SPEED OF SOUND AND AIR TEMPERATURE ESTIMATION USING THE TDOA-BASED LOCALIZATION FRAMEWORK

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ABSTRACT

Spatially distributed acoustic sensors find increasingly new applications in speech-based human-machine interfaces. One well researched topic is the localisation of an emitting source from Time-Difference-Of-Arrival (TDOA) measurements. This manuscript shows how to exploit the source localization framework and its state of the art techniques to accurately estimate the actual sound speed and the air temperature by measuring TDOAs of an unknown located acoustic source. Simulations and experiments show the validity of the proposed method.

1. INTRODUCTION

The dependence of the speed of sound on the temperature of the propagation medium is well known in acoustics. It has been often exploited for investigations on the average temperature and spatial temperature distributions. For instance in [1] by knowing emitter and receiver positions a tomography approach is used to measure temperature and wind velocity. In contrast, the approach taken here does not require to know the source position or the source signal, neither does it require synchronization between emitter and receiver. It starts from a well researched area, the localization of sound sources like human speakers. Due to the lack of synchronization between source and receiver, the absolute time of flight of the sound waves cannot be estimated. However, Time-Difference-Of-Arrival (TDOA) between different sensors can be estimated well by suitable signal correlation. TDOA-based methods are well established in source localization [2], a valuable review can be found e.g. in [3]. Most of these localization methods assume known propagation speed, which is a reliable assumption only under laboratory and controlled conditions. Methods which give a position estimate jointly with an estimate of the signal propagation speed are presented e.g. in [4–6]. Unfortunately in noisy conditions the so-obtained speed estimate turns out to be unsatisfactory and thus they are not suitable for temperature estimation purpose. The authors proposed in [7] a novel method to estimate the propagation speed and the corresponding air temperature from TDOAs measured by a sensor array.

This manuscript presents a comparison between such a novel method and related methods by means of simulations with the Cramer-Rao Bound (CRB) and real measurements with a new 3D microphone array. The manuscript is structured as follows. In Sec. 2 an overview of the source localization problem is given and some standard source localization methods are reviewed, also methods for joint localization and propagation speed estimation are presented. Sec. 3 describes thoroughly the proposed method for estimating the propagation speed. Sec. 4 is devoted to simulation and experimental results. Finally in Sec. 5 conclusions are drawn.

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2. SOURCE LOCALIZATION METHODS

When the absolute time delay between source and receiver is not available due to lack of synchronization the passive localization of an emitting source can be still performed by exploiting the Time-Difference-Of-Arrival (TDOA) between different receivers.

2.1. TDOA-Based Source Localization Problem

Consider the Euclidean space of $D = 2, 3$ dimensions. For sake of simplicity the following description refers to the case $D = 2$ depicted in Fig. 1. The acoustic source to be localized lies in an unknown position $\boldsymbol{x} = [x\ y]^{\mathrm{T}}$ and the M sensors of the array are distributed at the known positions $\boldsymbol{a}_i = [x_i\ y_i]^{\mathrm{T}}$ with $i = 0, \cdots, N$ and $N = M - 1$. Let $\tau_{i,j}$ indicate the time differences of arrival between the sensor $\boldsymbol{a}_i$ and the reference sensor $\boldsymbol{a}_j$. The $(M \times N)/2$ independent TDOAs $\tau_{i,j}$ for $0 \leq j < i \leq M$ are usually referred as the *full* TDOA set [8]. A special subset of the full set, the so-called *spherical* TDOA set, is mostly employed for localization. It can be obtained with respect to an arbitrary reference sensor $\boldsymbol{a}_j$, if the sensor $\boldsymbol{a}_0$ is chosen as reference, then the corresponding spherical set consists of the values $\tau_{i,0},\ i = 1, \cdots, N$. Here the vectors $\boldsymbol{\tau}_j,\ j = 0, \ldots, N$ represent spherical sets with respect to different reference sensors $\boldsymbol{a}_j,\ j = 0, \ldots, N$ (see colored graphs in Fig. 1).

The TDOA-based localization problem consists of finding $\boldsymbol{x}$ given the sensor positions $\boldsymbol{a}_i$ and one of the mentioned TDOA sets, e.g. the spherical set $\boldsymbol{\tau}_0 = [\tau_{1,0}\ \ldots\ \tau_{N,0}]^{\mathrm{T}}$. Each TDOA of the set

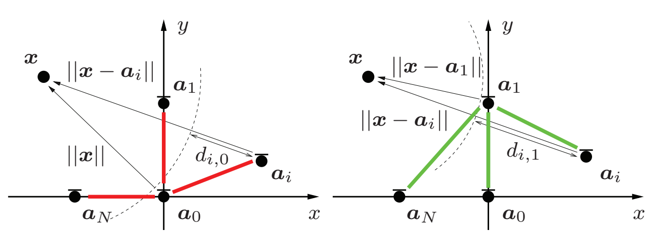

Fig. 1: Geometry of the two-dimensional source localization problem using a sensor array. The extension of the array is large enough to infer the source distance from the circular wave front. The left figure shows in red the graph corresponding to the set $\boldsymbol{\tau}_0$ while the right figure shows in green the graph corresponding to the set $\boldsymbol{\tau}_1$.

$\boldsymbol{\tau}_0$ can be expressed in terms of the travelled range difference as

$$\tau_{i,0} = \frac{1}{c}\, d_{i,0}\ , \qquad i = 1, \cdots, N\ , \tag{1}$$

where c is the signal propagation speed and $d_{i,0}$ denotes the source's range difference between the sensor $\boldsymbol{a}_i$ and the reference sensor $\boldsymbol{a}_0$ as shown in Fig. 1 left. Here the speed value c is assumed to be known, even though it might be unknown (see Sec. 2.2.3). Without loss of generality the reference $\boldsymbol{a}_0$ is assumed to be in the origin, then from geometrical reasoning (see Fig. 1 left) follows that

$$d_{i,0} = ||\boldsymbol{x}|| - ||\boldsymbol{x} - \boldsymbol{a}_i||\ , \qquad i = 1, \cdots, N\ , \tag{2}$$

where $||\cdot||$ denotes the Euclidean vector norm. The candidate position $\boldsymbol{x}$ must fulfill the above N equations.

2.2. Previous Works

It is well known, e.g. from [2,9], that by squaring Eq. (2) the following simpler set of equations can be obtained

$$\boldsymbol{a}_i^{\mathrm{T}}\boldsymbol{x} + d_{i,0}||\boldsymbol{x}|| = \frac{||\boldsymbol{a}_i^2|| - d_{i,0}^2}{2}\ , \qquad i = 1, \cdots, N\ . \tag{3}$$

The corresponding system of equations is described in matrix form as follows

$$\boldsymbol{\Phi}\boldsymbol{y}(\boldsymbol{x}) = \boldsymbol{b}\ , \tag{4}$$

$$\boldsymbol{y}(\boldsymbol{x}) = \begin{bmatrix} \boldsymbol{x} \\ ||\boldsymbol{x}|| \end{bmatrix}\ , \quad \boldsymbol{\Phi} = \left[\boldsymbol{A}\,\middle|\,\boldsymbol{d}_0\right]\ , \tag{5}$$

$$\boldsymbol{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_N \end{bmatrix}, \quad \boldsymbol{d}_0 = \begin{bmatrix} d_{1,0} \\ \vdots \\ d_{N,0} \end{bmatrix}, \quad \boldsymbol{A} = \begin{bmatrix} \boldsymbol{a}_1^{\mathrm{T}} \\ \vdots \\ \boldsymbol{a}_N^{\mathrm{T}} \end{bmatrix}\ , \tag{6}$$

$$b_i = \frac{1}{2}\left(||\boldsymbol{a}_i||^2 - d_{i,0}^2\right)\ . \tag{7}$$

Solving the system in (4) is an estimation problem since the elements of $\boldsymbol{A}$ and $\boldsymbol{d}_0$ are subject to uncertainties, i.e. $\boldsymbol{\Phi}\boldsymbol{y}(\boldsymbol{x}) \approx \boldsymbol{b}$. Typically the sensor positions $\boldsymbol{a}_i$ of a well constructed sensor array are considered to be exactly known, whereas the range differences $d_{i,0}$ are from (1) where the values $\tau_{i,0}$ are noisy correlation results. To avoid the direct solution of such a nonlinear estimation problem closed-form localization methods have been devised which provide approximate solutions in a linear fashion.

2.2.1. Unconstrained Least Squares Method (ULS)

Introducing a new scalar variable r independent of $\boldsymbol{x}$ in place of the norm $||\boldsymbol{x}||$ enables to address the problem as a linear least squares estimation of the unknown vector $\boldsymbol{y} = [\boldsymbol{x} \quad r]^{\mathrm{T}}$, where the last element no longer depends on the other vector elements. Provided that $N \geq D + 1$, such a least squares estimate is given in terms of the pseudo-inverse $\boldsymbol{\Phi}^{\dagger}$

$$\hat{\boldsymbol{y}} = \begin{bmatrix} \hat{\boldsymbol{x}} \\ \hat{r} \end{bmatrix} = \boldsymbol{\Phi}^{\dagger}\boldsymbol{b} = (\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{\Phi})^{-1}\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{b}\ . \tag{8}$$

This estimate should be considered an approximate solution of (4) since $\boldsymbol{y}$ is the *relaxed* or rather *unconstrained* [10] version of the vector $\boldsymbol{y}(\boldsymbol{x})$ and in general $\hat{r} \neq ||\hat{\boldsymbol{x}}||$.

In [11] it is given an alternative expression of the above estimate which separates range and position estimations and shows the dependency on the assumed propagation speed c

$$\hat{r}(c) = \frac{1}{c}\boldsymbol{\Theta}\boldsymbol{b}(c)\ , \quad \hat{\boldsymbol{x}}(c) = \boldsymbol{\Gamma}\boldsymbol{b}(c)\ , \tag{9}$$

where the matrices

$$\boldsymbol{\Theta} = (\boldsymbol{P}_{\boldsymbol{A}}^{\perp}\boldsymbol{\tau}_0)^{\dagger}\ , \quad \boldsymbol{\Gamma} = (\boldsymbol{P}_{\boldsymbol{\tau}_0}^{\perp}\boldsymbol{A})^{\dagger}\ , \tag{10}$$

are obtained from the orthogonal projectors shown in [11] and the vector $\boldsymbol{b}(c)$ is simply obtained by using (1) in (7).

2.2.2. Constrained Least Squares Method (CLS)

Constrained methods aim at a more accurate localization by finding an estimate $\tilde{\boldsymbol{y}} = [\tilde{\boldsymbol{x}} \quad \tilde{r}]^{\mathrm{T}}$ which obeys the constraint between range and position, i.e $\tilde{r} = ||\tilde{\boldsymbol{x}}||$. The constrained least squares solution of (4) may be obtained employing the Lagrange multipliers technique [10]. More attractive is its linear approximation which benefits from the closed-form estimate given in [3]. By defining the residual vector $\boldsymbol{\epsilon}(\hat{\boldsymbol{x}})$ and the Jacobian matrix $\boldsymbol{J} = \boldsymbol{\epsilon}'(\hat{\boldsymbol{x}})$

$$\boldsymbol{\epsilon}(\hat{\boldsymbol{x}}) = \boldsymbol{\Phi}\boldsymbol{\delta}\ , \quad \boldsymbol{J} = \boldsymbol{\epsilon}'(\hat{\boldsymbol{x}}) = \boldsymbol{\Phi}\boldsymbol{G}\ , \tag{11}$$

with

$$\boldsymbol{\delta} = \begin{bmatrix} \boldsymbol{0} \\ ||\hat{\boldsymbol{x}}|| - \hat{r} \end{bmatrix}\ , \quad \boldsymbol{G} = \begin{bmatrix} \mathbf{I} \\ \frac{\hat{\boldsymbol{x}}^{\mathrm{T}}}{||\hat{\boldsymbol{x}}||} \end{bmatrix}\ , \tag{12}$$

the linear approximation of the constrained estimate $\tilde{\boldsymbol{x}}$ reads

$$\tilde{\boldsymbol{x}} = \hat{\boldsymbol{x}} - \boldsymbol{J}^{\dagger}\boldsymbol{\Phi}\boldsymbol{\delta}\ . \tag{13}$$

2.2.3. Joint Position and Speed Estimation

For in-air propagation with strong temperature variations or in-solid scenarios the propagation speed might be unknown. Therefore localization methods have been devised which estimate the position jointly with the signal propagation speed.

Some authors [4–6] form from Eq. (2) a linear system in $D + 2$ unknowns, from which a speed estimate is obtained by means of an unconstrained least squares estimation of $D + 2$ unknowns. This method is basically an extension of the ULS from Sec. 2.2.1 but unfortunately the so-obtained system matrix might be easily ill-conditioned as shown e.g. in [12]. Moreover the speed estimate tends to be unreliable even if the localization result is decent.

In [12] it is shown that more robust results for the speed estimation can be obtained by assuming plane wave propagation. However the so-obtained estimate is affected by a bias when the source is close to the array and the plane wave approximation no longer applies.

3. NOVEL SPEED ESTIMATION METHOD

A different approach to estimate the propagation speed has been recently proposed by the authors in [7]. In ideal conditions and knowing exactly the actual propagation speed (denoted here as c°), the vector $\boldsymbol{\epsilon}(\hat{\boldsymbol{x}}) = \boldsymbol{\Phi}\boldsymbol{\delta}$ from (11) vanishes since $\hat{r} = ||\hat{\boldsymbol{x}}||$. On the other hand when $c \neq c^{\circ}$ such a vector engenders a systematic deviation between the unconstrained and constrained solutions. Using Eqns. (9) it may be expressed as function of the assumed speed c, i.e.

$$\boldsymbol{\epsilon}(c) = \boldsymbol{\Phi}(c)\begin{bmatrix} \boldsymbol{0} \\ \delta(c) \end{bmatrix} = c\delta(c)\boldsymbol{\tau}_0\ , \tag{14}$$

with

$$\delta(c) = ||\hat{\boldsymbol{x}}(c)|| - \hat{r}(c) = ||\boldsymbol{\Gamma}\boldsymbol{b}(c)|| - \frac{1}{c}\boldsymbol{\Theta}\boldsymbol{b}(c)\ . \tag{15}$$

Then the value $\hat{c}$ which annihilates the vector in (14) has to be equal to the actual propagation speed c° as Eqns. (9) provide compatible estimates $\hat{r}$ and $\hat{\boldsymbol{x}}$. Indeed the searched speed value has to be a zero of the scalar function $\delta(c)$ (the trivial solution $\hat{c} = 0$ of (14) is not of interest). The next section shows an efficient way to find such a speed value in acoustic applications.

3.1. Speed of Sound and Air Temperature Estimation

The function $\delta(c)$ involving the Euclidean norm of $\hat{\boldsymbol{x}}$ is nonlinear, therefore applying a root-finding algorithm might be demanding. Nonetheless near to the actual propagation speed c° it can be shown that such a function has a fairly linear behavior [7]. This means that given a reliable initial guess $\bar{c}$, the following first order Taylor expansion is a useful approximation

$$\delta(c) \approx \delta_{\text{lin}}(c) = \bar{\delta} + \bar{\delta}'(c - \bar{c}) \ , \tag{16}$$

with

$$\bar{\delta} = \delta(\bar{c}) \quad \text{and} \quad \bar{\delta}' = \left.\frac{\mathrm{d}\delta(c)}{\mathrm{d}c}\right|_{c=\bar{c}} . \tag{17}$$

Consider now an acoustic scenario and the problem of estimating the actual speed of sound from microphone measurements, in many cases a reliable initial guess for the speed of sound can be obtained from the standard air temperature value of $\bar{\theta} = 20\ ^\circ\mathrm{C}$ and the well-known formula

$$\bar{c} = 331\frac{\mathrm{m}}{\mathrm{s}} + 0,6\,\frac{\mathrm{m}}{\mathrm{s}\,^\circ\mathrm{C}}\,\bar{\theta} \ . \tag{18}$$

Then the actual speed can be inferred with enough accuracy from the zero-crossing of the linearized function $\delta_{\text{lin}}(c) = 0$. The searched sound speed value is given by

$$\hat{c} = \frac{\bar{\delta} - \bar{\delta}'\bar{c}}{\bar{\delta}'}. \tag{19}$$

The value of the first order derivative at $\bar{c}$ can be calculated with derivation rules from (15)

$$\bar{\delta}' = \frac{\hat{\boldsymbol{x}}(\bar{c})^{\mathrm{T}}}{||\hat{\boldsymbol{x}}(\bar{c})||}\boldsymbol{\Gamma}\bar{\boldsymbol{b}}' + \frac{\hat{r}(\bar{c})}{\bar{c}} - \boldsymbol{\Theta}\frac{\bar{\boldsymbol{b}}'}{\bar{c}} \ , \tag{20}$$

where $\hat{\boldsymbol{x}}(\bar{c})$ and $\hat{r}(\bar{c})$ are the unconstrained estimates of position and range obtained with the initial guess $\bar{c}$ while $\bar{\boldsymbol{b}}'$ is a vector containing the derivatives of (7) evaluated at $c = \bar{c}$.

Finally the so-obtained estimate $\hat{c}$ of the speed of sound can be converted using (18) into an average value of the air temperature in the volume enclosing the sensor array. This methodology is applied in the experimental part described in Sec. 4.

3.2. Extension to the Full TDOA Set

So far the used spherical TDOA set $\boldsymbol{\tau}_0$ has been derived with respect to the reference sensor $\boldsymbol{a}_0$ but actually the localization can be carried out in much the same way regarding an arbitrary spherical set $\boldsymbol{\tau}_j$ different from $\boldsymbol{\tau}_0$. As a consequence the scalar function $\delta(c)$ in (15) can be built for all TDOA sets $\boldsymbol{\tau}_j$. Then in noisy conditions an improved speed estimate can be found as the minimizer in the least squares sense of such functions, i.e. the corresponding least squares criterion is

$$\sum_{j=0}^{N} \delta_j(c)^2 = \sum_{j=0}^{N} \left(||\hat{\boldsymbol{x}}_j(c)|| - \hat{r}_j(c)\right)^2 \ , \tag{21}$$

where $\hat{\boldsymbol{x}}_j$ and $\hat{r}_j$ are unconstrained estimates of position and range obtained using the spherical set $\boldsymbol{\tau}_j$. In practice the proposed speed estimation method has been straightforwardly extended to exploit the full TDOA set since the required spherical sets $\boldsymbol{\tau}_j,\ j = 0, \ldots, N$ can be generated from the elements of the full TDOA set.

4. SIMULATION AND EXPERIMENTAL RESULTS

The simulated scenario considers a source lying in the xy-plane at a fixed distance of 1.5 m from the center of a sensor array. A regular distribution of $M = 10$ sensors simulates the real cross array used for the experiments (see Fig. 3). The standard deviation of the error corrupting the ideal TDOA vector $\boldsymbol{\tau}^\circ$ is set to be $\sigma = 10$ ms as in [13]. The source's bearing angle is sampled homogeneously at 48 positions and for each position bias and variance of the estimation are obtained over 1000 Monte Carlo trials. The speed estimation is carried out with the methods from [5] and [12] discussed in Sec. 2.2.3 and the novel one from Sec. 3. Here they are referred to as *ULS extended*, *ULS extended (plane wave)* and *proposed method* respectively. Fig. 2 shows the corresponding results along with the

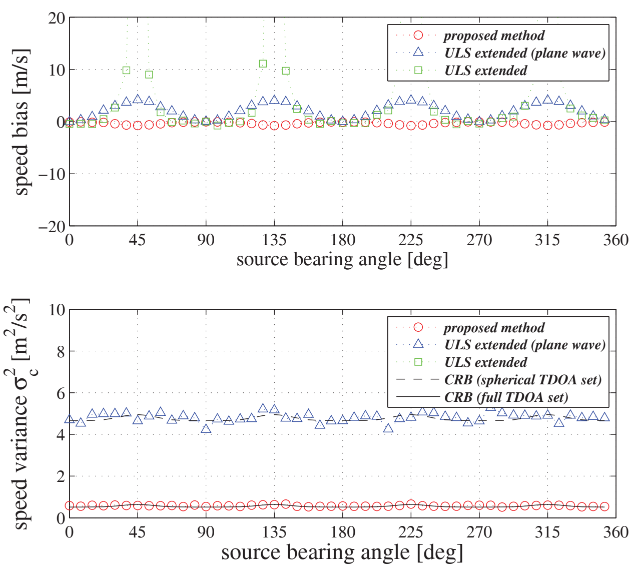

Fig. 2: Simulation results of the propagation speed estimation.

Cramer-Rao Bound (CRB) plotted against the source bearing. The CRB is employed here as a lower bound on the variance of any unbiased speed estimator according to [4]. The ULS extended suffers from a huge variance (which is out of the displayed range) and it is completely unreliable at certain angles. Thus it is not suited for speed estimation purpose and it will not be further considered in the experimental part. The ULS extended (plane wave) as expected suffers from a bearing-dependent bias since the source lies quite close to the sensors. This effect is evident at the odd multiples of $\frac{\pi}{4}$ due to the regular geometry of the array. The proposed method outperforms the others in terms of bias and variance. It is quite unbiased regardless of the source bearing and its variance attains the lower CRB corresponding to the full TDOA set.

Real measurements have been carried out in order to prove the practical validity of the presented theory. To this end room temperature estimates were derived from the TDOA-based sound speed estimation while the actual room temperature was measured by means of an electronic thermometer at the position of one loudspeaker. The experimental setup visible in Fig. 3 consists of a cross sensor array of $M = 10$ microphones with a maximum arm length of 0.35 cm positioned at the center of a circular 48-loudspeaker array with radius 1.5 m. White noise signals were sequentially emitted by the

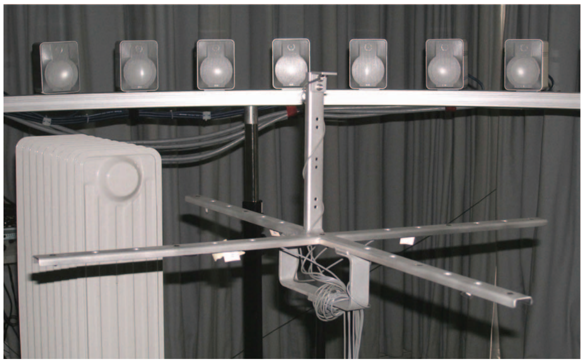

Fig. 3: Setup of temperature estimation experiment. The heater on the left was used to raise the room temperature from 22.1 °C to 27.0 °C.

loudspeakers in order to obtain by signal correlation (GCC-PHAT) TDOAs corresponding to 48 source's bearing angles. From these TDOAs a temperature estimate was derived for each bearing angle using the ULS extended (plane wave) [12] and the proposed method. The experiment was conducted twice. The first time the room temperature measured with the thermometer was 22.1 °C, the second time the room temperature was raised to 27.0 °C thanks to the heater visible in Fig. 3. The collected results are displayed in Fig. 4. In accordance with the simulations the plane wave approximation yields a strong bias at the odd multiples of $\frac{\pi}{4}$ which leads to unacceptable temperature values. On the other hand the proposed method provides reliable results regardless of the source bearing. The estimated temperatures are always around and close to the actual room temperatures (22.1 °C and 27.0 °C).

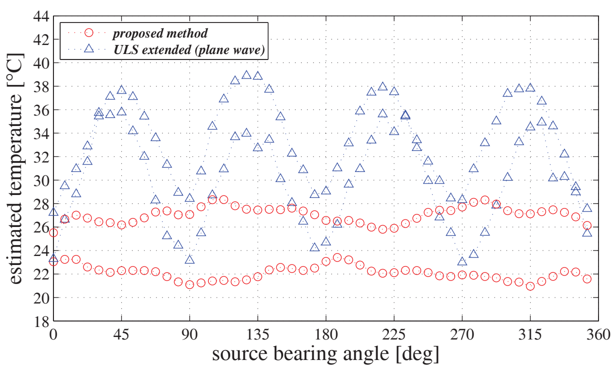

Fig. 4: Results of the temperature estimation experiments. The temperature estimates obtained with the proposed method are always around and close to the actual room temperatures (22.1 °C and 27.0 °C).

5. CONCLUSIONS

This paper addresses the problem of estimating the signal propagation speed from TDOAs measured by spatially distributed sensors. The presented theory relies on the source localization framework, as a practical case the estimation of the speed of sound and the corresponding air temperature by means of a microphone array has been considered. Simulations and experimental results show that the proposed method is by far the most reliable and accurate.

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