ANALYSIS OF ACOUSTIC MIMO SYSTEMS IN ENCLOSED SOUND FIELDS

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ABSTRACT

Methods for room acoustic analysis based on a single loudspeaker and a single microphone have been extensively studied, both theoretically and experimentally. Although measurement techniques for acquiring and processing spatial information in a room using arrays of loudspeakers and microphones have been recently proposed, the current literature in room acoustics does not describe the use of multiple-input multiple-output (MIMO) systems in a comprehensive manner. The aim of this paper is, therefore, to present an initial theoretical framework for the spatial analysis of enclosed sound fields using an acoustic MIMO system, based on a spherical loudspeaker array and a spherical microphone array. The fundamental characteristics of the system transfer matrix are shown to be invariant to rotation of the arrays, with its rank dependant on the number of reflections in the room. The paper concludes with a simulation study validating the theoretical models.

Index Terms— Room acoustic analysis, room impulse responses, MIMO systems, spherical arrays

1. INTRODUCTION

Characterizing acoustical properties of rooms and auditoria has been an important objective in a wide range of studies. Standard room acoustical parameters are computed from the room impulse responses that are measured with an omnidirectonal source and an omnidirectional microphone. The parameters obtained from the standard impulse responses mainly describe features of energy decay, such as reverberation time or clarity. Directional analysis of reverberant sound fields has gained prominence recently, introducing new spatial room acoustics measures such as directivity, lateral reflections of the sound field, and front-back ratio.

Gerzon discussed the task of recording the acoustics of a room with the aim of artificially reproducing sounds with similar acoustic properties [1]. His proposed technique was later extended by Farina [2], and implies that even more spatial information of an enclosure can be attained, once the concept of omnidirectional sources and receivers is extended. Farina's approach employed a spherical harmonic expansion of the sound field around the source and the receiver positions. Methods of acoustic path tracing that employ a highly directional source spanned over different spatial angles, increase the spatial separability of a room's impulse response and thus further enhance the spatial information of an enclosure. Methods of acoustic path tracing using a highly directional source have been proposed to increase the resolution of the spatial analysis, while, in another study, room transfer matrices were developed for improved sound field reproduction [3].

Recently, a variety of microphone arrays that provide detailed spatial information have been studied. In particular, spherical microphone arrays have been proposed, providing three-dimensional information by decomposing the sound field into spherical harmonics [4]. A spherical harmonics based design and analysis framework for spherical microphone arrays has also been proposed [5]. Direct sound and reflections recorded with spherical microphone arrays suggest that microphone arrays can be employed to identify the spatial characteristics of reverberant sound fields.

The methods mentioned above mainly describe the measurement techniques and not the analysis of the acquired data and mainly concern the use of an array only at the receiver position. However, analyzing data acquired from impulse responses measured by a MIMO system, employing the use of arrays at both the loudspeaker and the microphone ends, can lead to a more thorough understanding of the spatial properties of the sound field within an enclosure. This paper discusses the mathematical analysis of spatial impulse responses within the framework of a MIMO system. The results presented in this paper can facilitate the development of improved spatial analysis methods in rooms using acoustic MIMO systems.

The analysis of a free-field MIMO transfer-matrix and of a MIMO transfer-matrix for enclosed sound fields is presented, following the presentation of background on spherical microphone and loudspeaker arrays. The paper concludes with a simulation study.

2. BACKGROUND

This section presents the spherical Fourier transform and formulations related to spherical loudspeaker and microphone arrays. These will be used later in the paper in the derivation that describes the response between a transmitting spherical loudspeaker array and a recording spherical microphone array.

2.1. Spherical Fourier transform

The use of spherical arrays introduces a family of orthogonal functions, called the spherical harmonics, that solve the acoustic wave equation in spherical coordinates. A spherical Fourier transform (SFT) of a function, which is square integrable on the unit sphere, presents the function on a sphere as a linear combination of spherical harmonics. Consider a function $f(\theta,\phi)$ which is square integrable on the unit sphere; then the SFT of f, denoted by f_{nm}, and the inverse transform are given by [6]:

$$f_{nm} = \int_{\Omega \in S^2} f(\theta,\phi)\, Y_n^{m^*}(\theta,\phi) \sin(\theta)\, d\theta d\phi, \tag{1}$$

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$$f(\theta,\phi)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n} f_{nm}Y_n^m(\theta,\phi), \tag{2}$$

where "*" represents the complex conjugate and the integral covers the entire surface area of the unit sphere, denoted by S^2. $Y_n^m(\theta,\phi)$ represent the spherical harmonic functions, which are orthonormal and complete. Some further charateristics and relations can be found in [7].

2.2. Rotation of a spherical function

A function represented by its spherical Fourier transform has the same order after rotation and the new coefficients of the function, $f^r_{nm'}$, are given by [8]:

$$f^r_{nm'} = \sum_{m=-n}^{n} f_{nm}D^n_{m'm}(\alpha,\beta,\gamma), \tag{3}$$

where f_{nm} are the SFT coefficients of the original function, $D^n_{m'm}(\alpha,\beta,\gamma)$ is the Wigner-D function and (α,β,γ) are the Euler rotation angles. From the structure of eq. (3) it is seen that for a certain value of n the rotated coefficients are a linear transformation of the original coefficients for the same n. Wigner-D functions are unitary by definition; rotating the coordinate system about the origin does not change the function, but only presents it under a new coordinate system.

2.3. Spherical loudspeaker array

Consider a spherical source with a continuous radial velocity $u(\theta,\phi)$. The sound pressure outside this source is given by [9]:

$$p(k,r,\theta,\phi)=j\rho_0 c\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\frac{h_n(kr)}{h'_n(kr_t)}u_{nm}(k,r_t)Y_n^m(\theta,\phi), \tag{4}$$

where u_{nm} are the coefficients of the SFT of $u(\theta,\phi)$, ρ_0 is the air density, c is the speed of sound, $h_n(\cdot)$ is the spherical Hankel function with order n, $h'_n(\cdot)$ is the derivative of the function, r_t is the radius of the spherical source, k is the wavenumber, and $j=\sqrt{-1}$.

It has been shown in [9] that under some reasonable assumptions involving the frequency range of operation, sphere radius, and number of loudspeaker units in the array, eq. (4) provides a reasonable model of a spherical loudspeaker array.

2.4. Spherical microphone array

Given the sound pressure on a spherical surface, as measured by a spherical microphone array at the microphone positions, the SFT of the pressure can be typically calculated with negligible error. This requires that $kr < N$, with r the array radius and N the maximum spherical harmonics order [6].

The value of the pressure at a set of Q sampling points in space is defined by the $Q\times 1$ vector, $\mathbf{p}$:

$$\mathbf{p}=\left[p(\theta_1,\phi_1),p(\theta_2,\phi_2),\ldots,p(\theta_Q,\phi_Q)\right]^T. \tag{5}$$

In this paper we assume that the microphones are distributed according to the nearly-uniform sampling scheme [10], in which case all of the sampling weights are identical. Hence, the SFT of sampled pressure with a uniform sampling scheme is given by [10]:

$$p_{nm} = \frac{4\pi}{Q}\sum_{q=1}^{Q} p(\theta_q,\phi_q)Y_n^{m*}(\theta_q,\phi_q). \tag{6}$$

The latter can also be represented by a matrix form: $\mathbf{p}_{nm}=\frac{4\pi}{Q}\mathbf{Y}^*\mathbf{p}$ where $\mathbf{Y}$ is a matrix consisting of the spherical harmonic functions, with the corresponding orders at the different sampling points and $\mathbf{p}_{nm}=[p_{0,0},p_{1,-1},p_{1,0},\ldots,p_{N,N}]^T$. It can thus be shown, that $\sqrt{\frac{4\pi}{Q}}\mathbf{Y}$ is a unitary matrix.

3. FREE-FIELD MIMO TRANSFER-MATRIX

In this paper, the formulation of the MIMO transfer-matrix assumes an ideal order-limited spherical source located at the origin of the coordinate system. A spherical microphone array is positioned around a point different from the origin, and thus is non-concentric to the spherical loudspeaker source. A diagram of the MIMO system is presented in figure 1. Using the relations outlined in sec. 2.3, the

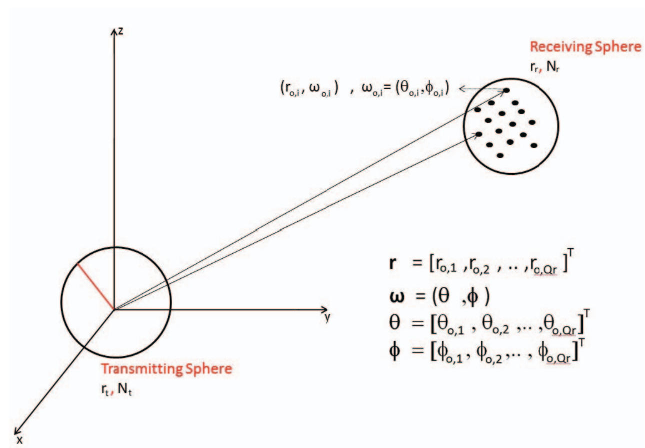

Fig. 1. Diagram of the MIMO system.

pressure at each omni-directional microphone of the microphone array, due to a continuous normal velocity function upon the source, can be computed by:

$$\begin{aligned} &p(k,r_{o,i},\theta_{o,i},\phi_{o,i}) \\ &= j\rho_0 c\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\frac{h_n(kr_{o,i})}{h'_n(kr_t)}u_{nm}(k,r_t)Y_n^m(\theta_{o,i},\phi_{o,i}), \end{aligned} \tag{7}$$

where $r_{o,i}$ denotes the distance between the sampling point and the origin of the coordinate system, $(\theta_{o,i},\phi_{o,i})$ denotes the elevation and azimuth angles, respectively, with regard to the origin and i denotes the i^{th} microphone, where $1\le i\le Q_r$ and Q_r is the number of microphones on the spherical microphone array.

Since we assume in this work that the normal surface velocity is order-limited, a vector of the pressure at the sampling points can be obtained with a transfer matrix, denoted $\mathbf{G}$, by $\mathbf{P}=\mathbf{G}\mathbf{u}_{nm}$. Here, $\mathbf{G}$ is defined by:

$$\mathbf{G} = [\mathbf{g}_1\ \mathbf{g}_2\ \cdots\ \mathbf{g}_{Q_r}]^T,$$

where

$$\mathbf{g}_i=[j\rho_0 c\frac{h_0(kr_{o,i})}{h'_0(kr_t)}Y_0^0(\omega_{o,i})\cdots j\rho_0 c\frac{h_{N_t}(kr_{o,i})}{h'_{N_t}(kr_t)}Y_{Nt}^{Nt}(\omega_{o,i})]^T,$$

$\mathbf{u}_{nm}=[u_{0,0},u_{1,-1},u_{1,0},\ldots,u_{N_t,N_t}]^T$ is a vector of the normal surface velocity coefficients of the spherical source and $\omega_{o,i} =$

$(\theta_{o,i}, \phi_{o,i})$. An additional matrix, relating $\mathbf{u}_{nm}$ to the SFT coefficients of the pressure on the spherical microphone array, can be constructed by assuming that the pressure on the recording sphere is order-limited and by transforming the pressure samples using the relations presented in sec. 2.4. This is performed by multiplying the original matrix $\mathbf{G}$ with $\frac{4\pi}{Q_r}\mathbf{Y}$ on the left.

In this section the system in free-field conditions is studied first. It is demonstrated how the rank of the transfer matrix degenerates to a unit rank when the distance between the arrays is large enough. The transfer matrix constructed earlier can be presented as a Hadamard product of two different matrices, denoted $\mathbf{Y}$ and $\mathbf{H}$; the elements of the matrices are given by:

$$[\mathbf{Y}]_{(n,m),i} = Y_n^m(\omega_{o,i}), \quad [\mathbf{H}]_{(n,m),i} = j\rho_0 c \frac{h_n(kr_{o,i})}{h_n'(kr_t)}.$$

The order of $(n, m), i$, is set in a manner that the product between every element of the two matrices gives the original transfer-matrix, $\mathbf{G}$. The first matrix, $\mathbf{Y}$, contains the spherical harmonic functions for different values of n and m and the arguments of the functions in each row are the elevation and azimuth angles for each microphone, $\omega_{o,i}$. In the second matrix, $\mathbf{H}$, every element is proportional to $\frac{h_n(kr_{o,i})}{h_n'(kr_t)}$ and in each row $r_{o,i}$ represents the distance between the i^{th} microphone from the origin of the system.

Both matrices tend to a unit rank when the distance between the two arrays is large enough. This is because the arguments of the functions composing the elements of the two matrices tend to the same constants in this case. Therefore, $\mathbf{G}$ itself tends to a unit rank following [11], rank $(\mathbf{G}) =$ rank $(\mathbf{Y} \circ \mathbf{H}) \leq$ rank $(\mathbf{Y})$ rank $(\mathbf{H})$. A criterion that measures a matrix singularity, or how close the rank of a matrix is to one, is defined by:

$$\Sigma = \left(\frac{\frac{1}{Q-1}\sum_{i=1}^{Q-1} \sigma_i}{\sigma_0} \right) \quad (8)$$

This criterion compares the strength of the first singular value to the rest of the other singular values. If Σ converges to zero, the rank converges to 1, and if it converges to a positive number (larger than zero) than the rank is larger than 1. We will use this criterion to compare the singularity of the transfer matrix for free-field conditions to that of a transfer matrix representing an enclosure.

4. ROTATION AND MIRRORING

Understanding the effects of rotation and mirroring of both the spherical source and microphone array are a crucial step before generalizing the analysis from a free-field condition problem to an actual enclosure. Under some conditions, the singular values of the transfer matrix are invariant to rotation of both the spherical source and the spherical microphone array, and even to the ”mirroring” of the spherical source. Invariance of the singular values does not mean that the matrix is not changed, but that only the gains, presented through the singular values, remain the same, while the ”directions” of the system, the right and left singular vectors, change.

The SFT of the sound pressure due to a rotated spherical source can be derived by using eqs. (3) and (4) as:

$$p_{nm'}^r = \frac{h_n(kr)}{h_n'(kr_t)} \sum_{m=-n}^{n} u_{nm}(k, r_t) D_{m'm}^n(\alpha, \beta, \gamma). \quad (9)$$

This equation suggests that rotation of the spherical source may only affect the singular vectors, but not the singular values of the transfer-matrix, due to the multiplication with the unitary Wigner-D matrix. This means that the fundamental characteristics of the system may remain the same after rotation, which is reasonable due to the spherical symmetry of the source.

Similar to source rotation, a rotation of the spherical microphone can be studied by deriving an expression for the SFT of the pressure measured by the microphone array, which after rotation is written as:

$$\tilde{\mathbf{G}} = \mathbf{Y}\mathbf{D}^r \left(\frac{4\pi}{Q_r} \tilde{\mathbf{Y}}^H \right) \mathbf{G} \quad (10)$$

where, $\left(\frac{4\pi}{Q_r} \tilde{\mathbf{Y}}^H \right)$ transforms the pressure samples to the pressure SFT coefficients, $\mathbf{D}^r$ rotates the microphone array, and $\mathbf{Y}$ transforms the rotated coefficients to the original sampling points. Since the multiplying matrices are unitary, the singular values stay invariant to the rotation of the microphone array.

Mirroring of the spherical source does not affect the singular values of the transfer matrix as well. Mirroring is easily accomplished in systems presented in a Cartesian coordinate system and is attained by flipping one of the axes. For polar coordinate systems, mirroring is immediate only for one axis, the y-axis; inserting $2\pi - \phi$ as the rotation angle argument to a spherical function instead of ϕ attains mirroring relative to the y-axis. The relation between a spherical harmonic function with the standard azimuth and elevation angles and the same function with $2\pi - \phi$ as the rotation angle, is given by conjugation [12]:

$$Y_n^m(\theta, 2\pi - \phi) = Y_n^{m*}(\theta, \phi). \quad (11)$$

When a function is presented by its SFT, the conjugation of the spherical harmonic functions can be presented by altering and rearranging the SFT coefficients, using:

$$Y_n^{m*}(\theta, \phi) = (-1)^m Y_n^{-m}(\theta, \phi). \quad (12)$$

Thus, mirroring of the array, changes the pressure in the following manner:

$$p(k, r, \theta, -\phi) = i\rho_0 c \sum_{n=0}^{\infty} \frac{h_n(kr)}{h_n'(kr_t)} \sum_{m=-n}^{n} \tilde{u}_{nm}(k, r_t) Y_n^m(\theta, \phi), \quad (13)$$

where $\tilde{u}_{nm} = u_{n(-m)}(k, r_t)(-1)^m$. In matrix form, rearranging and multiplying the corresponding coefficient by $(-1)^m$, is achieved by multiplying the input vector by a block diagonal and unitary matrix; in other words, the transfer matrix is multiplied by a unitary matrix on the right and the singular values remain the same.

Mirroring the other axes is achieved by rotating the sphere in a manner that the x-axis or z-axis becomes the y-axis, mirroring the y-axis as explained, and then rotating back. Since all of these operations are accomplished by multiplying by unitary matrices, the singular values are invariant to mirroring for all of the axes.

5. MIMO TRANSFER-MATRIX FOR ENCLOSED SOUND FIELDS

For omni-directional sources and microphones, one of the methods proposed to analyze the acoustic field within a rectangular room is the image method, proposed by Allen and Berkley [13]. The goal of this method is to obtain a good descritption of the room impulse response for omnidirectional sources and microphones. Hence a time domain model is used.

This method can be generalized for directional sources and microphone arrays; the only difference is that the new sources, which are not omni-directional, have to be mirrored according to their reflection path. For each image source a free-field transfer matrix is constructed, as explained in sec. 3, and now the transfer matrix of the room is constructed by summation of the image sources within the radius given by the speed of sound times the required analysis time window.

The far-field behavior of a free-field system was discussed earlier in sec. 3, where we saw that the rank of the matrix degenerates into a unit rank as the distance between the spherical source and spherical microphone array increases. For enclosures, the transfer matrix is formulated by a sum of free-field transfer matrices and, if the distance between the arrays is large enough, in a manner that the free-field matrix tends to a unit rank matrix, the image sources' transfer matrices all tend to unit rank as well. However, a matrix that is formulated as a sum of unit rank matrices is not necessarily a unit rank matrix itself. In the case of an enclosure different sets of output vectors are achievable due to the contributions from the different image sources, and so the rank can potentially increase, facilitating the use of this matrix for advanced array processing.

6. SIMULATION STUDY

The goal of the simulation study is to demonstrate how the measure of singularity, Σ, changes as more image sources are taken into account. This is done by starting with the influence of only the direct source, and gradually increasing the number of image sources by taking into account the image sources within an increasing radius around the spherical microphone array.

An arbitrary rectangular room has been chosen, with the dimensions of $[3, 4, 5]$ for its coordinates on the x-axis, y-axis, and z-axis respectingly. Wall pressure coefficients equal to 0.5 (assumed frequency-independent and angle-independent) have been chosen for all of the walls. The spherical microphone array, with a radius of 0.4m, is positioned at $[0.5, 1, 1]$ and the spherical source, with a radius of 0.3m is located at $[2, 2.5, 3]$. Following the image method described in sec. 5, image sources were constructed, rotated and mirrored and a transfer-matrix relating the spherical microphone array and each image source was constructed. The transfer-matrix for the enclosure was then constructed by summing all of the sources within the radius defined and the measure of singularity was calculated for a single frequency of 500Hz. The simulation study results are presented in fig. 2.

The figure shows only one significant singular value when only the direct sound is considered, and, as expected, the magnitude of the other singular values increase as more image sources are introduced.

7. CONCLUSIONS

In this paper a MIMO transfer-matrix representing the response from a spherical loudspeaker array to a spherical microphone array in a room was formulated and analyzed. The analysis show that the characteristics of the matrix, such as singular values, remain unchanged under rotation of the arrays. This is important when considering positioning of arrays in real rooms. The rank of the matrix was shown to depend on the number of reflections in the room. This suggests that advanced array processing methods, some of which require high or full rank matrices, may be possible using the proposed MIMO system when they are positioned in real rooms. The design of methods for spatial analysis using the proposed system is the topic of current research.

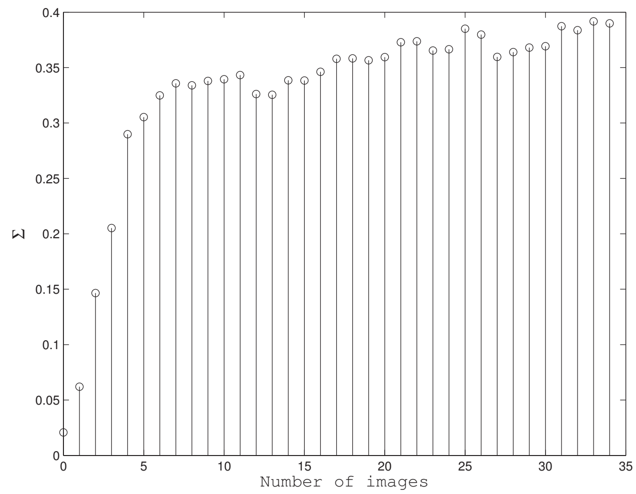

Fig. 2. Measure of singularity as a function of image sources.

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