DESIGN OF ROBUST POLYNOMIAL BEAMFORMERS FOR SYMMETRIC ARRAYS

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ABSTRACT

Polynomial broadband beamforming designs enable an easy, smooth, and dynamic steering of the main beam. A number of design methods based on constrained optimization have been proposed recently which allow for the control of the robustness of these designs. Of course, the addition of robustness constraints reduces the number of degrees of freedom of the design. In this paper, we present a method to enhance the spatial selectivity of the robust polynomial beamformer design by exploiting the structure of symmetric arrays while still satisfying the robustness constraints. The effectiveness of this method is shown in design examples for symmetric linear and circular arrays.

Index Terms— Robust Polynomial Beamformer, Symmetric Arrays

1. INTRODUCTION

With broadband beamforming for acoustic human-machine interfaces a beam of increased sensitivity has to be steered towards the desired and possibly moving source [1, 2, 3]. The polynomial beamforming (PB) method proposed in [4] enables dynamic and easy steering towards any desired look direction in a predefined angular range. It was also shown in [5] that PB can be combined efficiently with an acoustic echo canceller (AEC) resulting in AEC processing that is independent of beamsteering.

PB with an array of sensors is depicted in Fig. 1 and consists of two parts: $P+1$ fixed filter-and-sum units (FSUs) and a polynomial postfilter (PPF) of order P. The impulse response of the FIR filter of length L which processes the m-th microphone signal in the p-th FSU is denoted as $w_p(m,l)$ with $l = 0,\ldots,L-1$. The PPF weights and combines the different FSU output signals $\hat{y}_p(n)$ and, hence, the different FSU responses. The advantages of this method are that there is a fixed set of coefficients w_p and the steering direction of the beam is controlled by a single scalar D. Note that for $P = 0$ a single beamformer steered towards one look direction is obtained.

The original method proposed in [4] for designing the filters leads to noise sensitive beamformers and therefore we proposed a method in [6] which allows full control of the robustness of the polynomial beamformer design by applying a quadratic constraint on the white noise gain (WNG), which is a commonly used measure for the robustness of beamformer designs [7]. Other authors [8] have also proposed methods for robust designs. While the application of constraints is necessary for practically relevant designs it also reduces the number of degrees of freedom for the design.

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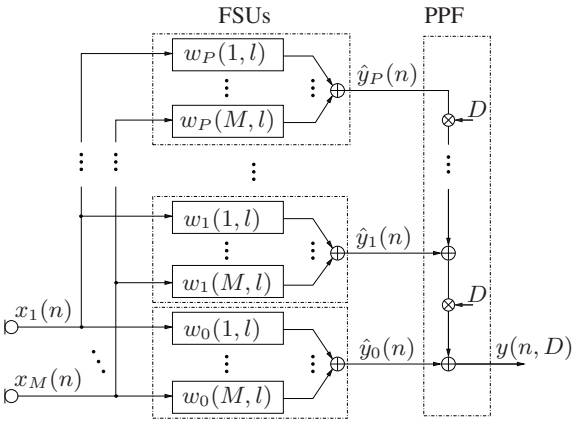

Fig. 1. Polynomial beamforming with an array of sensors

Typically, the PB is designed with prototype look directions (PLDs) distributed over the entire steering region, i.e., between $[0^\circ, \phi_{\max}]$, where $\phi_{\max} = 180^\circ$ and $\phi_{\max} = 360^\circ$ are the maximum steering angles for linear and circular arrays, respectively. The range over which the PLDs are distributed is termed PLD range here. The angular spacing between the PLDs has a direct bearing on the performance of the PB designs, i.e., large angular distances between PLDs lead to inferior performance in the adjoining angular regions. Therefore, in order to enhance the performance of PB designs, the angular distance between the PLDs should be reduced while still ensuring that steering across the entire steering region is still possible. It should be noted that simply increasing the number of PLDs in order to have a finer sampling grid over the entire steering region is often undesirable because this necessitates an increase in the PPF order P, which corresponds to an increase in the number of FSUs.

Lai et al., [8] proposed a method for enhancing the performance for uniformly spaced spiral arrays. The authors showed that it is sufficient to design the PB for uniform spiral arrays with the PLD range restricted to $[0^\circ, 360^\circ/M]$, where M is the total number of sensors, as opposed to $[0^\circ, 360^\circ]$. Steering the beam outside this range is achieved by rotating the sets of filters to the corresponding microphones [8].

In this paper a method which is more general and is applicable to any type of symmetric array for robust polynomial beamformer design is presented. The major advantage of this method over that proposed in [8] is that it is applicable for a larger set of symmetric arrays and it is capable of providing comparable spatial selectivity without compromising the robustness of the resulting beamformer.

Although the beamforming design is applicable to directional sensors and arbitrary source positions, we use the following com-

mon assumptions for beamforming designs for clarity: waves propagate in a free field, the sources are located in the farfield relative to the array, and all sensors are omnidirectional. Design examples for symmetric linear and circular arrays are presented.

2. POLYNOMIAL BEAMFORMER

A beamformer is characterized here by the beamformer response which describes the wavefield in the farfield produced for a given complex harmonic signal with frequency ω as parameter. The response of a polynomial beamformer for an array with M sensors, as depicted in Fig. 1, is given by

$$B_D(\omega,\phi) = \sum_{p=0}^{P} D^p \sum_{m=1}^{M} W_{m,p}(\omega) g_m(\omega,\phi), \qquad (1)$$

where $W_{m,p}(\omega) = \sum_{l=0}^{L-1} w_p(m,l)\exp(-jl\omega)$, $g_m(\omega,\phi)$ is the response of the m-th sensor located at position $\mathbf{r}_m$ to a plane wave with frequency ω traveling in the direction ϕ, L is the FIR filter length, and D denotes the steering direction. ϕ is the azimuth angle in a three-dimensional right-handed orthogonal coordinate system and elevation $\theta = 90^\circ$, i.e., ϕ lies in the Cartesian $x-y$ plane.

For introduction we briefly rederive the cost function of [6]. A desired frequency-invariant response $B_{\text{des}}(\phi,\phi_{\text{des}})$ is defined whose main beam points to the desired look direction ϕ_{des}. Consider an unconstrained least-squares beamformer which optimally approximates multiple desired responses, $B_{\text{des},i}(\phi,\phi_{\text{des}_i}), i = 0,\ldots,I-1$, each with a different look direction, by $B_{D_i}(\omega,\phi)$, $D_i = (\phi_{\text{des}_i} - \phi_{\max}/2)/(\phi_{\max}/2)$, in the least-squares sense. Note that ϕ_{des_i} are the PLDs which are typically uniformly distributed over the entire steering range. Typically, a numerical solution is obtained by discretizing the frequency range into Q frequencies ω_q, $q = 0,\ldots,Q-1$, the angular range into K angles $\phi_k, k = 0,\ldots,K-1$, and solving the resulting set of linear equations numerically. The beamformer design problem then reads [6]:

$$B_{\text{des},i}(\phi_k,\phi_{\text{des}_i}) \stackrel{!}{=} \sum_{p=0}^{P} D_i^p \sum_{m=1}^{M} W_{m,p}(\omega_q) g_m(\omega_q,\phi_k). \qquad (2)$$

Reformulating (2) in matrix notation the resulting beamformer design problem reads:

$$\min_{\mathbf{W}_{\text{f}}(\omega_q)} \sum_{i=0}^{I-1} \|\mathbf{G}(\omega_q)\mathbf{W}_{\text{f}}(\omega_q)\mathbf{d}_i - \mathbf{b}_{\text{des}_i}\|_2^2 \quad \forall q, \qquad (3)$$

where $\mathbf{b}_{\text{des}_i} = [B_{\text{des},i}(\phi_0,\phi_{\text{des}_i}),\ldots,B_{\text{des},i}(\phi_{K-1},\phi_{\text{des}_i})]^T$, $[\mathbf{G}(\omega_q)]_{km} = g_m(\omega_q,\phi_k)$, $[\mathbf{W}_{\text{f}}(\omega_q)]_{mp} = W_{m,p}(\omega_q)$, $\mathbf{d}_i = [D_i^0,\ldots,D_i^P]^T$, and $[\cdot]^T$ denotes the transpose.

Reformulating the problem and adding robustness and distortionless response constraints, a robust polynomial beamformer (RPB) design can be obtained by solving [6]

$$\min_{\mathbf{w}_{\text{f}}(\omega_q)} \|\boldsymbol{\mathcal{G}}(\omega_q)\mathbf{w}_{\text{f}}(\omega_q) - \mathbf{b}_{\text{des}}\|_2^2$$

subject to

$$\mathbf{a}_i^T(\omega_q)\mathbf{D}_i\mathbf{w}_{\text{f}}(\omega_q) = 1$$
$$\frac{|\mathbf{a}_i^T(\omega_q)\mathbf{D}_i\mathbf{w}_{\text{f}}(\omega_q)|^2}{\|\mathbf{D}_i\mathbf{w}_{\text{f}}(\omega_q)\|_2^2} \geq \gamma$$
$$\forall i = 0,\ldots,I-1, \qquad (4)$$

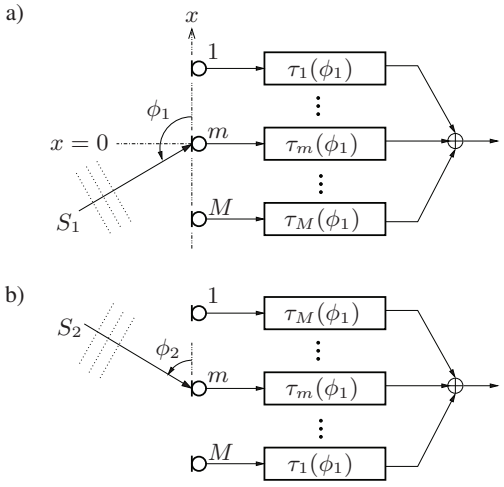

Fig. 2. Exploiting array symmetry for steering a DSB with $\phi_2 = 180^\circ - \phi_1$; Steering to a) ϕ_1 and b) ϕ_2

where $\boldsymbol{\mathcal{G}}(\omega_q) = [\mathbf{G}(\omega_q)\mathbf{D}_0,\ldots,\mathbf{G}(\omega_q)\mathbf{D}_{I-1}]^T$, $\mathbf{w}_{\text{f}}(\omega_q) = [W_{1,0}(\omega_q),\ldots,W_{1,P}(\omega_q),\ldots,W_{M,P}(\omega_q)]^T$, $\mathbf{D}_i = \mathbf{I}_M \otimes \mathbf{d}_i^T$, $\mathbf{a}_i(\omega_q) = [g_1(\omega_q,\phi_{\text{des}_i}),\ldots,g_M(\omega_q,\phi_{\text{des}_i})]^T$, and $\mathbf{b}_{\text{des}} = [\mathbf{b}_{\text{des}_0}^T,\ldots,\mathbf{b}_{\text{des}_{I-1}}^T]^T$. $\otimes$ denotes the Kronecker product, $\mathbf{I}_M$ is an $M \times M$ identity matrix, and γ is the lower bound for the WNG. (4) can straightforwardly be solved using `CVX`, a package for specifying and solving convex optimization problems [9, 10].

3. EXPLOITING ARRAY SYMMETRY

In PB designs the PLDs, ϕ_{des_i}, are typically uniformly distributed over the entire steering range in order to be able to steer the beam to any desired direction, i.e., the PLD range is equal to the entire steering range. It should be noted that the PLDs do not necessarily have to be uniformly distributed, however this would lead to larger errors in some areas and smaller ones in others. The idea behind this work is to limit the PLD range to only a part of the entire steering range by exploiting array symmetry. The same number of PLDs can then be used to cover a smaller angular region. As a consequence, the angular distance between these prototype look directions, which act as sampling points for interpolation, is decreased.

Although the following considerations are valid for all symmetric arrays, for the sake of simplicity let us first consider a symmetric linear array and a delay-and-sum beamformer (DSB).

Assume a source S_1 generates a plane wave which impinges on the array from ϕ_1 as depicted in Fig. 2a. A beam can be steered in this direction by computing the delay elements as

$$\tau_m(\phi_1) = \frac{r_m}{c}\cos\phi_1, \qquad (5)$$

where r_m is the position of the m-th sensor and c is the wave propagation speed. The beam can be steered toward another source S_2 located at $\phi_2 = 180^\circ - \phi_1$ by simply mirroring the delay elements about the center of the array ($x = 0$) as depicted in Fig. 2b. Thus, only the delays $\tau_m(\phi)$ for $\phi \in [0^\circ, 90^\circ]$ need to computed and mirroring can be applied to steer beyond 90°.

Although this result is trivial, we can apply exactly the same concept to limit the PLD range of a PB design for symmetric linear and circular arrays. Let the number of symmetry planes present on

an array be denoted by β. In the case of a polynomial beamformer design for a symmetric linear array $\beta = 1$ and $\phi_{\text{max}} = 180^\circ$. The PLD range can now be limited to $[0^\circ, 90^\circ]$ instead of $[0^\circ, 180^\circ]$. Steering beyond 90° is achieved simply by mirroring the filters.

Without loss of generality, let us assume one of the symmetry planes lies along the x-axis. In the case of a polynomial beamformer design for a symmetric circular array with non-uniform spacing, $\beta \geq 1$ depending on the sensor positions and $\phi_{\text{max}} = 360^\circ$. If $\beta = 1$ the PLD range can now be limited to $[0^\circ, 180^\circ]$ instead of $[0^\circ, 360^\circ]$. Steering beyond 180° is achieved simply by mirroring the filters about the symmetry plane. If $\beta = 2$ the PLD range is further limited to $[0^\circ, 90^\circ]$ and steering is achieved by mirroring about the two symmetry planes.

In the case of a circular array with M uniformly spaced sensors, which is a special case of a symmetric circular array, $\beta = M$. In this case the PLD range can be further limited to $[0^\circ, 360^\circ/(2M)]$ which is a significant reduction compared to the original range of $[0^\circ, 360^\circ]$.

From the considerations above the maximum angle that should be considered in the PLD range for symmetric arrays is equal to

$$\phi_{\text{PLD}} = \frac{\phi_{\text{max}}}{2\beta}. \tag{6}$$

An RPB design which exploits array symmetry is termed RPBS. It should be noted that if no symmetry exists for an array the PLD range has to cover the entire steering range.

4. EVALUATION

The proposed RPBS design was evaluated for microphone array broadband beamforming, by investigating symmetric linear and circular array geometries. FIR filters $w_p(m, l)$ with length $L = 512$ were used to approximate the frequency response vectors $[W_{m,p}(\omega_0), \ldots, W_{m,p}(\omega_{Q-1})]$ in the least squares sense. The main lobe of the desired frequency response was always defined with a 3-dB beamwidth of 20 degrees. Lower and upper cut-off frequencies of $0.3\,\text{kHz}$ and $3.4\,\text{kHz}$, respectively, were chosen with telephone speech signal capture in mind. A sampling frequency of $8\,\text{kHz}$, PPF order of $P = 3$ and a WNG lower limit of $\gamma = 0.001$ were chosen.

The mean squared error (MSE) between the desired responses $B_{\text{des}}(\vartheta_k, \phi_{\text{des}_i})$ and the actual responses $B_D(\omega_q, \phi_k)$ is computed for 5° steps in ϕ_{des}. The MSE is given by

$$MSE = \sum_{i=0}^{71} \sum_{q=0}^{Q-1} \sum_{k=0}^{K-1} \frac{(|B_D(\omega_q, \phi_k)| - |B_{\text{des}}(\phi_k, i5^\circ)|)^2}{72QK}.$$

Each of the design examples is represented by a figure containing multiple subfigures depicting the beamformer's beampattern, WNG, and magnitude response (MR) in the desired look direction, respectively.

The RPBS is first evaluated for a symmetric linear array ($\beta = 1$) consisting of seven microphones with a uniform spacing of $0.025\,\text{m}$. Note that a regular spacing is not required as long as the arrangement is symmetric. The PB designs are jointly optimized for $I = 6$ uniformly distributed PLDs. In case of the RPB design, the PLD range is $[0^\circ, 180^\circ]$, i.e., the PLDs are $[0^\circ, 36^\circ, 72^\circ, 108^\circ, 144^\circ, 180^\circ]$. This range is reduced in the RPBS design to $[0^\circ, 90^\circ]$ by exploiting the array symmetry, i.e., the PLDs are $[0^\circ, 18^\circ, 36^\circ, 54^\circ, 72^\circ, 90^\circ]$.

Fig. 3 depicts the results for $\phi_{\text{des}} = 120^\circ$ for both the RPB and RPBS designs. Both designs were not optimized for this look direction. The beampatterns clearly show that the RPBS design achieves

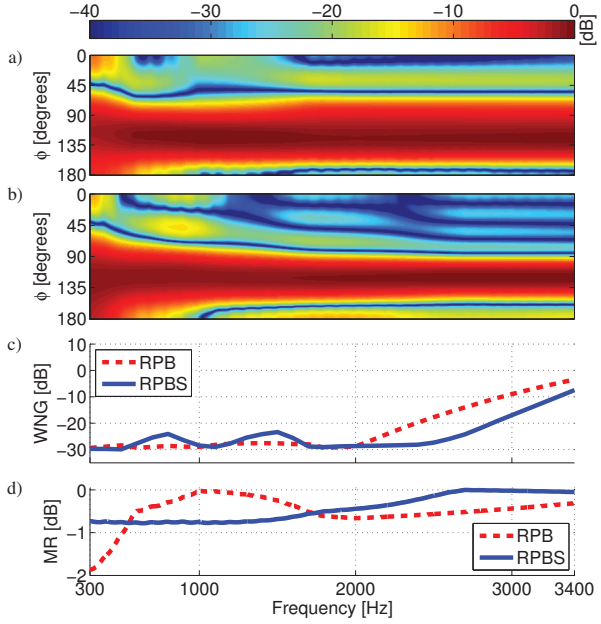

Fig. 3. 7-element uniform linear array; $0.025\,\text{m}$ spacing; $\phi_{\text{des}} = 120^\circ$; $I = 6$; Beampatterns for a) RPB and b) RPBS; c) WNGs; d) Magnitude responses

superior spatial selectivity compared to the RPB design by exploiting the symmetry of the array. The MSE of 0.04 for the RPBS design is significantly smaller than 0.1 obtained for the RPB design. The WNG is constrained successfully in both designs. Although the magnitude response deviations from $0\,\text{dB}$ for both designs are relatively small, the variations are smaller for the RPBS.

Next, we consider a symmetric circular array with six non-uniformly spaced microphones ($\beta = 1$) and radius $r = 0.03\,\text{m}$. The six microphones are placed at $\boldsymbol{\phi}_{\text{mic}} = [0^\circ, 50^\circ, 140^\circ, 180^\circ, 220^\circ, 310^\circ]$. The PB designs are optimized for $I = 6$ uniformly distributed PLDs, which are $[0^\circ, 71.8^\circ, 143.6^\circ, 215.4^\circ, 287.2^\circ, 359^\circ]$ for the RPB design and $[0^\circ, 36^\circ, 72^\circ, 108^\circ, 144^\circ, 180^\circ]$ for the RPBS.

Fig. 4 depicts the results for $\phi_{\text{des}} = 250^\circ$ for both designs. Both designs were not optimized for this look direction. The beampatterns show that both designs give a frequency-invariant main beam. However, the RPBS design results in superior spatial selectivity due to the narrower beam and the lower sidelobes. Note that due to the microphone placement, there is a reduction in spatial selectivity for both designs in the angular region about 0° and 180°. The MSE of 0.07 for RPBS design is also significantly smaller than 0.21 obtained for the RPB design. The WNG is constrained successfully. The deviations in the magnitude response are also smaller for the RPBS design.

Finally, we evaluate the performance of the RPBS design for a symmetric circular array with 6 uniformly spaced microphones ($\beta = M$) and a radius $r = 0.03\,\text{m}$. The microphones are placed at $\boldsymbol{\phi}_{\text{mic}} = [0^\circ, 60^\circ, 120^\circ, 180^\circ, 240^\circ, 300^\circ]$. We compare the performance with the method based on rotating filters proposed by Lai et al., [8] (here termed RPBL design) where the PLD range is reduced by a factor of M. Our method reduces the PLD range by a factor of $2M$. As the PLD range is significantly reduced we use $I = 4$ uniformly distributed PLDs, which are $[0^\circ, 20^\circ, 40^\circ, 60^\circ]$ for

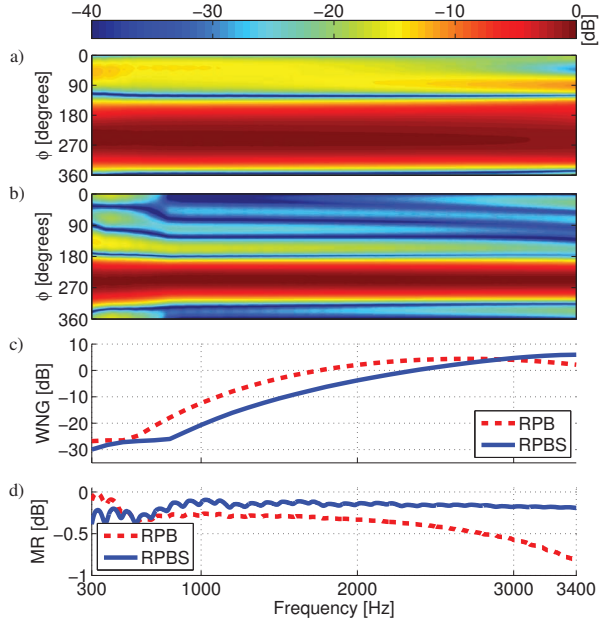

Fig. 4. 6-element non-uniform circular array; $r = 0.03\,\mathrm{m}$; $\boldsymbol{\phi}_{\mathrm{mic}} = [0^\circ, 50^\circ, 140^\circ, 180^\circ, 220^\circ, 310^\circ]$; $I = 6$; $\phi_{\mathrm{des}} = 250^\circ$; Beampatterns for a) RPB and b) RPBS; c) WNGs; d) Magnitude responses

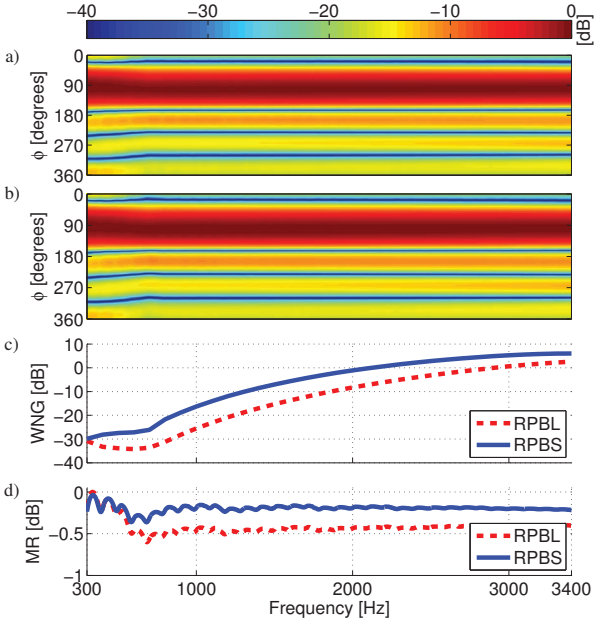

Fig. 5. 6-element uniform circular array; $r = 0.03\,\mathrm{m}$; $\boldsymbol{\phi}_{\mathrm{mic}} = [0^\circ, 60^\circ, 120^\circ, 180^\circ, 240^\circ, 300^\circ]$; $I = 4$; $\phi_{\mathrm{des}} = 95^\circ$; Beampatterns for a) RPBL and b) RPBS; c) WNGs; d) Magnitude responses

the RPBL design and $[0^\circ, 10^\circ, 20^\circ, 30^\circ]$ for the RPBS design.

Fig. 5 depicts the results for $\phi_{\mathrm{des}} = 95^\circ$ for both the RPBL and RPBS designs. Both designs were not optimized for this look direction. The beampatterns show very similar results. This is further supported by the fact that both designs have an MSE of 0.06. The differences in the designs become clearer by considering the WNGs and magnitude responses. It is clear to see that the WNG of the RPBL design has larger deviations than the RPBS and even violates the WNG constraint since it goes below $-30\,\mathrm{dB}$. The magnitude response deviations of the RPBS design are also smaller. Although the results for the RPB are not shown here due to space restrictions, they are significantly worse in terms of spatial selectivity and deviations in the WNG and magnitude response, due to the large angular distance of 120° between PLDs.

5. CONCLUSION

A novel method for the design of robust polynomial beamformers which exploits array symmetry has been presented. The beamformer design has been shown to offer superior spatial selectivity and improve the adherence to the WNG and distortionless response constraints compared to existing design methods.

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