

DIFFUSED SENSING FOR SHARP DIRECTIVITY MICROPHONE ARRAY

Kenta Niwa, Sumitaka Sakauchi, Ken'ichi Furuya, Manabu Okamoto and Yoichi Haneda

NTT Cyber Space Laboratories, NTT Corporation, Tokyo, Japan

ABSTRACT

We propose a method for achieving sharp directivity by sensing signals in a diffuse acoustic field. Directivity control based on a beamforming method has been studied to make it possible to extract the waveform and location of an identified target source even if there are many noise sources. Sharp directivity can be achieved by minimizing the output noise power of a beamforming filter. However, it is difficult to minimize the output noise power over a broad frequency ranges. Our approach for minimizing the output noise power is to control the spatial properties of the transfer functions and the spatial correlation matrix, by using a reflector that surrounds a microphone array. We investigated the relationships between the output noise power and the structure of the spatial correlation matrix and found that it was possible to minimize the output noise power by sensing diffuse acoustic signals and by designing filters taking the diffuseness of the acoustic field into consideration. In experiments, we observed diffusely reflected signals by placing a truncated-octahedral reflector near a spherical microphone array. We designed filters by using measured transfer functions and confirmed that the proposed method was effective for reducing the output noise power and forming a sharp directivity beamforming filter.

Index Terms— Microphone array, Beamforming filter, Spatial correlation matrix, Transfer function, Diffuse acoustic signal

1. INTRODUCTION

The use of a microphone array to control directivity has been studied to estimate the waveform and location of an identified target source [1] [2]. Beamforming methods are useful for facilitating teleconferencing systems [3], speech recognition [4], and natural human-robot communication [5]. Since these applications are expected to work well even when there are many noise sources, it is important to generate sharp directive beamforming in which the width of the main lobe is narrowed.

Sharp directivity can be achieved by minimizing the output noise power while constraining the response gain of the target source over a broad frequency ranges. Many methods for designing beamforming filters have been proposed to minimize the output noise power; these include the delay-and-sum method [6], the minimum variance distortionless response (MVDR) method [7], and the maximum signal-to-noise (S/N) ratio method [8]. In addition, Flanagan proposed the multi-beamforming method [9], which improves the S/N ratio by forming multiple delay and sum filters for emphasizing direct and reflected sounds. However, it is difficult to minimize the output noise power since the cross-correlation between observed signals is strong in some frequencies. Since the output noise power cannot be minimized then, our goal to form sharp directivity cannot be achieved.

We therefore propose a “diffused sensing method” to achieve sharp directivity. The method involves sensing diffuse acoustic sig-

nals by controlling the spatial properties of the transfer functions and the spatial correlation matrix [10]. Since the cross-correlation between signals observed in a diffuse acoustic field weakens [11], we presume that the output noise power can be reduced by observing the diffused acoustic signals. We positioned a reflector around a microphone array in order to observe diffusely reflected signals. The filters were designed taking the acoustic properties of the reflector into account. These properties are calculated by using measured transfer functions. Since the filters and the observed signals can be optimized to reduce the output noise power, sharp directivity is expected to be obtainable by using the proposed method.

This paper is organized as follows. The conventional beamforming technique is explained in Section 2. In Section 3, the structure of the spatial correlation matrix for achieving sharp directivity is discussed. We propose the diffused sensing method in Section 4, and a technique for implementing our method is described in Section 5. Experiments that were conducted by using an actual reflector are discussed in Section 6, and we conclude the paper in Section 7.

2. BEAMFORMING

Let us consider that M microphones receive a target and $K - 1$ noise sources. Our goal is to achieve sharp directivity to emphasize the target source arriving from an arbitrary direction even if K is a large number. The transfer function from the m -th microphone to the target and the k -th noise source at frequency ω are denoted by $A_{S,m}(\omega)$ and $A_{N_k,m}(\omega)$, respectively. When the source signal of the target and the k -th noise in the frequency domain are respectively described by $S(\omega, t)$ and $N_k(\omega, t)$, the observed signals $\mathbf{x}(\omega, t) = [X_1(\omega, t), \dots, X_M(\omega, t)]^T$ are given by

$$\mathbf{x}(\omega, t) = \mathbf{A}(\omega)\mathbf{s}(\omega, t), \quad (1)$$

$$\mathbf{A}(\omega) = [\mathbf{a}_S(\omega), \mathbf{a}_{N_1}(\omega), \dots, \mathbf{a}_{N_{K-1}}(\omega)],$$

$$\mathbf{a}_S(\omega) = [A_{S,1}(\omega), \dots, A_{S,M}(\omega)]^T,$$

$$\mathbf{a}_{N_k}(\omega) = [A_{N_k,1}(\omega), \dots, A_{N_k,M}(\omega)]^T,$$

$$\mathbf{s}(\omega, t) = [S(\omega, t), N_1(\omega, t), \dots, N_{K-1}(\omega, t)]^T,$$

where t and T denote a frame time and the transposition, respectively. It is assumed that $S(\omega, t)$ and $N_k(\omega, t)$ are temporally uncorrelated with each other as

$$\mathbb{E}\{\mathbf{s}(\omega, t)\mathbf{s}^H(\omega, t)\} = \mathbf{I}_K, \quad (2)$$

where $\mathbb{E}\{\cdot\}$ and H denote the expectation operator and Hermitian conjugate, respectively. The output signal of beamforming $Y(\omega, t)$ is obtained by convolving the observed signals and the filter $\mathbf{w}(\omega) = [W_1(\omega), \dots, W_M(\omega)]^T$.

$$Y(\omega, t) = \mathbf{w}^H(\omega)\mathbf{x}(\omega, t), \quad (3)$$

Here, the spatial correlation matrix denoted by $\mathbf{R}(\omega)$ is defined since it is required in designing $\mathbf{w}(\omega)$.

$$\begin{aligned}\mathbf{R}(\omega) &= \mathbb{E}\{\mathbf{x}(\omega, t)\mathbf{x}^H(\omega, t)\} \\ &= \frac{1}{K}\mathbf{A}(\omega)\mathbb{E}\{\mathbf{s}(\omega, t)\mathbf{s}^H(\omega, t)\}\mathbf{A}^H(\omega) \\ &= \frac{1}{K}\left[\mathbf{a}_S(\omega)\mathbf{a}_S^H(\omega) + \sum_{k=1}^{K-1}\mathbf{a}_{N_k}(\omega)\mathbf{a}_{N_k}^H(\omega)\right].\end{aligned}\quad (4)$$

Our goal can be achieved by minimizing the output noise power $P_{\text{out}}(\omega)$ over a broad frequency ranges as in Eq. (5), while the gain of the target source is subject to Eq. (6).

$$\min_{\mathbf{w}} \left(P_{\text{out}}(\omega) = \mathbb{E}\{|Y(\omega, t)|^2\} = \mathbf{w}^H(\omega)\mathbf{R}(\omega)\mathbf{w}(\omega) \right), \quad (5)$$

$$\text{subject to } \mathbf{w}^H(\omega)\mathbf{a}_S(\omega) = 1, \quad (6)$$

This filter designing method is known as the MVDR method with a constraint [12]. Therefore, $\mathbf{w}(\omega)$ is calculated by

$$\mathbf{w}(\omega) = \frac{\mathbf{R}^{-1}(\omega)\mathbf{a}_S(\omega)}{\mathbf{a}_S^H(\omega)\mathbf{R}^{-1}(\omega)\mathbf{a}_S(\omega)}. \quad (7)$$

3. BASIC PROPERTY OF OUTPUT NOISE POWER

The value of $P_{\text{out}}(\omega)$ is dependent on the spatial correlation matrix. Our basic idea for minimizing $P_{\text{out}}(\omega)$ is to control the spatial properties of the transfer functions e.g., by placing the reflector near the microphone array. Then, the spatial correlation matrix $\mathbf{R}(\omega)$ is also varied from Eq. (4). In this section, we derive the structure of $\mathbf{R}(\omega)$ for minimizing $P_{\text{out}}(\omega)$.

3.1. Relationships between output noise power and eigenspace of spatial correlation matrix

To investigate the relationships between $P_{\text{out}}(\omega)$ and the structure of the spatial correlation matrix, $\mathbf{R}(\omega)$ is decomposed by using the eigenvalue method as

$$\mathbf{R}(\omega) = \mathbf{V}(\omega)\mathbf{\Lambda}(\omega)\mathbf{V}^H(\omega), \quad (8)$$

where $\mathbf{V}(\omega) = [\mathbf{v}_1(\omega), \dots, \mathbf{v}_M(\omega)]$ is organized by M eigenvectors $\mathbf{v}_m(\omega) = [V_{m,1}(\omega), \dots, V_{m,M}(\omega)]^T$. Also, $\mathbf{\Lambda}(\omega) = \text{diag}\{[\Lambda_1^2(\omega), \dots, \Lambda_M^2(\omega)]\}$ is composed of M eigenvalues. If $L(\omega) (\leq M)$ eigenvalues are greater than 0, the eigenvalues are sorted as $\Lambda_1^2(\omega) \geq \dots \geq \Lambda_{L(\omega)}^2(\omega) > 0$. Since the eigenvectors are orthogonal to each other, the inverse matrix of $\mathbf{R}(\omega)$ is calculated by $\mathbf{R}^{-1}(\omega) = \mathbf{V}(\omega)\mathbf{\Lambda}^{-1}(\omega)\mathbf{V}^H(\omega)$. By substituting Eqs. (7) and (8) into $P_{\text{out}}(\omega)$, it is expanded as,

$$\begin{aligned}P_{\text{out}}(\omega) &= \frac{1}{\mathbf{a}_S^H(\omega)\mathbf{R}^{-1}(\omega)\mathbf{a}_S(\omega)} = \left[\sum_{m=1}^{L(\omega)} \frac{|\mathbf{v}_m^H(\omega)\mathbf{a}_S(\omega)|^2}{\Lambda_m^2(\omega)} \right]^{-1} \\ &= \left[\sum_{m=1}^{L(\omega)} \frac{Q_{S,m}(\omega)}{Q_{N,m}(\omega)} \right]^{-1},\end{aligned}\quad (9)$$

where $Q_{S,m}(\omega) = |\mathbf{v}_m^H(\omega)\mathbf{a}_S(\omega)|^2$ is the output power of the target source through the m -th eigenvector. On the other hand, $Q_{N,m}(\omega) =$

$\Lambda_m^2(\omega)$ is the output power of K sound sources through $\mathbf{v}_m(\omega)$ from Eq. (10).

$$\begin{aligned}Q_{N,m}(\omega) &= \Lambda_m^2(\omega) = \mathbf{v}_m^H(\omega)\mathbf{R}(\omega)\mathbf{v}_m(\omega) \\ &= \frac{1}{K} \left[|\mathbf{v}_m^H(\omega)\mathbf{a}_S(\omega)|^2 + \sum_{k=1}^{K-1} |\mathbf{v}_m^H(\omega)\mathbf{a}_{N_k}(\omega)|^2 \right]\end{aligned}\quad (10)$$

Therefore, $Q_{S,m}(\omega)/Q_{N,m}(\omega)$ is recognized as the S/N ratio through $\mathbf{v}_m(\omega)$.

3.2. Structure of spatial correlation matrix for minimizing output noise power

We derive the structure of $\mathbf{R}(\omega)$ for minimizing $P_{\text{out}}(\omega)$ even if the target source is arriving from an arbitrary direction. When the target and noise sources are assumed to be propagated as plane waves, the power of the transfer functions is normalized as,

$$\mathbb{E}\{|A_{S,m}(\omega)|^2\} = \mathbb{E}\{|A_{N_k,m}(\omega)|^2\} = 1. \quad (11)$$

The sums of $Q_{S,m}(\omega)$ and $Q_{N_k,m}(\omega)$ are constrained as,

$$\begin{aligned}\sum_{m=1}^{L(\omega)} Q_{S,m}(\omega) &= \sum_{m=1}^{L(\omega)} |\mathbf{v}_m^H(\omega)\mathbf{a}_S(\omega)|^2 = \sum_{m=1}^{L(\omega)} |A_{S,m}(\omega)|^2 = L(\omega), \\ \sum_{m=1}^{L(\omega)} Q_{N,m}(\omega) &= \frac{1}{K} \sum_{m=1}^{L(\omega)} \left[|A_{S,m}(\omega)|^2 + \sum_{k=1}^{K-1} |A_{N_k,m}(\omega)|^2 \right] = L(\omega).\end{aligned}\quad (12)$$

Then, the value range of $P_{\text{out}}(\omega)$ is limited by

$$\frac{Q_{N,L(\omega)}(\omega)}{L(\omega)} \leq P_{\text{out}}(\omega) \leq \frac{Q_{N,1}(\omega)}{L(\omega)}. \quad (14)$$

From Eq. (14), we see that the maximum value of $P_{\text{out}}(\omega)$ is minimized by increasing $L(\omega)$ and decreasing $Q_{N,1}(\omega)$. Those conditions are obtained when $L(\omega) = M$ and $Q_{N,1}(\omega) = \dots = Q_{N,M}(\omega) = 1$; then $P_{\text{out}}(\omega)$ can be minimized as,

$$P_{\text{out}}(\omega) = \frac{1}{M}. \quad (15)$$

Since $Q_{N,m}(\omega)$ is the eigenvalue of the spatial correlation matrix, the structure of $\mathbf{R}(\omega)$ for minimizing $P_{\text{out}}(\omega)$ is summarized in order to flatten the eigenvalue distribution as $\Lambda_1^2(\omega) = \dots = \Lambda_M^2(\omega) = 1$.

4. PRINCIPLE OF PROPOSED METHOD

Our strategy for flattening the eigenvalue distribution of $\mathbf{R}(\omega)$ is to observe diffuse acoustic signals. In a diffuse acoustic field, the cross-correlation between observed signals [11] is given as

$$\Gamma_{m,n}(\omega) = \text{sinc}\left(\frac{\omega\|\mathbf{p}_m - \mathbf{p}_n\|}{\nu}\right), \quad (16)$$

where \mathbf{p}_m and ν denote the position vector of the m -th microphone and the sound velocity, respectively. The sound sources are assumed to be positioned distantly, and $\Gamma_{m,n}(\omega)$ corresponds to the (m, n) -th components of $\mathbf{R}(\omega)$ as

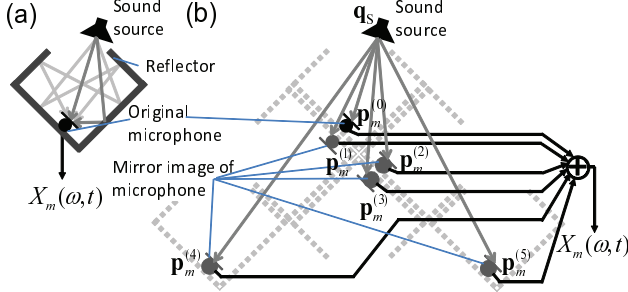


Fig. 1. Generation of diffusely reflected sounds using reflector

$$\mathbf{R}(\omega) = \begin{bmatrix} 1 & \Gamma_{1,2}(\omega) & \cdots & \Gamma_{1,M}(\omega) \\ \Gamma_{2,1}(\omega) & 1 & \cdots & \Gamma_{2,M}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ \Gamma_{M,1}(\omega) & \Gamma_{M,2}(\omega) & \cdots & 1 \end{bmatrix}. \quad (17)$$

Since $\Gamma_{m,n}(\omega)$ approaches asymptotically to 0 by positioning the microphones distantly, the spatial correlation matrix is approximated as $\mathbf{R}(\omega) = \mathbf{I}$. Then, the eigenvectors and the eigenvalues of $\mathbf{R}(\omega)$ are also obtained as $\mathbf{V}(\omega) = \mathbf{A}(\omega) = \mathbf{I}$. Then, $Q_{S,m}(\omega)$ and $Q_{N,m}(\omega)$ are calculated as

$$Q_{S,m}(\omega) = |\mathbf{v}_m^H(\omega) \mathbf{a}_S(\omega)|^2 = |A_{S,m}(\omega)|^2 = 1 \quad (m=1, \dots, M), \quad (18)$$

$$Q_{N,m}(\omega) = \Lambda_m^2(\omega) = 1 \quad (m=1, \dots, M). \quad (19)$$

Since the eigenvalue distribution of $\mathbf{R}(\omega)$ is flattened from Eq. (19), we see that $P_{\text{out}}(\omega)$ can be minimized by sensing signals in a diffuse acoustic field.

Our diffused sensing method is summarized as follows: i) Signals are sensed in a diffuse acoustic field to obtain optimum $\mathbf{R}(\omega)$ for reducing $P_{\text{out}}(\omega)$. This can be achieved through the use of a reflector, and the details are discussed in Sec. 5. ii) The $\mathbf{w}(\omega)$ is designed by taking the diffuseness of the acoustic field into consideration. Hereafter, it is assumed that the transfer functions are known and $\mathbf{w}(\omega)$ is designed by using them.

5. IMPLEMENTATION OF DIFFUSED SENSING USING MICROPHONE ARRAY AND REFLECTOR

An implementation of the diffused sensing method is explained. To control the spatial properties of the transfer functions and observe the diffuse acoustic signals, we placed a reflector near the microphone array. Figure 1 shows how the location of the reflector affects the transfer functions. When the microphone is surrounded by a reflector, direct and multiple reflected sounds are received, as shown in Fig. 1 (a). If the reflected sounds are assumed to be modeled using the image method [13], each reflected sound (image source) is regarded as the signal observed at the mirror image of the corresponding original microphone, as shown in Fig. 1 (b). If the microphones receive a direct (0-th image source) and D image sources, the transfer functions are calculated by summing the steering vectors [10]. When the d -th image source of the target source is denoted by $\mathbf{h}_S^{(d)}(\omega) = [H_{S,1}^{(d)}(\omega), \dots, H_{S,M}^{(d)}(\omega)]^T$, $\mathbf{a}_S(\omega)$ is given by

$$\mathbf{a}_S(\omega) = \sum_{d=0}^D \mathbf{h}_S^{(d)}(\omega), \quad (20)$$

$$H_{S,m}^{(d)}(\omega) = \frac{\kappa^{(d)}(\omega)}{\|\mathbf{p}_m^{(d)} - \mathbf{q}_S\|} \exp\left(-j\omega \frac{\|\mathbf{p}_m^{(d)} - \mathbf{q}_S\|}{\nu}\right), \quad (21)$$

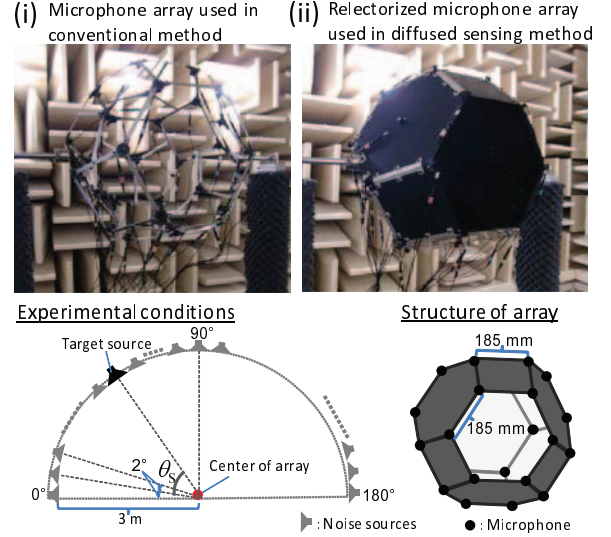


Fig. 2. Microphone arrays for measuring transfer functions

where $\mathbf{p}_m^{(d)}$, \mathbf{q}_S , and $\kappa^{(d)}(\omega)$ respectively denote the position vector of the mirror microphone of the d -th image source, that of the target source, and the reflection coefficient of the d -th image source ($\kappa^{(0)}(\omega) = 1$). Though the notations are omitted, $\mathbf{a}_{N_k}(\omega)$ is also composed of multiple image sources. By increasing D , the diffuseness of the observed signals will be increased. Since the eigenvalue distribution of $\mathbf{R}(\omega)$ will be then flattened, $P_{\text{out}}(\omega)$ is reduced.

The use of a reflector whose shape can fill the space, for example, a rectangular solid or truncated octahedron, is thought to be preferable for increasing D since the image sources are generated efficiently.

6. EXPERIMENTS

Experiments were conducted in an anechoic room to evaluate our diffused sensing method, which was implemented by placing a reflector near the microphone array. We investigated whether minimizing $P_{\text{out}}(\omega)$ and forming sharp directivity could be achieved using the proposed method.

6.1. Experimental conditions

We prepared two microphone arrays for the evaluation, as shown in Fig. 2. The difference between the two arrays was only in whether the truncated-octahedral reflector was used or not. The array in Fig. 2 (i) was used in the conventional method. There were 24 microphones positioned at the apexes of the truncated octahedron. Since the omni-directional microphones were supported by wires, few reflected sounds were included in the transfer functions. In contrast, Fig. 2 (ii) shows the array used in our diffused sensing method. The number, position, and directivity of microphones were the same as in Fig. 2 (i). The reflector was made of 8.0-mm-thick acrylonitrile butadiene styrene (ABS). Because the microphones captured signals inside the reflector, the transfer functions were composed of diffusely reflected sounds. The transfer functions of each array were measured by positioning a loudspeaker at 91 points, where the direction interval was 2.0 degrees. The arrival direction of the target source is denoted as θ_S . $\mathbf{w}(\omega)$ was calculated by substituting measured transfer functions into Eqs. (4) and (7). Table 1 lists the experimental parameters.

Table 1. Experimental parameters

Sampling frequency	48 kHz
Filter length	4096 taps
Number of microphones, M	24 ch
Number of sound sources, K	91
Direction interval of sound sources	2.0 deg
Minimum distance between microphones	185 mm
Minimum distance between sound sources	30 mm

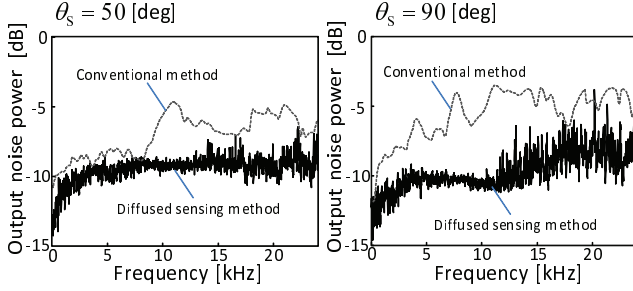


Fig. 3. Output noise power $P_{\text{out}}(\omega)$

6.2. Experimental results

Figure 3 shows the value of $P_{\text{out}}(\omega)$ when $\theta_s = 50$ and 90 degrees. It is clear that the $P_{\text{out}}(\omega)$ of our diffused sensing method was usually below that of the conventional method in all frequency ranges. Although the details have been omitted, the $P_{\text{out}}(\omega)$ of our diffused sensing method was well below that of the conventional method, especially when the target source was arriving from $\theta_s = 45$ to 135 degrees. We presume that $P_{\text{out}}(\omega)$ is reduced because the distribution of $Q_{s,m}(\omega)$ and $Q_{N,m}(\omega)$ are flattened when the diffused sensing method is used.

Figures 4 and 5 show the directivity of the conventional and the diffused sensing methods. As shown in Fig. 4, the beam width of the main lobe is a bit broad in the low frequency range, and spatial aliasing occurred in some directions in the high frequencies. On the contrary, sharp directivity to the arrival direction of the target source was achieved over a broad frequency ranges with the diffused sensing method, as shown in Fig. 5. We therefore confirmed that compared to the conventional method, the beam width of the main lobe was narrowed over a broad frequency ranges, and the spatial aliasing in the high frequency range was suppressed. We believe that the spatial aliasing was suppressed with the diffused sensing method because of the reduced cross-correlation between microphones.

7. CONCLUSION

We proposed the diffused sensing method to achieve sharp directivity by minimizing the output noise power. We found that the output noise power was minimized by sensing diffuse acoustic signals and by designing filters taking the transfer functions into account. To observe diffused signals, we placed a reflector near the microphone array. Through experiments using 24 microphones and the truncated-octahedral reflector, we confirmed that our diffused sensing method was effective for minimizing the output noise power and achieving sharp directivity.

Many other issues require further study; for example, we would like to investigate how well our method works in a reverberant environment, and we would like to reduce the number of transfer functions to be measured. Additionally, it is important to investigate the optimum reflector shape.

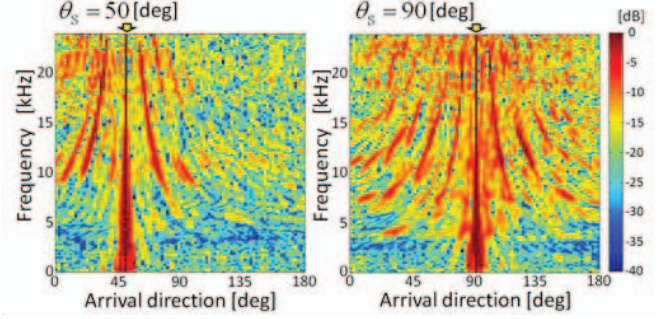


Fig. 4. Directivity of conventional method

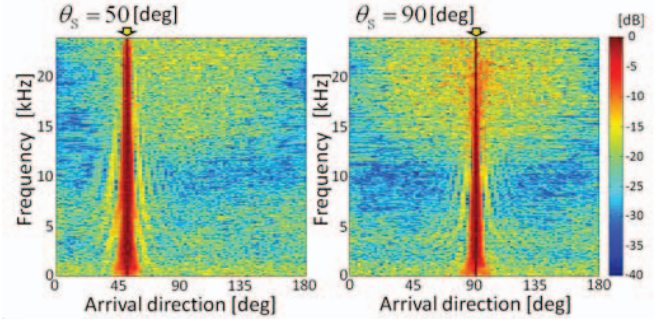


Fig. 5. Directivity of diffused sensing method

8. REFERENCES

- [1] S. Fischer et al., "Beamforming microphone arrays for speech acquisition in noisy environments," *Speech Communication*, vol. 20, Issues 3–4, pp. 215–227, 1996.
- [2] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propagat.*, vol. AP-34, pp. 276–280, 1986.
- [3] K. Kobayashi, K. Furuya, and A. Kataoka, "A talker-tracking microphone array for teleconferencing systems," *Audio Engineering Society Convention 113 (5642)*, 2002.
- [4] D. V. Compernelle, W. Ma, F. Xie, and M. V. Diest, "Speech recognition in noisy environments with the aid of microphone arrays," *Speech Communication*, vol. 9, Issues 5–6, pp. 433–442, 1990.
- [5] I. Hara, F. Asano, H. Asoh, J. Ogata, N. Ichimura, Y. Kawai, F. Kanehiro, H. Hirukawa, and K. Yamamoto, "Robust speech interface based on audio and video information fusion for humanoid hrp-2," *IROS 2004*, vol. 3, pp. 2404–2410, 2004.
- [6] J. L. Flanagan and D. A. Berkley, G. W. Elko, J. E. West, M. M. Sondhi, "Autodirective microphone systems," *Acoustica*, vol. 73, no. 2, pp. 58–71, 1991.
- [7] J. Capon, "High-resolution frequency-wavenumber spectrum analysis," *Proc. IEEE*, vol. 57, pp. 1408–1418, 1969.
- [8] S. P. Applebaum, "Adaptive arrays," *IEEE Trans. Antennas & Propag.*, vol. AP-24, no. 5, pp. 585–598, 1976.
- [9] J. L. Flanagan, A. C. Surendran, and E. E. Jan, "Spatially selective sound capture for speech and audio processing," *Speech Communication*, vol. 13, Issues 1–2, pp. 207–222, 1993.
- [10] D. H. Johnson and D. E. Dudgeon, *Array processing: concepts and techniques*, Englewood Cliffs, NJ: Prentice-Hall, 1993.
- [11] M. Tohyama, H. Suzuki, and Y. Ando, *The nature and technology of acoustic space*, Academic Press, 1995.
- [12] K. Takao, M. Fujita, and T. Nishi, "An adaptive antenna array under directional constraint," *IEEE Trans. Antennas & Propag.*, vol. AP-24, no. 5, pp. 662–669, 1976.
- [13] J. B. Allen and D. A. Berkley, "Image method for efficiently simulating small-room acoustics," *J. Acoust. Soc. Am.*, vol. 65, no. 4, pp. 943–950, 1979.