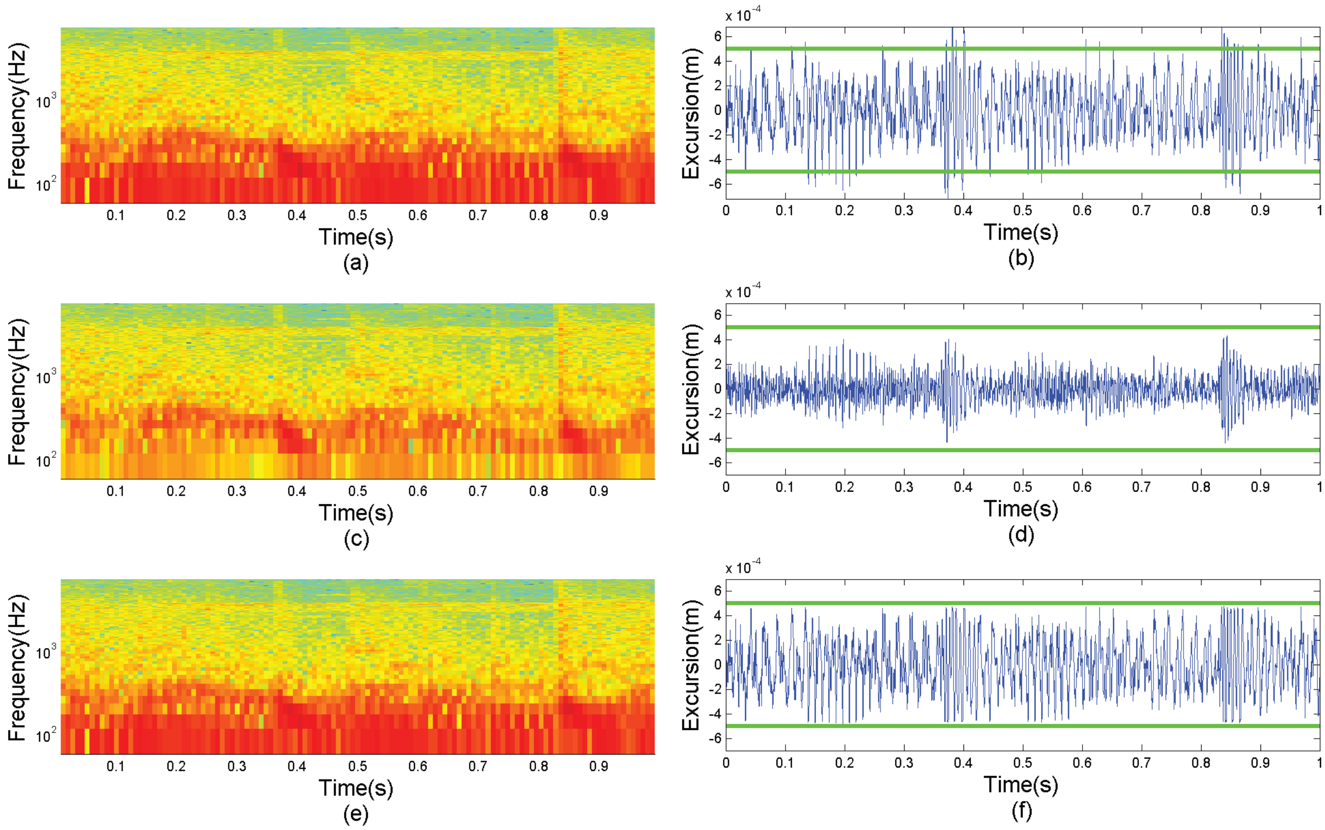

Fig. 5. A comparison of distortion and excursion protection effects when playing a music clip. (a) The original input voltage spectrogram and (b) diaphragm excursion without protection; (c) a spectrogram of the highpass filtered input voltage and (d) diaphragm excursion with the highpass filtering protection algorithm; and (e) a spectrogram of the input voltage processed by the proposed algorithm and (f) diaphragm excursion with the proposed algorithm.

digital loudspeaker model was established to predict the diaphragm excursion based on the input voltage signal. The predicted excursion is controlled using dynamic range compression in the excursion domain. The corresponding input voltage to the loudspeaker is then determined by an inverse loudspeaker model. Compared with a conventional highpass filtering algorithm, the proposed algorithm provides a more accurate and timely control of the loudspeaker diaphragm excursion while minimizing unnecessary interventions to the system.

6. REFERENCES

[1] Leung Kin Chiu, D.V. Anderson, and B. Hoomes, "Audio output enhancement algorithms for piezoelectric loudspeakers," in *Digital Signal Processing Workshop and IEEE Signal Processing Education Workshop (DSP/SPE), 2011 IEEE*, Jan. 2011, pp. 317 –320.

[2] Andrew Bright, *Active control of loudspeakers: An investigation of practical applications*, Ph.D. thesis, Technical University of Denmark, Denmark, 2002.

[3] Wolfgang Klippel, "The mirror filter-a new basis for linear equalization and nonlinear distortion reduction of woofer systems," in *Audio Engineering Society Convention 92*, 3 1992.

[4] Wolfgang Klippel, "Tutorial: Loudspeaker nonlinearitiesłcauses, parameters, symptoms," *J. Audio Eng. Soc*, vol. 54, no. 10, pp. 907–939, 2006.

[5] Andrew Bright, "Tracking changes in linear loudspeaker parameters with current feedback," in *Audio Engineering Society Convention 115*, 10 2003.

[6] Wolfgang Klippel, "Dynamic measurement and interpretation of the nonlinear parameters of electrodynamic loudspeakers," *J. Audio Eng. Soc*, vol. 38, no. 12, pp. 944–955, 1990.