

Pilot Design for Multi-User MIMO

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Abstract—In this paper, challenges regarding the provision of channel state information (CSI) on a multi-user (MU) MIMO-OFDM downlink are addressed. Reference symbols (pilots) support coherent detection at the receiver and enable link adaptation at the transmitter, but add overhead and consume transmit power. As spatial transmit processing combined with multiple access essentially requires the provision of dedicated and common pilots, i.e. pilots that include respectively exclude user-specific transmit processing, the potential gains of MU-MIMO may partly be canceled by the incurred pilot overhead. Fortunately, spatial correlation at the transmitter has the potential to substantially reduce overheads, by introducing spatial reuse of dedicated pilots, and by utilizing spatial interpolation of common pilots. We demonstrate that a reduction in overhead is traded with compromised channel estimation accuracy, which particularly limits the attainable spectral efficiency in the high SNR regime. This implies that a bandwidth efficient pilot design is to be complemented by advanced channel estimation schemes, which enhance accuracy by utilizing data symbols together with pilots.

Index Terms—OFDM, MIMO, multiple access, pilot aided channel estimation (PACE)

I. INTRODUCTION

Systems employing multiple antennas in combination with orthogonal frequency division multiplexing (OFDM) have been adopted in several wireless standards, such as IEEE 802.11n [1], and beyond 3rd generation (B3G) mobile communication systems [2]. A multiple-input multiple-output (MIMO) OFDM downlink is considered where one base station serves multiple mobile users. As the number of base station antennas typically exceeds the number of antennas at the mobile receiver, the signal streams of several users are to be spatially multiplexed so to achieve the capacity of the multi-user (MU) MIMO-OFDM downlink [3].

Reference symbols known to the receiver, referred to as pilots, support coherent detection at the receiver. Channel state information (CSI) is also essential to allow for link adaptation and spatial precoding at the transmitter. A fundamental requisite for MU-MIMO with spatial precoding is the provision of the *effective channel* at the receiving end, whereas the unweighted MIMO channel matrix is required at the transmitter to generate the spatial precoder.

The provision of various kinds of CSI at the receiver and transmitter makes pilot design for MU-MIMO a challenging task. To estimate the effective channel, *dedicated pilots* that include user-specific spatial processing are to be inserted. In addition, *common pilots* that do not contain user-specific processing are needed to generate the MIMO channel matrix, as well as to measure the channel over the entire frequency band, so to exploit multi-user diversity by channel aware multi-user slot assignment [4]. A low rate feedback link conveys the CSI to the base station. With sophisticated compression and quantization algorithms a large portion of the potential gains of MU-MIMO can be realized [5, 6].

Another fundamental issue is the pilot overhead; if transmit antennas are mutually uncorrelated, the incurred pilot overhead grows in proportion to the number of transmit antennas [7], and may consume up to half of the available resources [8]. Fortunately, in case of spatially correlated transmit antennas, pilot overhead can be substantially reduced. To this end, dedicated pilots associated to well spatially separated beams may be multiplexed in space, i.e. pilots of different spatial streams are placed on the same subcarriers. Likewise, common pilots may only be inserted on selected transmit antennas to facilitate channel estimation by spatial interpolation [9] and/or to obtain the relevant information for spatial precoding [10].

In this paper, pilot design enabling multi-user MIMO with spatial transmit processing is addressed. Using the capacity lower bound of [8] as performance metric, the trade-off between pilot overhead and channel estimation accuracy is established. Specifically, we elaborate on the benefits of exploiting spatial correlation to reduce pilot overhead.

II. SYSTEM MODEL

An OFDM system with N_c subcarriers, and L OFDM symbols per frame is considered. A number of subcarriers and OFDM symbols are grouped into time-frequency resource blocks (chunks). A number of N_{st} signal streams are spatially multiplexed per chunk. The transmit symbol vector on subcarrier n at OFDM symbol block ℓ is denoted by $\mathbf{x}_{n,\ell} = (x_{n,\ell}^{(1)}, x_{n,\ell}^{(2)}, \dots, x_{n,\ell}^{(N_{st})})^T$, where $x_{n,\ell}^{(i)}$ denotes spatial stream i with energy $E[x_{n,\ell}^{(i)}] = E_d/N_{st}$. For MU-MIMO spatial streams may be associated with different users. The N_{st} spatial streams are processed by the spatial precoder $\mathbf{V} = [\mathbf{v}^{(1)}, \dots, \mathbf{v}^{(N_{st})}]$ of dimension $N_T \times N_{st}$, where the spatial precoder for stream i is $\mathbf{v}^{(i)} = [v_1^{(i)}, \dots, v_{N_T}^{(i)}]^T$. In frequency division duplex (FDD) mode the selection of precoding vectors is driven by the receiver through a feedback process. The output of the precoder $\mathbf{V} \mathbf{x}_{n,\ell}$ is passed to an antenna array with N_T elements, and transmitted over a multipath fading channel. Assuming perfect orthogonality in time and frequency, the received signal yields

$$y_{n,\ell} = \mathbf{h}_{n,\ell}^T \mathbf{V} \mathbf{x}_{n,\ell} + z_{n,\ell} \quad (1)$$

where $z_{n,\ell}$ accounts for additive white Gaussian noise (AWGN) with zero mean and variance N_0 .

The received signal (1) applies to a mobile receiver equipped with a single receive antenna; the extension to multiple receive antennas is treated in [11–13]. In case of MIMO-OFDM with uncorrelated receive antennas, where channel estimation for each receive antenna is carried out independently, (1) directly applies. When receive antennas are correlated, the accuracy of the channel estimates may be improved by spatial smoothing of the received pilots over the receive antennas [11–13].

The *effective channel* observed at the receiver, including spatial transmit precoding, is given by the $N_{st} \times 1$ vector $\mathbf{g}_{n,\ell} =$

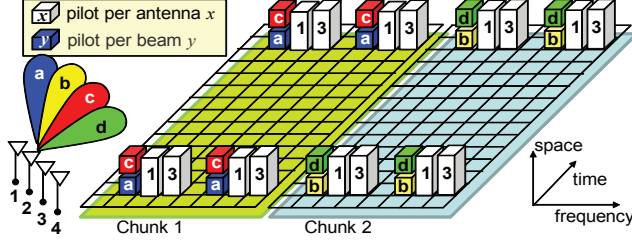


Fig. 1. MU-MIMO pilot design: dedicated pilots are spatially multiplexed and common pilots are inserted only on selected antennas.

$\mathbf{V}^T \mathbf{h}_{n,\ell}$. Its mean $G^{(i)} = E[|g_{n,\ell}^{(i)}|^2]$ represents the average beamforming gain of the i -th beam $\mathbf{v}^{(i)}$ in the direction of the observed receiver. The channel transfer function (CTF) $\mathbf{h}_{n,\ell}$ of dimension $N_T \times 1$ represents the unweighted channel, i.e. the channel response without spatial precoding. A normalized average channel gain, $E\{|h_{n,\ell}^{(\mu)}|^2\} = 1$ is assumed.

Spatial precoding separates the N_{st} spatial streams. In the following we assume that for detection of spatial stream i the interference from other streams is treated as noise. Then (1) can be re-written as

$$y_{n,\ell} = g_{n,\ell}^{(i)} x_{n,\ell}^{(i)} + \underbrace{\sum_{j=1, j \neq i}^{N_{st}} g_{n,\ell}^{(j)} x_{n,\ell}^{(j)}}_{\zeta_{n,\ell}^{(i)}} + z_{n,\ell} \quad (2)$$

where $\zeta_{n,\ell}^{(i)}$ denotes interference due to spatial multiplexing with variance

$$\sigma_{\zeta}^2[i] = E[|\zeta_{n,\ell}^{(i)}|^2] = \sum_{j=1, j \neq i}^{N_{st}} G^{(j)}. \quad (3)$$

The average beamforming gain $G^{(j)} = E[|g_{n,\ell}^{(j)}|^2]$ accounts for the side-lobes of the j -th beam $\mathbf{v}^{(j)}$ in the direction of the receiver of the desired stream i . According to (2), coherent detection of the data symbol $x_{n,\ell}^{(i)}$ only requires the effective channel $g_{n,\ell}^{(i)} = \{g_{n,\ell}^{(i)}\}_i$ of stream i .

III. PILOT DESIGN

For MU-MIMO two types of pilots are distinguished: dedicated pilots to support detection of transmitted data, and common pilots to enable link adaptation at the transmitter. These two pilot types are described in the following.

Dedicated pilots per beam (DPB) undergo the same user-specific spatial processing as the associated data symbols of that chunk. For DPB N_{st} pilot sets are inserted according to the rank of the spatial precoder \mathbf{V} , as illustrated in Fig. 1. Given that ϱ pilots beams are spatially multiplexed on one subcarrier,¹ $\alpha_{DPB} = \lceil N_{st}/\varrho \rceil$ orthogonal pilot sets in time and/or frequency are required. For DPB the received signal of pilot $\tilde{x}_{n,\ell}^{(i)}$ benefit from the beamforming gain $G^{(i)}$, and is given by²

$$\tilde{y}_{n,\ell}^{(i)} = \tilde{g}_{n,\ell}^{(i)} \tilde{x}_{n,\ell}^{(i)} + \underbrace{\sum_{j=1, j \neq i}^{\varrho} \tilde{g}_{n,\ell}^{(j)} \tilde{x}_{n,\ell}^{(j)}}_{\tilde{\zeta}_{n,\ell}^{(i)}} + \tilde{z}_{n,\ell}, \quad 1 \leq i \leq \varrho \quad (4)$$

¹The ceiling operator $\lceil x \rceil$ denotes the smallest integer equal or larger than x .

²Variables related to pilot symbols are marked with a $\tilde{\cdot}$ in the following.

where $\tilde{\zeta}_{n,\ell}^{(i)}$ denotes the interference due to ϱ spatially multiplexed pilots. Provided that transmit beams \mathbf{V} are typically selected such that the spatial interference is reasonably small, a spatial pilot reuse factor of $\varrho = N_{st}$ is feasible, so that $\alpha_{DPB} = 1$.

In order to estimate the effective channel of stream i , a number of M_t and M_f received pilots (4), cast into the $M_t M_f \times 1$ vector $\tilde{\mathbf{y}}^{(i)}$, are processed by a 2D interpolation filter \mathbf{w} , to produce the estimate $\hat{g}_{n,\ell}^{(i)} = \mathbf{w}^H \tilde{\mathbf{y}}^{(i)}$ [14]. Due to the user-specific spatial processing, dedicated pilots may vary from chunk to chunk. Hence, interpolation over multiple chunk is not possible, so that M_t and M_f are upper bounded by the number of pilots per chunk.

Common pilots per antenna (CPA) exclude spatial precoding and measure the CTF $h_{n,\ell}^{(\mu)}$ over the entire frequency band. CPA support link adaptation, adaptive chunk selection and spatial precoding, by feedback of the compressed and quantized CTF to the transmitter [5]. If transmit antennas are uncorrelated, common pilots add overhead that grows in proportion to N_T [7]. In case of correlated transmit antennas, spatial correlation can be exploited to improve channel estimation accuracy [11]. Alternatively, spatial correlation allows pilots to be inserted on selected antennas only [9, 10], as illustrated in Fig. 1. Then $\alpha_{CPA} = N_T/D_s$ orthogonal pilot sets are inserted, where D_s denotes the pilot spacing in space.

Pilot Overhead: Pilot symbols consume resources that are constraint by bandwidth and power. One spatial stream is assigned N_p pilot and N_d data symbols, which amounts to

$$N_c L = N_d + \alpha N_p \quad (5)$$

with $\alpha = \alpha_{CPA} + \alpha_{DPB}$. The resulting pilot overhead yields

$$\Omega_p = \frac{\alpha N_p}{N_c L}. \quad (6)$$

The accuracy of the channel estimates may be improved by a pilot boost. However, potential gains of a pilot boost marginalize as the number of transmit antennas increases [8, 15]. We therefore restrict the discussion to a fixed power allocation per subcarrier of E_d for both data and pilot symbols.

IV. ESTIMATION ERROR MODEL FOR DEDICATED PILOTS

In [15] a model for common pilots is developed to assess the effect of channel estimation errors on the system performance. This estimation error model is applied to dedicated pilots in [16], including spatial reuse of pilots. Channel estimation errors $\varepsilon_{n,\ell}^{(i)} = g_{n,\ell}^{(i)} - \hat{g}_{n,\ell}^{(i)}$ are quantified by the mean squared error (MSE) $\sigma_{\varepsilon}^2[n, \ell, i] = E[|\varepsilon_{n,\ell}^{(i)}|^2]$. For dedicated pilots (4) the channel estimates are generated by a 2D interpolation filter $\hat{g}_{n,\ell}^{(i)} = \mathbf{w}^H \tilde{\mathbf{y}}^{(i)}$.

In order to allow for a tractable model, we choose to average the MSE over frequency and time (indices n and ℓ), so $\sigma_{\varepsilon}^2[n, \ell, i] \rightarrow \bar{\sigma}_{\varepsilon}^2[i]$. However, averaging over the beam index i is in general not possible, as the spatial interference strongly depends on beams $\mathbf{v}^{(i)}$.

It is assumed that the signal and noise parts are mutually uncorrelated. Furthermore, $\tilde{g}_{n,\ell}^{(i)}$ and $\tilde{\zeta}_{n,\ell}^{(i)}$ are uncorrelated if pilots of spatially multiplexed streams are scrambled with uncorrelated pilot sequences. Then the MSE, can be split into a noise error $\bar{\sigma}_n^2[i]$, an interpolation error $\bar{\sigma}_{ip}^2$, and a component

due to spatial interference over pilot symbols $\tilde{\sigma}_\zeta^2[i]$. The MSE for DPB can be expressed as [16]:

$$\bar{\sigma}_\varepsilon^2[i] = \bar{\sigma}_n^2[i] + \bar{\sigma}_{\text{itp}}^2 + \tilde{\sigma}_\zeta^2[i] \quad (7)$$

The variance of the noise error is

$$\bar{\sigma}_n^2[i] = \frac{N_{\text{st}}N_0}{E_d} \mathbf{w}^H \mathbf{w} = \frac{N_{\text{st}}N_0}{E_d} \cdot \frac{1}{\bar{W}} \quad (8)$$

where $\bar{W} = 1/(\mathbf{w}^H \mathbf{w})$ defines the estimator gain.

The interpolation error accounts for impairments between $g_{n,\ell}^{(i)}$ and imperfect interpolation between received pilots. Its variance $\bar{\sigma}_{\text{itp}}^2$ depends on the 2nd order statistics of $g_{n,\ell}^{(i)}$ [16].

Unlike CPA, the estimator dimension $M_t \times M_f$ for DPB is bounded by the number of pilots per chunk, so that both \bar{W} and $\bar{\sigma}_{\text{itp}}^2$ are typically significantly poorer than for CPA. The resulting error floor caused by $\bar{\sigma}_{\text{itp}}^2$ at high SNR severely affects the performance of DPB [17]. Fortunately, iterative channel estimation schemes, where decoded symbols fed back from the channel decoder output are utilized as auxiliary pilots, retain a performance close to a perfectly known channel [17].

The mean interference due to spatial reuse of pilots is

$$\tilde{\sigma}_\zeta^2[i] = \frac{E[|\tilde{\zeta}_{n,\ell}^{(i)}|^2]}{\bar{W}} = \sum_{j=1, j \neq i}^{\varrho} \frac{G^{(j)}}{\bar{W}} \quad (9)$$

where $\tilde{\zeta}_{n,\ell}^{(i)}$ is defined in (4) and $G^{(j)}$ represents the average interference of beam $j \neq i$ due to spatial reuse of pilots ($\varrho > 1$) in direction of the receiver of desired stream i . Clearly, $\tilde{\sigma}_\zeta^2[i] = 0$ when all pilots are orthogonally separated ($\varrho = 1$).

For DPB with $\varrho = N_{\text{st}}$ the interference due to spatial pilot reuse becomes $\tilde{\sigma}_\zeta^2 = \sigma_\zeta^2 / \bar{W}$. As the interpolation error $\bar{\sigma}_{\text{itp}}^2$ for DPB is relatively large, it is not expected that $\tilde{\sigma}_\zeta^2$ dominates. Hence, spatial reuse of pilots with $\varrho = N_{\text{st}}$ appears to be particularly attractive.

V. SPECTRAL EFFICIENCY ANALYSIS

In [8] a capacity lower bound is derived that determines the attainable spectral efficiency of pilot aided channel estimation schemes. To give a realistic capacity bound for MU-MIMO, apart from channel estimation errors also the effects of the compressed and quantized CSI on the generation of the spatial precoder \mathbf{V} are to be taken into account [6]. A comprehensive study of all imperfections in MU-MIMO is beyond the scope of this paper, and also strongly depends on the chosen MU-MIMO scheme. We therefore restrict the spectral efficiency analysis to the effect of channel estimation errors at the receiver.

The effect of channel estimation errors on the received signal (2) is described by the equivalent system model [18]

$$y_{n,\ell} = g_{n,\ell}^{(i)} x_{n,\ell}^{(i)} + \eta_{n,\ell}^{(i)} \quad (10)$$

where the effective noise term $\eta_{n,\ell}^{(i)} = z_{n,\ell} + \varepsilon_{n,\ell}^{(i)} x_{n,\ell}^{(i)} + \zeta_{n,\ell}^{(i)}$ accounts for AWGN, channel estimation errors $\varepsilon_{n,\ell}^{(i)} = g_{n,\ell}^{(i)} - \hat{g}_{n,\ell}^{(i)}$, and interference from other beams $\zeta_{n,\ell}^{(i)}$ in (2). In the following we assume that the estimation error $\varepsilon_{n,\ell}^{(i)}$ is a Gaussian random variable with zero mean and variance equal to the MSE, $\bar{\sigma}_\varepsilon^2[i]$. Then the variance of the effective noise term $\eta_{n,\ell}^{(i)}$ yields

$$\bar{\sigma}_\eta^2[i] = N_0 + \frac{E_d}{N_{\text{st}}} \bar{\sigma}_\varepsilon^2[i] + \frac{E_d}{N_{\text{st}}} \sigma_\zeta^2[i] \quad (11)$$

where $\sigma_\zeta^2[i]$ is given by (3).

Effective SNR: The SNR of the equivalent system model (10) describes the effective SNR including spatial processing and channel estimation errors. Inserting the parametrized MSE of Section IV, the effective SNR yields

$$\gamma_{n,\ell}^{(i)} = \frac{|g_{n,\ell}^{(i)}|^2}{\bar{\sigma}_\eta^2[i]} = \frac{|g_{n,\ell}^{(i)}|^2}{\frac{N_{\text{st}}N_0}{E_d} \left(1 + \frac{1}{\bar{W}}\right) + \sigma_{\text{floor}}^2} \quad (12)$$

In the high SNR regime, the effective SNR (12) is dominated by the SNR independent term

$$\sigma_{\text{floor}}^2 = \sigma_\zeta^2[i] + \tilde{\sigma}_\zeta^2[i] + \bar{\sigma}_{\text{itp}}^2 \quad (13)$$

giving rise to an error floor.

The effective SNR (12) is a measure for the performance degradation due to channel estimation errors, and allows to assess the achieved error probability [18] or the capacity [8].

Capacity of MU-MIMO including channel estimation errors: The ergodic channel capacity that includes channel estimation and pilot insertion losses can be lower bounded by [8]

$$C = \sum_{i=1}^{N_{\text{st}}} (1 - \Omega_p) E[\log_2(1 + \gamma_{n,\ell}^{(i)})] \quad (14)$$

where the expectation is taken over frequency and time (indices n and ℓ). The effect of channel estimation errors on the capacity is twofold: the degradation of the effective SNR $\gamma_{n,\ell}^{(i)}$, and the loss in bandwidth efficiency due to pilot insertion by Ω_p in (6).

VI. NUMERICAL EXAMPLE

Assessment Methodology: An OFDM system with $N_c = 1024$ subcarriers and $L = 12$ OFDM symbols per frame is considered. Groups of $8 \times 12 = 96$ symbols in frequency and time constitute one chunk. The signal bandwidth is 40 MHz, which corresponds to a sampling duration of $T_{\text{sp}} = 25$ ns. The base station is equipped with $N_T = 8$ transmit antennas with $\lambda/2$ -spaced elements. The spatial precoder selects N_{st} beams out of a number of $N_{\text{bf}} = 8$ fixed beamforming vectors according to [19]. The sidelobe level of the beams can be lowered by tapering. Tapering reduces inter-beam interference $\zeta_{n,\ell}^{(i)}$ in (2), at the expense of a decreased maximum beamforming gain $G^{(j)}$ [19]. In case of spatial multiplexing, $N_{\text{st}} > 1$, Chebyshev tapering [20] with a sidelobe suppression of 22 dB is selected.

A multi-path fading channel of a typical urban macro-cell (model C2 in [21]), specified within the IST-WINNER project is considered. The channel model determines the power-delay profile and angle of departures [21]. Time variations due to mobile velocities follow Jakes' model [22], with maximum Doppler frequency of $f_D = 0.0067$, normalized to the OFDM symbol duration, corresponding to a velocities up to 50 km/h at 5 GHz carrier frequency.

The pilot overhead Ω_p is composed of common and dedicated pilots, with pilot spacings in frequency and time set to $D_f = 4$ and $D_t = 10$. To keep the pilot overhead of common pilots to an acceptable level the pilot spacing in space is set to $D_s = 4$. Assuming $N_{\text{st}} = 4$ spatial streams and allowing for spatial reuse of pilots with $\varrho = N_{\text{st}}$, 4 and 8 dedicated and common pilots are inserted per chunk, so that the overall pilot overhead amounts to $\Omega_p = 12.5\%$. This compares to a 4-fold overhead of conventional pilot design with $D_s = 1$ and $\varrho = 1$

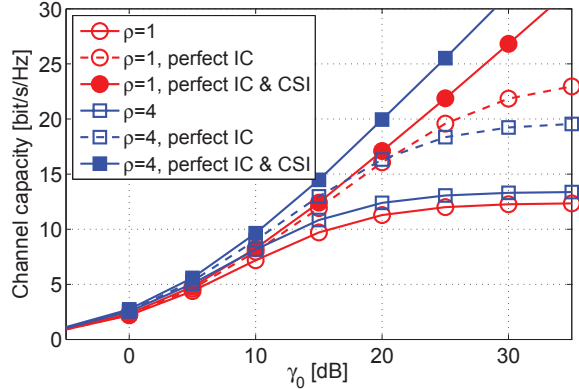


Fig. 2. Capacity of MU-MIMO vs γ_0 with $N_{st}=4$ spatial streams. The impact of spatial pilot reuse on the capacity is elaborated. In addition, the capacity of a system that is able to cancel the spatial interference on data symbols (marked “perfect IC”), together with perfect channel knowledge (marked “perfect IC & CSI”), is also included.

of $\Omega_p=50\%$. As the impact of common pilots is difficult to quantify, due to additional distortions of the low rate feedback link to the transmitter, only estimation errors of dedicated pilots are considered.

Channel estimation is carried out using a 2D Wiener filter with model mismatch as specified in [14]. To generate the $M_f=2$ by $M_t=2$ filter coefficients, an uniformly distributed power delay profile and Doppler power spectrum non zero within the range $[0, \tau_{max}]$ and $[-f_D, f_D]$ are assumed.

Results: The channel capacity versus the SNR $\gamma_0=E_d/N_0$ for $N_{st}=4$ spatially multiplexed streams is shown in Fig. 2. We observe that the capacity is increased by $\approx 10\%$ when allowing for a spatial pilot reuse from $\rho=1$ to 4; due to a reduction in DPB pilot overhead. Inter-beam interference on the data symbols dominates the effective SNR (12), so that the reduced overhead offered by spatial pilot reuse with $\rho=4$ outweighs the increased estimation error due to the additional spatial interference on the pilots. An advanced receiver that is able to cancel the interference from other beams further boosts capacity. When considering perfect interference cancellation (IC), where $\sigma_\zeta^2=0$, spatial pilot reuse ($\rho=4$) is only superior to orthogonal pilots ($\rho=1$) in the low SNR regime ($\gamma_0 < 20$ dB). On the other hand, by also allowing for advanced channel estimation schemes, has the potential to retain the superior performance of spatial pilot reuse, as indicated by the upper bound in Fig. 2 of perfect IC and CSI, with $\sigma_\zeta^2=0$ and $\bar{\sigma}_\varepsilon^2=0$.

VII. CONCLUSIONS

Conventional pilot design for multi-user (MU) MIMO schemes may incur high pilot overheads, severely affecting the spectral efficiency. Specifically, two types of pilots are needed: dedicated pilots to facilitate estimation of the effective channel at the receiver; as well as common pilots for spatial precoding, conveyed through low rate feedback link to the transmitter. Means to cut the overhead of conventional pilot design, that grows proportionally with the number of transmit antennas, are therefore essential. Fortunately, the pilot overhead can be grossly reduced by spatially multiplexing of dedicated pilots and by spatial interpolation of common pilots that are inserted on selected antennas only. This however implies that pilot aided

schemes may fail to deliver sufficient accuracy, especially in the high SNR regime. Advanced channel estimation at the receiver, where data symbols are used to generate refined estimates, is therefore identified as a natural complement of bandwidth efficient MU-MIMO pilot design.

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