# DECENTRALIZED DYNAMIC SPECTRUM ALLOCATION BASED ON ADAPTIVE ANTENNA ARRAY INTERFERENCE MITIGATION DIVERSITY: ALGORITHMS AND MARKOV CHAIN ANALYSIS

Alexandr M. Kuzminskiy\*

Bell Laboratories, Alcatel-Lucent The Quadrant, Swindon SN5 7DJ, UK ak9@alcatel-lucent.com

### ABSTRACT

Decentralized dynamic spectrum allocation (DSA) that exploit adaptive antenna array interference mitigation (IM) diversity at the receiver, is proposed for interference-limited environments with high level of frequency reuse. The system consists of base stations (BSs) that may belong to different providers, who can optimize uplink frequency allocation to their subscriber stations (SSs) to achieve the least impact of IM on the useful signal, assuming no control over band allocation of other BSs sharing the same bands. "Selfish" and "good neighbor" decentralized DSA strategies are considered. Convergence and convergence rate of the introduced techniques are investigated by means of the theory of absorbing Markov chains.

*Index Terms*— Decentralized dynamic spectrum allocation, interference mitigation, "selfish" and "good neighbor" strategies, absorbing Markov chain.

#### 1. INTRODUCTION

Dynamic spectrum or channel allocation can be an effective way to increase spectral efficiency of wireless communications systems [1]. In the license-exempt spectrum, channel allocation must be performed by each provider in a decentralized autonomous way, e.g., as currently considered for ad hoc [3] or WIMAX [2] networks. In this case, DSA strategy is focused on maximal interference avoidance. For example, in [2] a multichannel version of the carriersense multiple-access collision-avoidance (CSMA/CA) algorithm operates by selectively activating or deactivating groups of OFDM sub-carriers separated by the guard bands.

In the most interesting scenario, where the total number of SSs that belong to different closely located but not explicitly cooperating subsystems exceeds the number of available bands, joint interference avoidance/suppression may be required. One such system is analyzed in [3], where adaptive transmit/receive beamforming is considered for each ad hoc node pair. These node pairs communicate with each other on a selected frequency basis, whereby the transmit beamformer replicates the adaptive receive antenna beampattern. Clearly, by reducing energy transmitted to directions occupied by the interference, the self performance can be improved simultaneously with reducing the interference load for the neighboring nodes. Global

\*Part of this work has been done in the context of the IST FP7 PHYDYAS project.

Yuri I. Abramovich

## Defence Science and Technology Organization PO Box 1500, Edinburgh SA 5111, Australia Yuri.Abramovich@dsto.defence.gov.au

convergence of a decentralized DSA algorithm in such a reciprocal environment, supported by game theory methodology, e.g., [4], can be established. Global convergence of linear precoding algorithms in non-cooperative resource sharing systems is studied in [5].

A more challenging scenario is considered in this paper with no such reciprocity. The global convergence cannot be guaranteed in this case. Instead, we propose a technique that significantly reduces probability of undesirable non(slow)-convergent behavior. Specifically, we consider DSA in the uplink interference-limited environment with a number of wireless systems consisting of multipleantenna BSs and associated single-antenna SSs. These systems may belong to different providers and do not explicitly cooperate in a centralized fashion. Frequency channels in this case can be formed in an OFDM-based, e.g., WIMAX, system by an appropriate subcarrier allocation with guard bands for preventing energy leakage between channels allocated to unsynchronized users [2] or in spectrally efficient filter bank based multicarrier (FBMC) systems by using frequency selective filters for adjacent channels [6].

Since the number of available bands is less than the total number of SSs, some of these SSs belonging to different subsystems have to share the same frequency. We show that an IM-based DSA algorithm at each subsystem should allocate bands to its users, such that the propagation channels from the users to their BSs are as orthogonal as possible to the active interference propagation channels. The main problem here is that any decision made by a given BS regarding frequency allocation of its users may have an arbitrary impact on interference scenarios for other BSs, due to the non-reciprocal nature of propagation channels from the SSs of a given subsystem to other BSs. Therefore, in this non-reciprocal scenario, the performance, convergence, and convergence rate of decentralized DSA is far from obvious and must be investigated.

The two types of decentralized IM-based DSA techniques, namely, the "selfish" and "good neighbor" ones, are introduced in the paper. A Markov model is developed for the considered problem and the theory of absorbing Markov chains is used for convergence analysis.

#### 2. SYSTEM MODEL AND PROBLEM FORMULATION

The considered system consists of N independent subsystems containing base stations  $BS_n$ , n = 1, ..., N and corresponding users  $SS_{nm}$ , m = 1, ..., M, where M is the number users per BS. Users transmit data to their BSs using one of the  $F \ge M$  available frequency channels. BSs have full information and control of their own users. In particular, they can estimate propagation channels in all the available bands and prescribe the individual bands and transmit powers to their own users. Assuming for simplicity narrowband channels, the signal received by an antenna array of K elements for the nth subsystem can be expressed as follows:

$$\mathbf{x}_{nf}(t) = \sum_{l=1}^{N} \sum_{m=1}^{M} \delta_{fd_{lm}} q_{lm} \mathbf{h}_{d_{lm}mln} s_{lm}(t) + \mathbf{z}_{nf}(t), \quad (1)$$

where  $\mathbf{x}_{nf}(t)$  is the  $K \times 1$  vector of the signal received at  $BS_n$ in the *f*th band at the *t*th time instant,  $\mathbf{h}_{fmln}$  is the  $K \times 1$  vector of propagation channel to  $BS_n$  in the *f*th band from the *m*th user of the *l*th subsystem,  $s_{nm}(t)$  is the  $SS_{nm}$  transmitted signal with  $E\{|s_{nm}(t)|^2\} = 1$  and  $q_{nm}^2$  is its constrained power  $\sum_{m=1}^{M} q_{nm}^2 = M$ , n = 1, ..., N,  $\mathbf{z}_{nf}(t)$  is a  $K \times 1$  vector of AWGN with  $E\{\mathbf{z}_{nf}(t)\mathbf{z}_{nf}(t)^*\} = \sigma^2 \mathbf{I}_K$ ,  $d_{nm}$  is the *nm*th element of the  $N \times M$  decision matrix  $\mathbf{D}$  denoting the frequency band assigned to  $SS_{nm}$ ,  $E\{\cdot\}$  is the averaging operator,  $(\cdot)^*$  is the conjugate transpose operation,  $\mathbf{I}_K$  is the  $K \times K$  unity matrix, and  $\delta ij$  is the Kronecker function.

In this study we assume an interference limited scenario  $\sigma^2 \ll 1$ , different bands for all the users in one subsystem, i.e., all the rows in matrix **D** contain different elements, and constant power  $q_{nm} = 1$  for all users in the system, i.e., locally selected frequency bands are the only adjustable parameters. Power control in the IM-based DSA is addressed in [7].

We define a global performance metric, which cannot be estimated locally at each  $BS_n$ , as the data rate for the weakest link in the system

$$\gamma = \min_{m=1,\ldots,M, n=1,\ldots,N} \log_2 \left[ 1 + \text{SINR}(\mathbf{D}) \right], \quad (2)$$

where  $SINR(\mathbf{D}) = \mathbf{h}_{d_{nm}mnn}^* \mathbf{R}_{d_{nm}n}^{-1} \mathbf{h}_{d_{nm}mnn}$  is the SINR at the output of the optimal spatial filter for the *nm*th user and

$$\mathbf{R}_{d_{nm}n} = \sum_{i \neq n}^{N} \sum_{j=1}^{M} \delta_{d_{nm}d_{ij}} \mathbf{h}_{d_{ij}jin} \mathbf{h}_{d_{ij}jin}^{*} + \sigma^{2} \mathbf{I}_{K}$$
(3)

is the  $K \times K$  interference covariance matrix at BS<sub>n</sub> in the band occupied by SS<sub>nm</sub>.

In this paper, we concentrate on cognitive radio related issues, rather than on non-stationary propagation channel, and a finite amount of data effects. Thus, the propagation channels for all users in all bands are assumed to be stationary and known at the corresponding BS, e.g., BS<sub>n</sub> knows  $\mathbf{h}_{fmnn}$  for  $f = 1, \ldots, F$ ,  $m = 1, \ldots, M$ , and  $n = 1, \ldots, N$ .

Space-time spectrum sensing is required at each  $BS_n$  to obtain the interference covariance matrices (3) in all the available bands. To do this, we assume that all users can transmit data signals or stay silent during data and sensing intervals controlled by the BSs. Furthermore, focusing on the cognitive radio effects, we assume that the sensing intervals for different subsystems do not overlap and the interference covariance matrices are estimated accurately during corresponding sensing intervals. A low probability of overlapping of the sensing intervals can be achieved, for example, by means of random duration of the data intervals.

The problem is to develop and analyze decentralized algorithms for selection of the decision matrix  $\mathbf{D}$  that with high probability achieve reasonably fast convergence to acceptable steady-state global performance (2).

#### 3. IM-BASED DSA ALGORITHMS

A basic element of an IM-based DSA algorithm is a local search of the band assignment. In the considered system, a natural "selfish" search can be based on local maximization of the minimum SINR independently for each  $BS_n$ :

$$\mathbf{d}_n = \arg \max \min_{f_i \neq f_j \in \mathcal{F}} \mathbf{h}_{f_m m n n}^* \mathbf{R}_{f_m n}^{-1} \mathbf{h}_{f_m m n n}, \qquad (4)$$

where  $\mathbf{R}_{fn}$  is defined in (6),  $\mathbf{d}_n = [d_{n1}, \ldots, d_{nM}]$  is the  $1 \times M$  vector of different elements representing the *n*th row of the global allocation matrix  $\mathbf{D}$ , and  $\mathcal{F} = 1, \ldots, F$  is the set of all available bands. This algorithm will be referred to as *maximum minimum* (MaxMin) search. If exhaustive local search in (4) is not feasible, simplified algorithms can be applied as studied in [7].

The "selfish" algorithm can be summarized for the *n*th subsystem as follows:

Sensing interval

Step 1: Estimate  $\mathbf{R}_{fn}, f = 1, \ldots, F$ ;

Step 2: Find  $d_n$  according to (4) or simplified search algorithms, and assign bands  $d_n$  to  $SS_{nm}$ ;

Step 3: Calculate the optimal weight vectors

$$\mathbf{w}_{nm} = \frac{\mathbf{R}_{d_{nm}n}^{-1} \mathbf{h}_{d_{nm}mnn}}{\mathbf{h}_{d_{nm}mnn}^{*} \mathbf{R}_{d_{nm}n}^{-1} \mathbf{h}_{d_{nm}mnn}}, \ m = 1, \dots, M.$$
(5)

Data interval

 $SS_{nm}, m = 1, \ldots, M$  transmit data in the bands assigned in  $d_n$ ;

BS<sub>n</sub> receives data with the optimal weight vectors  $\mathbf{w}_{nm}$ ,  $m = 1, \ldots, M$ .

The main disadvantage of the "selfish" algorithm is that in pursuing the best results for its own BS, the interference environment of other BSs keeps changing, leading to poor convergence for the whole system. Furthermore, it does not allow any control of the convergence properties, such as a trade-off between convergence probability and speed, and the global performance. To overcome these drawbacks, we introduce a "good neighbor" threshold-regulated IMbased DSA algorithm.

The main idea is to prevent selection of new bands at some BS if its performance is already above some threshold  $\gamma_0$  and minimize the number of new band allocations to achieve the given threshold. It is expected that local minimization of the new band allocations may reduce non-stationary interference to other subsystems and improve convergence properties compared to the "selfish" approach. Indeed, if only a few users have SINR below the threshold and actually need re-allocation to other bands, then application of the conventional search algorithms as in (4) may still cause re-allocation of many or even all the users, which creates a difficult non-stationary environment.

The new search problem can be formulated as follows:

$$\mathbf{d}_{n} = \arg \min_{f_{i} \neq f_{j} \in \mathcal{F}} \sum_{m=1}^{M} |\operatorname{sign}(f_{m} - d_{nm}^{(0)})|, \tag{6}$$

subject to

$$\log_2\left(1 + \mathbf{h}_{f_m m n n}^* \mathbf{R}_{f_m n}^{-1} \mathbf{h}_{f_m m n n}\right) \ge \gamma_0, \tag{7}$$

where  $d_{nm}^{(0)}$  is the *m*th element af the current band allocation vector  $\mathbf{d}_n^{(0)}$  before the current sensing interval for BS<sub>n</sub> and sign(a) =  $\{-1, 0, 1\}$  is the sign function. Algorithm (6), (7) will be referred to as *minimum switch* (MinSwitch) search.

It is worth emphasizing that a threshold-regulated approach also can be implemented based on the MaxMin search, where the best local bands can be reallocated only if some of the user's SINRs fall below the threshold. However, even in this case, the MaxMin search may reallocate many or all the users even if only a few of them actually need that to satisfy the threshold. Thus, it is expected that the MinSwitch search may show better convergence, especially for high-dimension systems.

The threshold-regulated algorithm can be specified by adding two more steps to the DSA algorithm above after Step 1 and modifying Step 2:

• Sensing interval

Step 1a: Calculate

$$\gamma_n = \log_2 \left( 1 + \min_{m=1,\dots,M} \mathbf{h}_{d_{nm}mnn}^* \mathbf{R}_{d_{nm}mn}^{-1} \mathbf{h}_{d_{nm}mnn} \right);$$
(8)

Step 1b: If  $\gamma_n \geq \gamma_0$ , then go to the "Data interval" stage without updating  $\mathbf{d}_n$  and  $\mathbf{w}_{nm}$ ; otherwise, go to Step 2;

Step 2: Find  $d_n$  according to (6), (7) or simplified local search, then assign bands  $d_n$  to  $SS_{nm}$ .

#### 4. USING ABSORBING MARKOV CHAINS FOR ANALYSIS OF THE IM-BASED DSA ALGORITHMS

Now, our goal is to analyze the performance of the decentralized IM-based DSA algorithms for given stationary propagation channels. The theory of Markov chains, e.g., [8], provides us with a tool to do this.

To formulate a Markov model we assume that all possible  $I = (A_M^F)^N$  different allocation matrices  $\mathbf{D}_i$ ,  $i = 1, \ldots, I$  form states of the Markov chain. For a given state  $\mathbf{D}_i$ , sensing of the *n*th subsystem transfers the system to state  $\mathbf{D}_{j_n}$  depending on the given channel realization and DSA algorithm, where  $j_n \in [1, I]$ , including  $j_n = i$ . Repeating this procedure for  $n = 1, \ldots, N$ , a set of  $\mathbf{D}_{j_n}$  can be found, where not all  $j_n$  may be different.

Assuming that, at each sensing interval, one randomly selected subsystem is sensed with probability  $p_{\text{sens}} = N^{-1}$ , the nonzero elements of the  $I \times I$  transition probability matrix  $\mathbf{P} = \{p_{ij}\}$  can be defined as follows:

$$p_{ij} = g_j p_{\text{sens}}, \, i = 1, \dots, I,\tag{9}$$

where  $1 \leq g_j \leq N$  is the number of outcomes of sensing trials at  $BS_n$ , n = 1, ..., N, leading to  $D_{j_n} = D_j$ . For example, if sensing each of N subsystems leads to different states for the given initial state, then all the corresponding states get equal probabilities  $p_{\text{sens.}}$ . If some of the sensing trials lead to the same outcome, then this state gets increased probability according to (9).

The transition probability matrix  $\mathbf{P} = \{p_{ij}\}$  is a sparse stochastic matrix with maximum N nonzero elements in a row, such that  $\sum_{j=1}^{I} p_{ij} = 1$  for  $i = 1, \ldots, I$ , which completely defines the Markov model of the considered system.

The Markov theory, e.g., [8], provides us with analytical expressions for the convergence probabilities and speed for an absorbing Markov chain, which has at least one absorbing point with transition probability  $p_{ii} = 1$  and all other states are transient with non-zero probabilities to transit to one of the absorbing points not necessarily in one step. One difficulty is that in the general case, the Markov chain may contain ergodic subchains with states that can transit only within corresponding subchains. Obviously, in the considered application of Markov theory, situations with no absorbing states and/or with ergodic subchains lead to a non-zero probability of undesirable non-convergent behavior.

To apply the theory of absorbing Markov chains to our problem, we need the following: calculate a transition probability matrix; classify all the states into three groups: transient, absorbing, and ergodic, e.g., as in [9]; estimate the global performance for the absorbing states; if ergodic subchains are found, then transform the initial Markov chain to the reduced size absorbing Markov chain by means of replacing the ergodic subchains with the corresponding absorbing states; calculate probabilities of absorption by the absorbing states (desirable behavior) and ergodic subchains (undesirable nonconvergent behavior) and average convergence speed.

When all the states are classified, then the absorbing Markov chain with the  $(I_t + I_a) \times (I_t + I_a)$  transient probability matrix Pa can be formed by replacing all the ergodic subchains, if they exist, with absorbing states, where  $I_a \ge 0$  is the number of absorbing states including the actual ones and the collapsed ergoding subchains if they exist,  $I_t$  is the number of transient states, and  $I_t + I_a \le I$ .

For a given  $P_a$ , the probabilities of convergence to the corresponding absorbing states can be found as follows [8]:

$$\mathbf{E} = \mathbf{C}\mathbf{B} \tag{10}$$

where **E** is the  $I_t \times I_a$  matrix of convergence probabilities from each transient state to each absorbing point, **A** and **B** are  $I_t \times I_t$ and  $I_t \times I_a$  components of the canonical form  $\bar{\mathbf{P}}_a$  of the transition matrix

$$\bar{\mathbf{P}}_{a} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{0} & \mathbf{I} \end{bmatrix},\tag{11}$$

and  $\mathbf{C} = (\mathbf{I} - \mathbf{A})^{-1}$  is the  $I_t \times I_t$  fundamental matrix of  $\bar{\mathbf{P}}_a$ .

The average number of iterations (sensing intervals in our case) before absorption is

$$\mathbf{t} = \mathbf{C1},\tag{12}$$

where t is the  $I_t \times 1$  vector of the average number of iterations before absorption from each initial transient state, and 1 is a  $I_t \times 1$  vector of all ones.

Now, for the given system configuration and propagation channels, we can analyze the steady- state and convergence performance of the algorithms presented in Section 3. Let us illustrate that for  $M = 2, F = 3, N = 5, K = 4, \sigma^2 = 10^{-2}$ , and independent random Gaussian vectors  $\mathbf{h}_{fmln} \sim \mathcal{CW}(0, \mathbf{I}_K)$  as stationary propagation channels.

The probabilities  $p_e$  of absorption by ergodic subchains (nonconvergence) from a randomly selected initial state averaged over 100 channel realizations are presented in Tabl. 1 together with probabilities  $p_{na}$  to find a chain with no absorbing points. The cumulative distributed functions (CDF) of the number of absorption states, convergence speed, and global performance for the absorption states are presented in Fig. 1 for the "selfish" MaxMin-based algorithm and the "good neighbor" threshold-regulated MinSwitch solution.

The following observations can be made:

- The number of absorbing points grows significantly with introduction of the threshold, compared to the "selfish" solution.

- The probability of non-convergence can be controlled by selection of the threshold.

- The most important observation is that threshold selection allows significant improvement of the convergence speed.

Similar results were obtained for the same system configuration with K = 3 for a lower level of global performance. The main difference is that a much lower number of absorbing points and slower convergence were observed in the K = 3 case compared with 4 BS antennas for thresholds selected at the same relative distance from the global performance. Fig. 2 presents a comparison of the number of absorbing points and convergence speed for 3 and 4 BS antennas for thresholds at 60% of the global performance shown in Figs. 3 and 4: 4 bits/symbol for K = 4 and 2 bits/symbol for K = 3. One can see that for a similar relative performance, the K = 3 case shows approximately 10 times fewer absorbing points and at least twice longer convergence compared with the case of 4 BS antennas. A possible explanation of this behavior is that if the number of antennas is not enough to cancel all interference components, then the number of good solutions should be much lower because they require a reduced dimension of the interference subspace additionally to avoiding colinearity between propagation channels of the desired signal and interference. This makes decentralized algorithms less efficient compared to the case of complete interference suppression. One can expect that this situation may be even more complicated for higher dimension systems.

### 5. CONCLUSION

DSA techniques are addressed that operate in a non-reciprocal environment, where any changes in frequency allocation of a certain subsystem introduces a non-stationary interference scenario for other subsystems in the network. "Selfish" and "good neighbor" threshold-regulated IM-based DSA strategies are introduced. Their convergence and convergence rate are studied by means of the theory of absorbing Markov chain for low-dimension system configuration. Further investigation is given in [7], including power control and simplified algorithms for higher-dimension systems.

#### 6. REFERENCES

- Q. Zhao, B. M. Sadler, "A survey of dynamic spectrum access," IEEE Signal Processing Magazine, vol. 24, no. 3, pp. 79-89, May 2007.
- [2] O. Ashagi, S. Murphy, L. Murphy, "A distributed approach to interference mitigation between OFDM-based 802.16 systems operating in license-exempt spectrum," in Proc. ICC, pp. 4855-4860, June 2007.
- [3] E. Zeydan, D. Kivanc-Tureli, U. Tureli, "Joint iterative channel allocation and beamforming algorithm for interference avoidance in multiple-antenna ad hoc networks," in Proc. MILCOM, Oct. 2007.
- [4] V. Srivastava, J. Neel, A. B. MacKenzie, R. Menon, L. A. DaSilva, J. E. Hicks, J. H. Reed, R. P. Gilles, "Using game theory to analyze wireless ad hoc networks," IEEE Communications Surveys and Tutorials, vol. 7, no. 4, pp. 46-56, 2005.
- [5] G. Scutari, D. P. Palomar, S. Barbarossa, "Optimal linear precoding strategies for wideband non-cooperative systems based

on game theory - Part II: Algorithms," IEEE Trans. Signal Processing, vol. 56, no. 3, pp. 1250-1267, March 2008.

- [6] "Physical layer for dynamic access and cognitive radio," PHY-DYAS, INFSO-ICT-211887, http://www.ict-phydyas.org.
- [7] A. M. Kuzminskiy, Y. I. Abramovich, "Adaptive antenna array interference mitigation diversity for decentralized dynamic spectrum allocation in license-exempt spectrum," to appear in Proc. ICC, June 2009.
- [8] J. G. Kemeny, G. L. Thompson, Introduction to finite mathematics, Prentice-Hall, N. J., 1966.
- [9] B. L. Fox, D. M. Landi, "An algorithm for identifying the ergodic subchains and transient states of a stochastic matrix," Communications of the ACM, vol. 11, no. 9, pp. 619-621, Sept. 1968.

Tabl.	1.	Proba	bilit	v of	und	lesiral	ble	non-convergen	t be	havior
				/ ~ ~						

Exhaustive	No th	reshold	5 bits	s/symbol	4 bits/symbol		
Search	pna	$p_{e}$	$p_{na}$	$p_{e}$	pna	$p_{e}$	
MaxMin	2%	4.3%	0%	0.12%	0%	0.005%	
MinSwitch	-	-	0%	0.08%	0%	0.002%	



Fig. 1. Number of absorbing points, convergence rate and steady-state performance for K = 4 antennas at BS



**Fig. 2**. Comparison of the number of absorbing points and convergence speed for 3 and 4 antennas at a BS for thresholds at 60% of the global performance: 4 bits/symbol for K = 4 antennas and 2 bits/symbol for K = 3 antennas