A VARIABLE STEP-SIZE FILTERED-X GRADIENT ADAPTIVE LATTICE ALGORITHM FOR ACTIVE NOISE CONTROL

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ABSTRACT

The gradient adaptive lattice (GAL) algorithm is very attractive choice for active noise control of multiple sinusoidal interferences. In the GAL algorithm, a selection of step-size parameters trades off between convergence speed and steady-state performance. In this paper, we develop a variable step-size scheme for the filtered-x GAL (VSS-FxGAL) algorithm. This proposed algorithm achieves a good compromise between fast convergence speed and low steady-state mean-square error (MSE). In addition, comparing to the filtered-x affine projection (FxAP) algorithm, the proposed algorithm performs better when the filter input consists of multiple sinusoids.

Index Terms— Active noise control (ANC), adaptive filters, gradient adaptive lattice (GAL) algorithm, variable step-size filtered-x gradient adaptive lattice (VSS-FxGAL) algorithm

1. INTRODUCTION

Active noise control (ANC) system works on the principle of destructive interference between an original "primary" disturbance sound field $d(n)$ measured at the location "error" sensors (typically microphones), and a "secondary" sound filed $y(n)$ that is generated by control actuators [1]. Conventional adaptive algorithms are likely to be unstable in ANC due to the phase shift (delay) introduced by the secondary path [1]. However, the well-known filtered-x structure [1] is suitable for ANC because of phase shift compensation. Different types of adaptive filters using the filtered-x structure have been developed for ANC [2].

Among these, the filtered-x least mean square (FxLMS) algorithm is widely used due to its computational simplicity and robustness [3]. However, non-white input signals can deteriorate its convergence speed [4]. In order to overcome this problem, the filtered-x affine projection (FxAP) algorithm [5] was developed, which was found to be efficient due to a good tradeoff between convergence speed and computational complexity [6]. However, interferences are often consist of multiple sinusoids in ANC applications. When multiple sinusoidal interferences with wide amplitude dynamic range are considered, the convergence speed of the AP algorithm is degraded.

For improving stealth ability of submarines, self-noises generated by machinery components should be damped [7]. These self-generated noises consist of dominant multiple sinusoids and background colored noise, so ANC system must be able to damp the dominant multiple sinusoids.

The gradient adaptive lattice (GAL) algorithm [8] has certain theoretical properties which promise superior performances over transversal filters, especially for sinusoidal filter inputs [9]. In particular, convergence speed of the GAL algorithm is fairly independent of the statistics of the reference signals. However, in stability bound, an initial convergence speed of the GAL algorithm is slower than
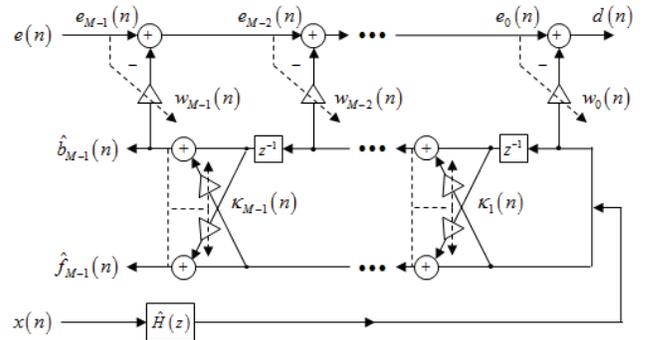
Fig. 1. A block diagram of the active noise control system based on the lattice structure [10].

that of the AP algorithm. In order to improve the initial convergence speed of the GAL algorithm, the step-size parameter needs to be controlled.

In this paper, we design a variable step-size (VSS) scheme for the filtered-x GAL (FxGAL) algorithm. Previously, the ANC system based on the lattice structure was proposed [10]. In this algorithm, the order update of the estimation error was modified to avoid the estimation of the primary noise, so that local estimation errors were directly obtained from the residual error. Fig. 1 shows the structure of the lattice-based ANC system proposed in [10]. The VSS-FxGAL algorithm is developed by applying the VSS scheme to the lattice-based ANC system in Fig. 1. The lattice predictor in the structure decouples the filtered reference input so that the system delivers good performances even with non-white input signals. Furthermore, by using the VSS scheme, the proposed algorithm provides fast convergence speed and low steady-state mean-square error (MSE) as compared to the FxGAL algorithm.

2. FILTERED-X GRADIENT ADAPTIVE LATTICE ALGORITHM FOR ANC

Consider the lattice predictor that transforms the filtered reference signal $v(n) = x(n) * \hat{h}(n)$ into the orthogonal filtered backward prediction errors $\hat{b}_m(n)$, $0 < m < M - 1$, where $\hat{h}(n)$ denotes the estimated secondary path, and M denotes the order of the lattice predictor [8]. This orthogonalization is carried out in the lattice through formulas:

$$\hat{f}_{m+1}(n) = \hat{f}_m(n) - \kappa_m(n-1)\hat{b}_m(n-1), \qquad (1)$$

$$\hat{b}_{m+1}(n) = \hat{b}_m(n-1) - \kappa_m(n-1)\hat{f}_m(n). \qquad (2)$$

The orthogonality of the filtered backward prediction errors $\hat{b}_m(n)$ is expressed as [8]

$$E\left\{\hat{b}_i(n)\hat{b}_j(n)\right\} = 0, \, i \neq j. \tag{3}$$

In the GAL algorithm for ANC [10], the reflection coefficients $\kappa_m(n)$ are adjusted using a recursive formula

$$\kappa_m(n) = \kappa_m(n-1) + \frac{\tilde{\mu}}{\xi_m(n)+\delta}\left[\hat{f}_{m+1}(n)\hat{b}_m(n-1) + \hat{f}_m(n)\hat{b}_{m+1}(n)\right] \tag{4}$$

where $\tilde{\mu}$ is the step-size parameter, δ is the regularization constant, and $\xi_m(n)$ is the power of both the forward prediction error and the delayed backward prediction error at the mth stage, which can be recursively estimated as $\xi_m(n) = \beta\xi_m(n-1) + (1-\beta)\left[\hat{b}_m^2(n) + \hat{f}_m^2(n)\right]$ where β is the smoothing parameter. The mth stage adaptive coefficient $w_m(n)$ is updated to minimize the power of the mth stage local estimation error:

$$e_m(n) = e_{m-1}(n) + h(n) * [w_m(n)b_m(n)] \tag{5}$$

where

$$w_m(n) = w_m(n-1) - \frac{\mu}{\sigma_{\hat{b}_m}^2(n)+\delta}\hat{b}_m(n)e_m(n) \tag{6}$$

and $e_{-1}(n) = d(n)$ is the primary noise. $h(n)$ is the secondary path and $*$ denotes convolution. $b_m(n)$ is the mth stage backward prediction error of the reference signal $x(n)$ obtained by the reflection coefficients in Eq. (4). μ denotes the step-size parameter, and $\sigma_{\hat{b}_m}^2(n)$ denotes the power of the mth stage filtered backward prediction error which also can be recursively estimated similarly to $\xi_m(n)$. To apply the GAL algorithm in Eqs. (5), (6) to ANC system, the primary noise $d(n)$ should be available. However, in ANC situation, the primary noise is measured by using an error sensor in a combined form with the control signal. This requirement can be satisfied by exploiting a modular structure of the lattice filter. From the order-update in Eq. (5), a modified order-update equation can be obtained as [10]

$$e_{M-m-1}(n) = e_{M-m}(n) - w_{M-m-1}(n)\hat{b}_{M-m-1}(n) \tag{7}$$

where $e_{M-1}(n) = e(n)$ is the residual error at the error sensor. In the above equation, the mth stage local estimation error is computed in reverse order from the residual error. This process can also be viewed as a process for synthesizing the primary noise from the residual error.

3. VARIABLE STEP-SIZE FXGAL

In the following, we derive a VSS-FxGAL algorithm by combining the FxGAL algorithm presented in the previous section with a variable step-size (VSS) scheme.

For convenience of the derivation, we use an approximation $\hat{b}_m(n)e_m(n) \approx \hat{b}_m(n)e_{M-1}(n)$ which is justified by the orthogonality of the filtered backward prediction errors. Using vector notations, the update equation in Eq. (6) can be rewritten as

$$\mathbf{w}(n) = \mathbf{w}(n-1) - \mu(n)\mathbf{\Sigma}^{-1}(n)\hat{\mathbf{b}}(n)e_{M-1}(n), \tag{8}$$

where the residual error $e_{M-1}(n)$ is defined as

$$e_{M-1}(n) = d(n) + h(n) * \left[\mathbf{w}^T(n-1)\mathbf{b}(n)\right] \tag{9}$$

and the primary noise $d(n)$ consists of the target noise $p(n)$ that needs to be controlled and the system noise $s(n)$ that are uncorrelated to the reference signal $x(n)$. The vector $\mathbf{w}(n) = [w_0(n), \ldots, w_{M-1}(n)]^T$ is the adaptive filter coefficients vector. The matrix $\mathbf{\Sigma}(n) = \text{diag}\left\{\sigma_{\hat{b}_0}^2(n), \ldots, \sigma_{\hat{b}_{M-1}}^2(n)\right\}$ is a diagonal matrix with diagonal elements given by the filtered backward prediction error powers. The vector $\hat{\mathbf{b}}(n) = \left[\hat{b}_0(n), \ldots, \hat{b}_{M-1}(n)\right]^T$ is the filtered backward prediction errors vector. The vector $\mathbf{b}(n) = [b_0(n), \ldots, b_{M-1}(n)]^T$ is the backward prediction errors vector. The positive scalar $\mu(n)$ in Eq. (8) denotes the variable step-size parameter. Using the adaptive filter coefficients at time n, the *a posteriori* error can be defined as

$$\varepsilon(n) = d(n) + h(n) * \left[\mathbf{w}^T(n)\mathbf{b}(n)\right]. \tag{10}$$

We first apply Eq. (8) into Eq. (10) and eliminate $\mathbf{w}(n-1)$ using Eq. (9). Then, by assuming that the secondary path is well estimated, i.e., $h(n) \approx \hat{h}(n)$, we have

$$\varepsilon(n) = \left[1 - \mu(n)\hat{\mathbf{b}}^T(n)\mathbf{\Sigma}^{-1}(n)\hat{\mathbf{b}}(n)\right]e_{M-1}(n). \tag{11}$$

Due to uncorrelation between the system noise $s(n)$ and the reference signal $x(n)$, the ideal ANC systems will leave only the system noise $s(n)$ in the primary noise $d(n)$. Therefore, the ideal step-size parameter $\mu(n)$ should satisfy the condition

$$\sigma_\varepsilon^2(n) = \sigma_s^2(n), \, \forall n \tag{12}$$

where $\sigma_\varepsilon^2(n) = E\left\{\varepsilon^2(n)\right\}$ is the power of the *a posteriori* error and $\sigma_s^2(n) = E\left\{s^2(n)\right\}$ is the power of the system noise. Squaring Eq. (11) and taking the expectations, we obtain

$$E\left\{\left[1 - \mu(n)\hat{\mathbf{b}}^T(n)\mathbf{\Sigma}^{-1}(n)\hat{\mathbf{b}}(n)\right]^2\right\}\sigma_{e_{M-1}}^2(n) = \sigma_s^2(n), \tag{13}$$

where the independent assumption in [11] is used and the condition in Eq. (12) is applied. Using the orthogonality of the filtered backward prediction errors and the approximation $E\left\{\hat{b}_m^4(n)\right\} \approx E\left\{\hat{b}_m^2(n)\right\}^2$, Eq. (13) is further simplified to

$$[1 - \mu(n)M]^2\sigma_{e_{M-1}}^2(n) = \sigma_s^2(n), \tag{14}$$

where $E\left\{e_{M-1}^2(n)\right\} = \sigma_{e_{M-1}}^2(n)$ is the power of the residual error. From Eq. (14), it is straightforward to obtain a variable step-size parameter in the nth iteration:

$$\mu_{vss}(n) = \frac{1}{M}\left[1 - \sqrt{\frac{\sigma_s^2(n)}{\sigma_{e_{M-1}}^2(n)}}\right]. \tag{15}$$

Now the obtained variable step-size parameter can be used to replace the step-size parameters in Eqs. (4) and (6). The residual error power can be estimated in a recursive form:

$$\hat{\sigma}_{e_{M-1}}^2(n) = \lambda\hat{\sigma}_{e_{M-1}}^2(n-1) + (1-\lambda)e_{M-1}^2(n), \tag{16}$$

where $0 < \lambda < 1$ is the smoothing parameter.

However, the system noise power $\sigma_s^2(n)$ is not available in ANC system. To solve this problem, we first assume that the adaptive filter has converged fairly enough, thereby the secondary path is well

estimated, i.e., $p(n) \approx \left[\mathbf{w}^T(n)\,\hat{\mathbf{b}}(n) \right]$. Under this assumption, the local estimation error at the $(M-2)$th stage can be written as

$$e_{M-2}(n) \approx s(n) + \hat{y}_{M-1}(n), \qquad (17)$$

where $\hat{y}_{M-1}(n) = w_{M-1}^T(n)\,\hat{b}_{M-1}(n)$ is the $(M-1)$ estimated stage adaptive filter output. The system noise power can be estimated as

$$\sigma_s^2(n) \approx \sigma_{e_{M-2}}^2(n) - \sigma_{\hat{y}_{M-1}}^2(n). \qquad (18)$$

It is important to note that this estimates depends only on the signals available within ANC system, i.e., the $(M-2)$th stage local estimation error, $e_{M-2}(n)$, and the $(M-1)$th stage estimated adaptive filter output, $\hat{y}_{M-1}(n)$. Based on these findings, Eq. (15) can be rewritten as

$$\mu_{vss}(n) = \frac{1}{M}\left[1 - \sqrt{\frac{\hat{\sigma}_{e_{M-2}}^2(n) - \hat{\sigma}_{\hat{y}_{M-1}}^2(n)}{\hat{\sigma}_{e_{M-1}}^2(n)}} \right], \qquad (19)$$

where $\hat{\sigma}_{e_{M-2}}^2(n)$ is the $(M-2)$ stage estimated local estimation error power and $\hat{\sigma}_{\hat{y}_{M-1}}^2(n)$ is the $(M-1)$ stage estimated adaptive filter output power that can be recursively calculated similarly to $\hat{\sigma}_{e_{M-1}}^2(n)$.

Finally, some practical issues need to be considered. First, a very small positive number δ should be added to the denominator in Eq. (19) to avoid division by zero. Second, under the our assumptions that the adaptive filter has converge fairly enough, we have $\sigma_{e_{M-2}}^2(n) \geq \sigma_{\hat{y}_{M-1}}^2(n)$ and $\sigma_{e_{M-2}}^2(n) - \sigma_{\hat{y}_{M-1}}^2(n) \approx \sigma_{e_{M-1}}^2(n)$. Nevertheless, the estimates of these parameters could lead to some deviation from the previous theoretical condition. Therefore, we will take absolute values in Eq. (19). Hence, the variable step-size parameter is rewritten as

$$\mu_{vss}(n) = \frac{1}{M}\left| 1 - \sqrt{\frac{\left| \hat{\sigma}_{e_{M-2}}^2(n) - \hat{\sigma}_{\hat{y}_{M-1}}^2(n) \right|}{\hat{\sigma}_{e_{M-1}}^2(n) + \delta}} \right|. \qquad (20)$$

Third, we use the smoothing parameter $\beta(n) = 1 - \mu_{vss}(n)$ for the estimates of the parameters such as $\xi(n)$, $\sigma_{\hat{b}}^2(n)$ in the proposed algorithm.

4. SIMULATION RESULTS

Computer simulations were performed to assess the performance of the proposed VSS-FxGAL algorithm. The sampling rate was 2 kHz. All measured acoustic echo paths (primary and secondary ones) are plotted in Fig. 2(a), (b). We assumed that the secondary path was known, i.e., $h(n) = \hat{h}(n)$. The adaptive filter order was set to $M = 50$. In vehicle/ship environments, the reference signal often consists of multiple sinusoids and another colored noise. Thus, in the computer simulations, the reference signal $x(n)$ was assumed as the sum of sinusoids with colored noise as given by

$$x(n) = \sum_{i=1}^{I} C_i \sin(2\pi f_i n + \theta_i) + \phi(n) \qquad (21)$$

where I is the number of the sinusoids, C_i, f_i and θ_i are amplitude, frequency and phase of the sinusoids, respectively, and $\phi(n)$ is colored noise. The power spectrum density of the reference signal is plotted in Fig. 2(c). The system noise is a colored Gaussian signal generated by filtering white Gaussian noise (of zero mean and unit
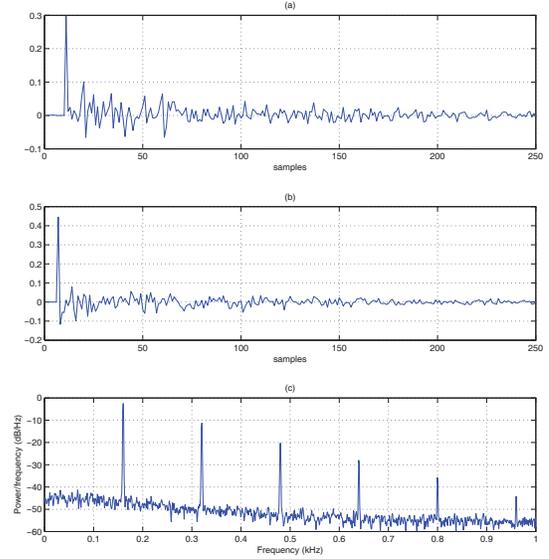
Fig. 2. Impulse response of (a) the primary path, (b) the secondary path, and (c) the power spectrum density of the reference signal.
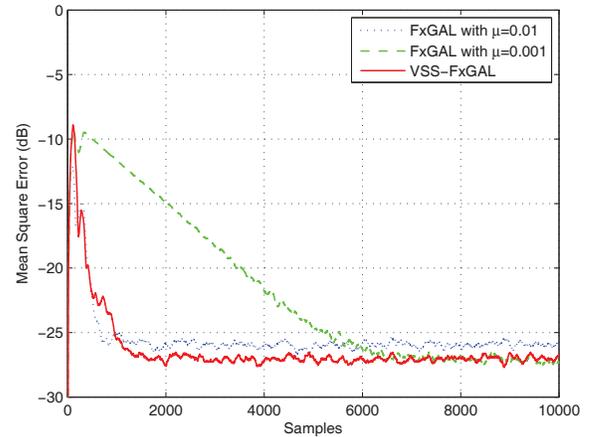
Fig. 3. MSE of the FxGAL algorithms ($\mu = 0.01$, $\mu = 0.001$) and VSS-FxGAL algorithm.
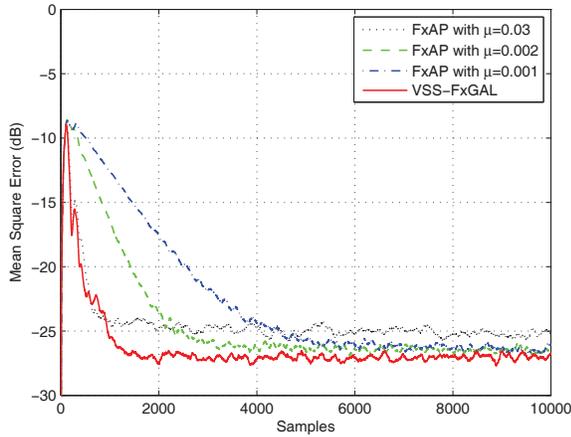
Fig. 4. MSE of the FxAP algorithms ($\mu = 0.03$, $\mu = 0.002$, $\mu = 0.001$) and VSS-GAL algorithm.

variance) with a first order autoregressive filter of the transfer function $1/(1 - az^{-1})$ where $a = 0.95$. The target noise-to-system noise ratio (TSR) was 30dB. In all experiments, the results are averaged over 30 independent trials.

In Fig. 3, the learning curve of the VSS-FxGAL algorithm was compared with those of the FxGAL algorithm for two different step-size parameter cases, i.e., $\mu = 0.01$ and $\mu = 0.001$. It can be noticed that the VSS-FxGAL algorithm has convergence speed similar to the FxGAL algorithm with $\mu = 0.01$, and it achieves low steady-state MSE, which is closer to the one obtained by FxGAL algorithm with $\mu = 0.001$.

In Fig. 4, the learning curve of the VSS-FxGAL algorithm was compared with those of the FxAP algorithm for three different step-size parameter cases, i.e., $\mu = 0.03$, $\mu = 0.002$, and $\mu = 0.001$. The projection order was set to $p = 4$. It can be noticed that the VSS-FxGAL algorithm outperforms the FxAP algorithm in terms of convergence speed and steady-state MSE.

Finally, in Fig. 5, the steady-state residual error power spectrum density of the VSS-FxGAL algorithm was compared with that of the FxAP algorithm with $\mu = 0.001$, $p = 4$. The VSS-FxGAL, FxAP algorithms accurately recover the system noise.

5. CONCLUSIONS

A new VSS-FxGAL algorithm for ANC has been proposed in this paper. Being compared to the classical FxGAL and FxAP algorithms, the VSS-FxGAL algorithm has superior performance when the primary noise consists of multiple sinusoids. Computer simulations in the context of ANC showed that the proposed method achieved faster convergence speed as well as smaller steady-state MSE than the FxAP algorithms.

6. REFERENCES

[1] S. J. Elliott and P. A. Nelson, "Active noise control." *IEEE Singal Processing Mag.*, vol. 10, no. 4, pp. 12-35, Oct. 1993.

[2] S. M. Kuo and D. R. Morgan, "Active noise control: A tutorial review," *Proc. IEEE*, vol. 87, no. 6, pp. 943-973, Jun. 1999.
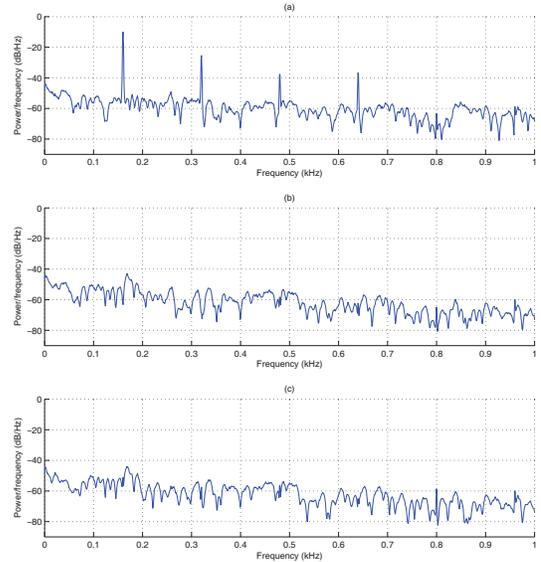
Fig. 5. Power spectrum density of (a) the primary noise (ANC off), (b) the steady-state residual error of the VSS-FxGAL algorithm, and (c) the steady-state residual error of the FxAP algorithm ($\mu = 0.001$ and $p = 4$).

[3] P. A. Nelson and S. J. Elliott, *Active Control of Sound.* Academic Press, New York, 1992..

[4] S. Haykin, *Adaptive Filter Theory*, 4th ed. Upper Saddle RIver, NJ: Prentice-Hall, 1996.

[5] S. C. Douglas, "The fast affine projection algorithm for active noise control," *Proceedings of the* 29*th Asilomar Conference on Signlas, Systems and Components*, Pacific Grove, CA, vol. 2, pp. 1245-1249, Oct. 1995.

[6] A. Carini and G. Sicuranza, "Transient and steady-state analysis of filtered-x affine projection algorithm," *IEEE Trans. on Signal Process.*, vol. 54, no. 2, Feb. 2006.

[7] S. Zimmerman, *Submarine technology for the 21st century*, 2nd ed. Trafford, 2006.

[8] L. J. Griffiths, "An adaptive lattice structure for noise-cancelling applications," *Proc. IEEE Int. Conf. on ASSP*, Tulsa, Okla., pp. 87-90, April 1978.

[9] R. C. North, J. R. Zeidler, T. R. Albert, and W. H. Ku, "Comparison of adaptive lattice filters to LMS transversal filters for sinusoidal cancellation, " *Proc. IEEE Int. Conf. on ASSP*, vol. 4, pp. 33-36, Mar. 1992.

[10] Y. C. Park and S. D. Sommerfeldt, "A fast adaptive noise control algorithm based on the lattice structure," *Applied Acoustics*, Elsevier Science Limited, Vol. 40, 1996.

[11] S. Makino, Y. Kaneda, and N. Koizumi, "Exponentially weighted step-size NLMS adaptive filter based on the statistics of a room impulse response," *IEEE Trans. Speech Audio Process.*, vol. 1, pp. 101-108, Jan. 1993.