A SPARSE BLOCKING MATRIX FOR MULTIPLE CONSTRAINTS GSC BEAMFORMER

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ABSTRACT

Modern high performance speech processing applications incorporate large microphone arrays. Complicated scenarios comprising multiple sources, motivate the use of the linearly constrained minimum variance (LCMV) beamformer (BF) and specifically its efficient generalized sidelobe canceler (GSC) implementation. The complexity of applying the GSC is dominated by the blocking matrix (BM). A common approach for constructing the BM is to use a projection matrix to the null-subspace of the constraints. The latter BM is denoted as the eigen-space BM, and requires M^2 complex multiplications, where M is the number of microphones. In the current contribution, a novel systematic scheme for constructing a multiple constraints sparse BM is presented. The sparsity of the proposed BM substantially reduces the complexity to $K \times (M-K)$ complex multiplications, where K is the number of constraints. A theoretical analysis of the signal leakage and of the blocking ability of the proposed sparse BM and of the eigen-space BM is derived. It is proven analytically, and tested for narrowband signals and for speech signals, that the blocking abilities of the sparse and of the eigen-space BMs are equivalent.

***Index Terms*—** Beamforming, Generalized sidelobe canceler

1. INTRODUCTION

A wide range of applications, such as home entertainment, audio conferences and hearing aids, utilize spatial filtering with microphone arrays for obtaining high performance signal enhancement. Over the past years, technology advances in processing power, communication bandwidth and power consumption, have extended the use cases to more complicated scenarios, involving multiple speakers and interferences. By increasing the number of microphones, more control on the desired spatial response is available. The LCMV BF is a beampattern design criterion for minimizing the noise power at the output of the BF under a set of linear constraints. The closed-form LCMV-BF has an equivalent GSC implementation. The GSC form separates the objectives of constraining the beampattern (performed by the fixed beamformer (FBF)) and of minimizing the noise level (performed by the BM followed by the noise canceler (NC)). Here are two examples where LCMV-BFs are used in a challenging multiple sources scenarios. Markovich-Golan et al. [1] have utilized the LCMV criterion implemented as GSC in the short time Fourier transform (STFT) domain for enhancing a group of desired speakers, and mitigating a group of competing speakers in a reverberant environment. Bertrand and Moonen [2] proposed a distributed LCMV algorithm for a network of microphone arrays sharing a wireless communication channel. Node dependent desired responses to a global Kth order constraints set are determined. Iteratively, each node broadcasts K channels to all other nodes, and updates its local BF. Each local BF utilizes the local microphones and the broadcasted channels from other nodes.

Consider the complexity of applying the GSC. Assuming that M microphones are used, and K constraints are defined, the complexity per frame and frequency bin of the GSC in the STFT domain is attributed to: 1) M complex multiplications at the FBF; 2) the complexity of applying the BM; 3) $M-K$ complex multiplications at the NC. There is no unique design for the BM. For the single constraint scenario, Gannot et al. [3] propose a sparse $M \times (M-1)$ BM which requires only $M-1$ complex multiplications. Herbordt and Kellerman [4] provided an efficient adaptive BM utilizing also only $M-1$ complex multiplications. A commonly used $M \times (M-K)$ BM is comprised of the basis vectors spanning the null-subspace of the constraints columns-space. The basis vectors can be obtained by applying the singular value decomposition (SVD) to the constraints matrix. Applying the SVD based BM involves $M \times (M-K)$ complex multiplications. Markovich-Golan et al. [1] use an $M \times M$ projection matrix to the null-subspace of the constraints matrix as the BM. The latter scheme requires M^2 complex multiplications. Reuven et al. [5] and Krueger et al. [6] propose a 2-stage projection procedure for designing a BM for the case of a single desired speaker and a single interfering speaker. The resulting BM is an $M \times M$ matrix, and its application also requires M^2 complex multiplications. Clearly, the complexity of the GSC is mainly dominated by the BM (in the general K constraints case). Tseng and Griffiths [7] propose to construct the BM by recursively projecting the received signals to the null-space of the constraints, one by one. The resulting procedure requires MK complex multiplications (provided that $K < \frac{M}{2}$).

In the current contribution a novel systematic procedure for designing a K constraints sparse $M \times (M-K)$ BM is proposed. The BM requires only $(M-K)K$ complex multiplications. The blocking ability of the sparse BM, defined as the robustness to the acoustic transfer functions (ATFs), is analyzed and compared with the blocking ability of the commonly used eigen-space BM. For low estimation errors, it is proven that blocking ability of the sparse BM and of the eigen-space BM are equivalent.

The paper is organized as follows. In Sec. 2 the problem is formulated. In Sec. 3 the eigen-space BM and the sparse BM are formally derived. Then, in Sec. 4 the blocking ability and the signal leakage of the BMs are analyzed. A comprehensive experimental study of narrowband signals as well as of speech signals is described in Sec. 5.

2. PROBLEM FORMULATION

Consider a microphone array comprising M microphones. The received signals in the STFT domain are:

$$\mathbf{z}(\ell,\zeta) = \mathbf{H}(\zeta)\mathbf{s}(\ell,\zeta) + \mathbf{v}(\ell,\zeta) \quad (1)$$

where ℓ is the frame index and ζ is the frequency bin index. The received signals comprise two contributions. The first contribution $\mathbf{H}(\zeta)\mathbf{s}(\ell,\zeta)$ is related to the constrained sources, where $\mathbf{s}(\ell,\zeta) = \begin{bmatrix} s_1(\ell,\zeta) & \cdots & s_K(\ell,\zeta) \end{bmatrix}^T$ is a $K \times 1$ vector of coherent signals, $\mathbf{H}(\zeta) = \begin{bmatrix} \mathbf{h}_1(\zeta) & \cdots & \mathbf{h}_K(\zeta) \end{bmatrix}$ is an $M \times K$ constraints matrix comprised of the K ATFs relating the constrained sources and the microphones. The second contribution $\mathbf{v}(\ell,\zeta)$ is related to the non-constrained contribution. Without loss of generality we assume that the ATFs are normalized, i.e. $\|\mathbf{h}_k(\zeta)\|^2;\ k = 1,..,K$. Henceforth, the frequency bin index ζ is omitted for brevity. The derived formulas correspond to either a single frequency, or one frequency bin for wideband signals.

3. DESIGNING THE BM

In Sec. 3.1 the eigen-space based BM is defined, as in [1]. In Sec. 3.2 the proposed sparse BM is derived.

3.1. Eigen-space based BM

The eigen-space BM is given by the projection matrix to the null-subspace of the constraint matrix $\mathbf{H}$:

$$\mathbf{B}_e = \mathbf{I}_{M\times M} - \mathbf{H}\left(\mathbf{H}^\dagger\mathbf{H}\right)^{-1}\mathbf{H}^\dagger \quad (2)$$

where $\mathbf{I}_{M\times M}$ is the $M \times M$ identity matrix. It can be verified that $\mathbf{B}_e^\dagger\mathbf{H} = \mathbf{0}$. Application of the eigen-space BM involves M^2 complex multiplications per frame and frequency bin.

3.2. Sparse BM

The contribution of the constrained signals to the received signals in (1), i.e. $\mathbf{Hs}(\ell)$, lies in a rank-K subspace in the M dimensional space. By a proper transformation, $\mathbf{Hs}(\ell)$ can be expressed as a linear combination of the constrained signal contributions of K reference received signals. Without loss of generality we consider the first K microphones as the reference signals. Denote the reference microphones by $\mathbf{z}_r(\ell) = \begin{bmatrix} z_1(\ell) & \cdots & z_K(\ell) \end{bmatrix}^T$. The reference microphones are given by:

$$\mathbf{z}_r(\ell) = \mathbf{H}_r\mathbf{s}(\ell) + \mathbf{v}_r(\ell) \quad (3)$$

where $\mathbf{H}_r = \mathbf{H}_{1:K,1:K}$, and $\mathbf{v}_r(\ell) = \begin{bmatrix} v_1(\ell) & \cdots & v_K(\ell) \end{bmatrix}^T$. Assuming that $\mathbf{H}_r$ is invertible, $\mathbf{Hs}(\ell)$ can be expressed in terms of $\mathbf{H}_r\mathbf{s}(\ell)$ as $\mathbf{Hs}(\ell) = \begin{bmatrix} \mathbf{I}_{K\times K} & \boldsymbol{\beta}_{K+1} & \cdots & \boldsymbol{\beta}_M \end{bmatrix}^\dagger \mathbf{H}_r\mathbf{s}(\ell)$ where $\boldsymbol{\beta}_m = \left(\mathbf{H}_r^{-1}\right)^\dagger \mathbf{H}_{m,:}^\dagger$ for $m = K+1,..,M$ and $\mathbf{H}_{m,:}$ is the mth row of $\mathbf{H}$. Utilizing the latter representation, a noise reference (non-constrained part) based on the mth microphone (for $m = K+1,..,M$), is extracted by subtracting a linear combination of the reference microphones $\mathbf{z}_r(\ell)$ from $z_m(\ell)$. I.e.

$$u_m(\ell) = z_m(\ell) - \boldsymbol{\beta}_m^\dagger\mathbf{z}_r(\ell) = v_m(\ell) - \boldsymbol{\beta}_m^\dagger\mathbf{v}_r(\ell). \quad (4)$$

The corresponding BM is denoted as $\mathbf{B}_s$ and is given by:

$$\mathbf{B}_s = \begin{bmatrix} -\boldsymbol{\beta}_{K+1} & \cdots & -\boldsymbol{\beta}_M \\ & \mathbf{I}_{(M-K)\times(M-K)} & \end{bmatrix}. \quad (5)$$

Please note that $\mathbf{B}_s$ has $(M-K)\times K$ non-zero entries in its first K rows and $M-K$ entries equal to 1 in the lower $M-K$ rows. Hence, the proposed BM can be denoted as the sparse BM. Its application requires $(M-K)\times K$ complex multiplications per frame per frequency, which is much lower than the $M\times(M-K)$ complex multiplications required by the eigen-space BM (assuming that $K \ll M$).

In the special case of $K=1$ the proposed sparse BM equals to the BM proposed by Gannot et al. [3], which is based on the relative transfer function (RTF) with respect to a single (arbitrarily chosen) microphone.

4. PERFORMANCE ANALYSIS

In the current section, the blocking ability and the signal leakage criteria are defined and analyzed for the eigen-space and for the sparse BMs.

Consider a noisy estimate of $\mathbf{H}$:

$$\tilde{\mathbf{H}} = \mathbf{H} + \boldsymbol{\Delta} \quad (6)$$

where $\boldsymbol{\Delta} = \begin{bmatrix} \boldsymbol{\delta}_1 & \cdots & \boldsymbol{\delta}_K \end{bmatrix}$ comprises the $M\times 1$ dimensional vectors $\boldsymbol{\delta}_1,..,\boldsymbol{\delta}_K$ of independent identically distributed (IID) complex Normal random variables with a zero mean, and a variance of λ_u. Since $\mathbf{h}_k;\ k=1,..,K$ are assumed to be normalized, the estimation accuracy defined as $\|\mathbf{H}\|_{\mathrm{F}}^2/\mathrm{E}\left\{\|\boldsymbol{\Delta}\|_{\mathrm{F}}^2\right\}$ equals $(M\lambda_u)^{-1}$, where $\|\cdot\|_{\mathrm{F}}^2$ is the squared Frobenius norm.

The ability of the noisy BM $\tilde{\mathbf{B}}_b$ to block $\mathbf{h}_k$, the ATF of the kth source, is denoted by η_b^k, and equals the ratio between the leakage of kth ATF to the output of the BM, $\lambda_b^{s,k}$, and the power of a unit variance spatially white noise filtered by the BM, λ_b^n:

$$\eta_b^k = \frac{\lambda_b^{s,k}}{\lambda_b^n} \quad (7)$$

where

$$\lambda_b^{s,k} = \mathrm{E}\left\{\|\tilde{\mathbf{B}}_b^\dagger\mathbf{h}_k\|^2\right\} \quad (8a)$$

$$\lambda_b^n = \mathrm{E}\left\{\|\tilde{\mathbf{B}}_b^\dagger\mathbf{w}\|^2\right\} \quad (8b)$$

$b \in \{e,s\}$ stands for sparse BM (s), or eigen-space BM (e), and $\mathbf{w}$ is an $M\times 1$ vector of zero mean, and unit variance complex Normal IID random variables (RVs). Substituting $\mathbf{h}_k = \tilde{\mathbf{h}}_k - \boldsymbol{\delta}_k$ in (8a) and noticing that $\tilde{\mathbf{B}}^\dagger\tilde{\mathbf{h}}_k = \mathbf{0}$ by construction yields:

$$\lambda_b^{s,k} = \mathrm{E}\left\{\|\tilde{\mathbf{B}}_b^\dagger\boldsymbol{\delta}_k\|^2\right\}. \quad (9)$$

The total blocking ability is defined as the sum of the blocking abilities of all constrained ATFs:

$$\eta_b = \sum_{k=1}^{K}\eta_b^k. \quad (10)$$

4.1. Blocking ability and signal leakage of the eigen-space BM

The noisy eigen-space BM is given by substituting the noisy ATFs (6) in (2):

$$\tilde{\mathbf{B}}_e = \mathbf{I}_{M\times M} - \tilde{\mathbf{H}}\left(\tilde{\mathbf{H}}^\dagger\tilde{\mathbf{H}}\right)^{-1}\tilde{\mathbf{H}}^\dagger. \quad (11)$$

The noise power at the output of the eigen-space BM, λ_e^n, is given by substituting (11) in (8b):

$$\lambda_e^n = \mathrm{E}\left\{\mathbf{w}^\dagger \tilde{\mathbf{B}}_e \tilde{\mathbf{B}}_e^\dagger \mathbf{w}\right\} = \mathrm{trace}\left\{\mathrm{E}\left\{\tilde{\mathbf{B}}_e \tilde{\mathbf{B}}_e^\dagger\right\}\right\}. \tag{12}$$

As $\tilde{\mathbf{B}}_e$ is a hermitian projection matrix, the following equation holds: $\tilde{\mathbf{B}}_e \tilde{\mathbf{B}}_e^\dagger = \tilde{\mathbf{B}}_e \tilde{\mathbf{B}}_e = \tilde{\mathbf{B}}_e$. And after some matrix manipulation λ_e^n equals:

$$\lambda_e^n = M - K. \tag{13}$$

The signal leakage of the kth ATF at the output of the eigen-space BM, $\lambda_e^{s,k}$, is given by substituting $\tilde{\mathbf{B}}_e$ in (9):

$$\lambda_e^{s,k} = \mathrm{E}\left\{\boldsymbol{\delta}_k^\dagger \tilde{\mathbf{B}}_e \tilde{\mathbf{B}}_e^\dagger \boldsymbol{\delta}_k\right\} \tag{14}$$

Expanding (14) to a Taylor series around $\mathbf{H}$ as a function of $\boldsymbol{\Delta}$, and neglecting elements of order $\boldsymbol{\Delta}^n$ for $n > 2$, and using $\mathrm{E}\{\boldsymbol{\Delta}\} = \mathbf{0}$, the following approximation holds:

$$\lambda_e^{s,k} \approx \mathrm{E}\left\{\boldsymbol{\delta}_k^\dagger \mathbf{B}_e \mathbf{B}_e^\dagger \boldsymbol{\delta}_k\right\} = \lambda_u \,\mathrm{trace}\left\{\mathbf{B}_e \mathbf{B}_e^\dagger\right\}. \tag{15}$$

And similarly to the derivation of (13), λ_e^s equals:

$$\lambda_e^{s,k} = (M-K)\lambda_u. \tag{16}$$

Therefore, the ability of the noisy eigen-space BM $\tilde{\mathbf{B}}_e$ to block $\mathbf{h}_k$, the ATF of the kth source is given by:

$$\eta_e^k = \frac{\lambda_e^{s,k}}{\lambda_e^n} = \lambda_u. \tag{17}$$

And the total blocking ability of the eigen-space BM is:

$$\eta_e = K\lambda_u. \tag{18}$$

4.2. Blocking ability and signal leakage of the sparse BM

The noisy sparse BM is constructed by substituting $\tilde{\mathbf{H}}$ (6), the noisy estimate of $\mathbf{H}$, in (5):

$$\tilde{\mathbf{B}}_s = \begin{bmatrix} -\tilde{\boldsymbol{\beta}}_{K+1} & \cdots & -\tilde{\boldsymbol{\beta}}_M \\ & \mathbf{I}_{(M-K)\times(M-K)} & \end{bmatrix} \tag{19}$$

where

$$\tilde{\boldsymbol{\beta}}_m = \left(\tilde{\mathbf{H}}_r^{-1}\right)^\dagger \tilde{\mathbf{H}}_{m,:}^\dagger \tag{20a}$$

$$\tilde{\mathbf{H}}_r = \mathbf{H}_r + \boldsymbol{\Delta}_r. \tag{20b}$$

and $\boldsymbol{\Delta}_r = \Delta_{1:K,1:K}$.

Similarly to the derivation in (12), the noise power at the output of the sparse BM is given by:

$$\lambda_s^n = \mathrm{trace}\left\{\mathrm{E}\left\{\tilde{\mathbf{B}}_s \tilde{\mathbf{B}}_s^\dagger\right\}\right\}. \tag{21}$$

Following the definition in (19), the latter expression is:

$$\lambda_s^n = M - K + \sum_{m=1}^{M-K} \mathrm{E}\left\{\|\tilde{\boldsymbol{\beta}}_m\|^2\right\}. \tag{22}$$

Consider a single term of the sum in (22):

$$\mathrm{E}\left\{\|\tilde{\boldsymbol{\beta}}_m\|^2\right\} = \mathrm{E}\left\{\tilde{\mathbf{H}}_{m,:}\tilde{\mathbf{H}}_r^{-1}\left(\tilde{\mathbf{H}}_r^\dagger \tilde{\mathbf{H}}_r\right)^{-1}\tilde{\mathbf{H}}_{m,:}^\dagger\right\}. \tag{23}$$

Assuming again that $\mathbf{H}_r$ is invertible and high estimation accuracy, i.e. $\|\mathbf{H}_r\|^2 \gg \|\boldsymbol{\Delta}_r\|^2$, and by replacing the expression $\left(\tilde{\mathbf{H}}_r^\dagger \tilde{\mathbf{H}}_r\right)^{-1}$ with its first term Taylor series expansion around $\mathbf{0}$, we obtain:

$$\left(\tilde{\mathbf{H}}_r^\dagger \tilde{\mathbf{H}}_r\right)^{-1} \approx \left(\mathbf{I} - \left(\mathbf{H}_r^\dagger \mathbf{H}_r\right)^{-1}\left(\mathbf{H}_r^\dagger \boldsymbol{\Delta}_r + \boldsymbol{\Delta}_r^\dagger \mathbf{H}_r + \boldsymbol{\Delta}_r^\dagger \boldsymbol{\Delta}_r\right)\right) \cdot \left(\mathbf{H}_r^\dagger \mathbf{H}_r\right)^{-1}. \tag{24}$$

Next, substituting the approximation (24) in (23) and neglecting terms $\boldsymbol{\Delta}^n$ of order $n > 2$ and using $\mathrm{E}\{\boldsymbol{\Delta}\} = \mathbf{0}$, the following approximation holds:

$$\mathrm{E}\left\{\|\tilde{\boldsymbol{\beta}}_m\|^2\right\} \approx \|\boldsymbol{\beta}_m\|^2 + \lambda_u \,\mathrm{trace}\left\{\left(\mathbf{H}_r^\dagger \mathbf{H}_r\right)^{-1}\right\} - \lambda_u \mathbf{H}_{m,:}\left(\mathbf{H}_r^\dagger \mathbf{H}_r\right)^{-2}\mathbf{H}_{m,:}^\dagger. \tag{25}$$

Finally, substituting (25) in (22) yields:

$$\begin{aligned}\lambda_s^n =& M - K + \lambda_u \,\mathrm{trace}\left\{\left(\mathbf{H}_r^\dagger \mathbf{H}_r\right)^{-1}\right\}(M-K) \\ &+ \sum_{m=1}^{M-K} \|\boldsymbol{\beta}_m\|^2 - \lambda_u \mathbf{H}_{m+K,:}\left(\mathbf{H}_r^\dagger \mathbf{H}_r\right)^{-2}\mathbf{H}_{m+K,:}^\dagger \\ \approx& M - K + \sum_{m=1}^{M-K}\|\boldsymbol{\beta}_m\|^2 \end{aligned} \tag{26}$$

where the approximation in the last transition is due to the high estimation accuracy.

Similarly to the derivation of (14), the leakage of the kth ATF to the output of the sparse BM is given by:

$$\begin{aligned}\lambda_s^{s,k} =& \lambda_u \,\mathrm{trace}\left\{\mathbf{B}_s \mathbf{B}_s^\dagger\right\} \\ =& \left(M - K + \sum_{m=1}^{M-K}\|\boldsymbol{\beta}_m\|^2\right)\lambda_u. \end{aligned} \tag{27}$$

The ability of the noisy sparse BM $\tilde{\mathbf{B}}_s$ to block $\mathbf{h}_k$, the ATF of the kth source is given by:

$$\eta_s^k = \frac{\lambda_s^{s,k}}{\lambda_s^n} = \lambda_u \tag{28}$$

and the total blocking ability of the sparse BM is therefore:

$$\eta_s = K\lambda_u. \tag{29}$$

Please note that the blocking ability of the proposed sparse BM is equivalent to the blocking ability of the eigen-space BM (17,18).

5. EXPERIMENTAL STUDY

The performance of the proposed sparse BM, and of the eigen-space BM is presented for narrowband signal scenarios in Sec. 5.1, and for wideband speech signals in Sec. 5.2.

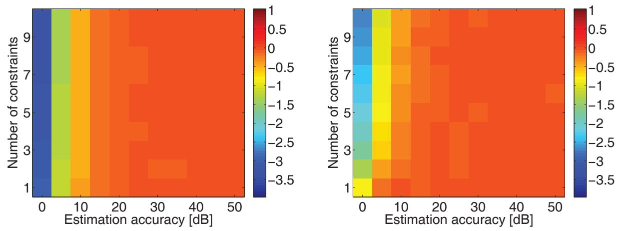

Fig. 1. Difference in dB between theoretical and empirical blocking abilities for narrowband signals simulation with $M = 20$ microphones, for the eigen-space BM (left) and sparse BM (right)

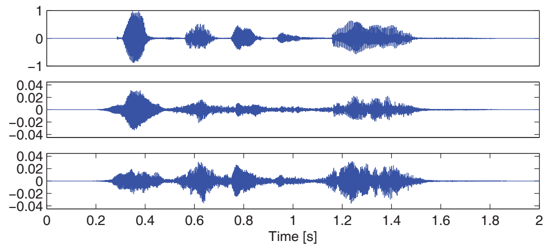

Fig. 2. Source number 1 as received by the 1st microphone (top), its contribution to the leakage at the output of the eigen-space BM (middle) and at the output of the sparse BM (bottom)

5.1. Narrowband signals

A comprehensive Monte-Carlo simulation was performed for validating the theoretical analysis derived in Sec. 4. A total of 561 scenarios were tested, the parameters of the scenario were: 1) the number of microphones was set to $M = 5, 10, .., 30$; 2) the number of constraints was set to $K = 1, 2, .., \lfloor \frac{M}{2} \rfloor$; 3) the estimation accuracy level was set to $0\text{dB}, 5\text{dB}, .., 50\text{dB}$. At each scenario the performance was averaged over 100 randomly generated ATFs ($\mathbf{H}$), times 1000 random estimation errors per instance. Altogether, the blocking abilities of 56.1×10^6 sparse and eigen-space BMs were evaluated and compared with the theoretical analysis. In Fig. 1 the average differences between the theoretical blocking ability and of the empirical blocking ability for the sparse and eigen-space BMs are depicted. In these figures the number of microphones was set to $M = 20$, while the numbers of constraints varied in the range of $K = 1, 2, .., \lfloor \frac{M}{2} \rfloor$ and the signal to noise ratio (SNR) levels varied in the range of $0\text{dB}, 5\text{dB}, .., 50\text{dB}$. The results validate the theoretical analysis as the average deviation from the theory for estimation accuracies higher than 5dB is lower than 0.5dB.

5.2. Speech signals

The eigen-space and sparse BMs were tested on wideband speech signals in a simulated $4\text{m} \times 3\text{m} \times 3\text{m}$ room environment with a reverberation time of $T_{60} = 150\text{ms}$. A uniform linear microphone array comprising 9 microphones with 5cm spacing was placed next to one of the walls. Three speakers and a stationary interference were located in the room, at a distance of 1.8m in front of the microphone array, at angles $-60^o, -20^o, 20^o, 60^o$. The received signals were sampled at a sample rate of 8KHz and transformed to the STFT domain with 4096 discrete Fourier transform (DFT) points and a 50% overlap between frames. The three speakers were constrained. The BMs were calculated in the STFT domain based on the normalized ATFs of the three speakers contaminated by a -30dB error level. High estimation accuracy can be obtained by applying the subspaces based estimation in [1]. In order to keep the power at the output of the sparse BM at a constant level over frequency, a normalized BM $\tilde{\mathbf{B}}_s / \|\tilde{\mathbf{B}}_s\|_{\text{F}}$ was used rather than $\tilde{\mathbf{B}}_s$. Note that the latter scaling does not affect the blocking ability as the signal leakage and the spatially white noise gain are multiplied by the same factor. The total blocking ability of the eigen-space BM was -28dB while the total blocking ability of sparse BM was slightly worse at -26.5dB. The 1st source as received by the microphone array and its contribution to the leakage at the outputs of the BMs are depicted in Fig. 2. Note the different scale in the microphone and leakage figures. It can be verified that the proposed sparse BM, and the eigen-space BM obtain similar performance for wideband speech signals.

6. CONCLUSIONS

A novel systematic scheme for constructing a K constraints sparse BM for the LCMV-BF was derived. The signal leakage and the blocking ability of the proposed sparse BM and of the commonly used eigen-space BM are analyzed and compared. It is analytically proven that the blocking abilities of both BMs are equivalent, provided that the estimation accuracy is high. The computational complexity of the proposed sparse BM is $K \times (M - K)$, which is substantially lower than the computational complexity of the eigen-space BM, which is M^2. The theoretical analysis is experimentally verified for both narrowband signals and wideband speech signals.

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