A DYNAMIC GAME MODEL FOR AMPLIFY-AND-FORWARD COOPERATIVE COMMUNICATIONS

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ABSTRACT

Cooperative wireless communication protocols are designed with the assumption that users always behave in a socially efficient manner. This assumption may not be valid in commercial wireless networks where users may violate rules of cooperation to reap benefits of cooperation at no cost. Disobeying the rules of cooperation creates a social-dilemma where well-behaved users exhibit uncertainty about intention of other users. Cooperation in social-dilemma is characterized by a non-cooperative Nash equilibrium which indicates *the difficulty of maintaining a socially optimal cooperation without establishing a mechanism to detect and mitigates effects of misbehavior*. In this paper, we formulate interaction of users in cooperative Amplify-and-Forward as a dynamic game with incomplete information. We show the existence of a perfect Bayesian equilibrium.

Index Terms— Ad hoc network, Communication system security, Cooperative diversity, Game theory

1. INTRODUCTION

Cooperative diversity envisages to provide some of the benefits of multiple input multiple output systems (MIMO) to single-antenna users. The envisaged performance benefits are realized through cooperative diversity protocols such as Amplify-and-Forward (**AF**) which is shown to significantly enhance system performance[1]. Cooperative diversity protocols are designed based on the principle of direct reciprocity where a user will be motivated to help others attain cooperative diversity gain with anticipation to reap those same benefits when the helped users reciprocate. When every user obeys the rules of cooperation (i.e., follow direct reciprocity), a stable and socially efficient cooperation is realizable. This may be true in wireless networks under the control of a single entity since cooperation is confined to users with a common objective. On the other hand, users in commercial wireless networks may intentionally fail to reciprocate violating rules of cooperation to reap benefits of cooperation without bearing the cost. Note that the intention to deviate from rules of cooperation is motivated by the desire to save energy. Thus, in commercial wireless networks a socially efficient cooperation may not be easily achieved without introducing a mechanism that fosters cooperation by disincentivizing misbehavior.

Various mechanisms have proposed to ensure that users adhere to the rules of cooperation. In [2, 3] a Generous Tit for Tat strategy is proposed that would drive the operating point of a cooperation game to a Pareto optimal Nash equilibrium. Pricing based cooperation scheme is proposed in [4] where a user is charged for channel use whenever it transmits its own data and gets reimbursed when it forwards for other users. In [5] the pricing scheme in [4] is extended to cooperative diversity. The game theoretic formalization of cooperation in the aforementioned works in particular and in the literature in general, assumes a static game model where players make decisions simultaneously. In other words, users do not observe actions of their partners when they make decisions. Thus, a static game model do not capture well the dynamics of users interactions in cooperative diversity. Recently a dynamic Bayesian game framework has been proposed to model routing in energy constrained wireless ad hoc networks in [6]. The approach proposed in this paper is motivated by the work in [6].

In this paper, we formulate interaction of users in cooperative **AF** as a dynamic game with incomplete information. The dynamic game formulation captures temporal and information structure of cooperative communications. Temporal structure of a dynamic game defines order of play : cooperative transmissions occur in sequential manner wherein a source user transmits first and then potential cooperators decide to either cooperate (process received information using **AF** and forwards it) or deviate from cooperation. The sequential nature of cooperative transmissions is dictated by the half-duplex constraint of wireless devices, i.e., a relay terminal can not receive and transmit at the same time in the same frequency band. The information structure of a dynamic game characterizes what each player knows when it makes a decision : in commercial wireless networks intention of each user is not known a priori, hence, incomplete information specification of the game represents the uncertainty each user has about the intention of other users in the network. Since relay terminals make decisions after observing actions of source terminals, cooperative interaction of users is a Stackelberg (leader-follower) game. A dynamic game with incomplete information is studied within Bayesian framework, thus, we refer to the proposed game model as dynamic Bayesian game. We show that the proposed dynamic game model satisfies conditions for the existence of a perfect Bayesian equilibrium.

The rest of the paper is organized as follows. In Section 2, we describe system model for cooperative **AF**. In Section 3, we introduce a dynamic game model for cooperative diversity using **AF**. Finally, in Section 4, we present concluding remarks.

2. SYSTEM MODEL

We consider N-user TDMA based cooperative diversity system wherein users forward information for each other using cooperative **AF**, We assume that users (source terminals) select utmost one potential cooperator (relay) among all their neighboring users. We assume that transmission between each user (including the intended receiver) undergoes independent Rayleigh fading. We denote by $\gamma_1, \gamma_2, \gamma_3$ instantaneous signal to noise ratio (SNR) of channel be-

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tween source and intended receiver, source and relay, relay and intended receiver. Information is transmitted at the rate of R b/s in a frame length of M-bits. We assume that all users transmit at the same power level and modulation rate.

2.1. Cooperative AF

In cooperative **AF** a user amplifies faded and noisy version of signal received from its partner before forwarding it to the intended receiver. The amplification factor, denoted by β, is a function of inter-user channel gain and is subject to relay's power constraint [1].

2.2. Rules of Cooperation

We assume that users form a cooperative partnership where each user affirms its willingness to cooperate via a protocol handshake. A willingness to cooperate may indicate that a user (relay) has enough available power to expend for cooperation. It may also indicate an intent to economize on the other user's cooperative behavior.

2.3. Benefit and Costs of Cooperation

The benefit of cooperation is measured by the average frame success rate (FSR)

$$FSR = [1 - BER]^M \tag{1}$$

where BER is average received bit error probability given by $BER = \int\int\int_0^\infty Q[\sqrt{2(\gamma_1 + f(\gamma_2, \gamma_3))}]\,p(\gamma_1)p(\gamma_2)p(\gamma_3)d\gamma_1\,d\gamma_2\,d\gamma_3$ assuming BPSK modulation, $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{z^2}{2}}dz$. It has been shown that **AF** exhibits significant error performance gains over SISO [7].

Cost of cooperation p_R incurred by a relay terminal is sum of energy expended for protocol handshake and to forward information bits,

$$p_R = p_{R,data} + p_{handshake} \tag{2}$$

where $p_{R,data}$ energy expended to forward data and $p_{handshake}$ energy expended to establish cooperative partnership. Total energy expended for cooperative transmission of information bits is given by $p_I = p_{R,data} + p_{S,data}$ where $p_{S,data}$ is energy expended by source terminal. Note $p_{handshake} \ll (p_{S,data}, p_{R,data})$.

2.3.1. Utility Function

In [8] utility function of a wireless network is defined as a measure of the number of information bits received per joule of total energy expended,

$$u_i = \frac{T_i(p)}{p} \ bits/Joule \tag{3}$$

where $T_i(p) = W \times FSR$ is throughput of user u_i, W is the bandwidth, $p = p_I + p_{handshake}$ is total cost of cooperation. Note that $p_{handshake}$ contributes $zero$ utility since no information bits are transmitted during protocol handshake. Thus, (3) defines a well-behaved utility function, i.e., $p_I \to 0, u_i \to 0$ and $p_I \to \infty, u_i \to 0$. We verify behavior of the utility function as shown in Fig. 1.

3. DYNAMIC GAME MODEL FOR COOPERATIVE AMPLIFY-AND-FORWARD

We assume that the benefits of cooperation and the cost it incurs are common knowledge. That is, users are willing to expend their own resources to help other users achieve reliable communication with
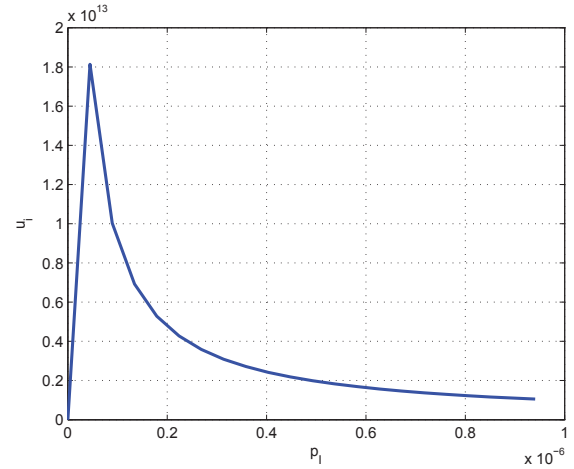

Fig. 1. Utility function.

the expectation to achieve those same benefits when their partners reciprocate. We assume rational and intelligent users that are intent on maximizing their individual utilities.

We consider stage games that occur in time periods $t_k, k = 0, 1, \ldots$, where within each stage game t_k, source and relay terminals interact repeatedly for a duration of T seconds. The assumption of multiple cooperative interactions within a stage game is intuitively valid since cooperative transmissions may span multiple time slots. The period T for each stage game t_k may be defined as the time it takes for a cooperative transmission to reach its intended destination. We assume that duration of a stage game T is long enough to average out effects of channel variation. It is obviously clear that a new stage game starts when a source terminal i ($i \in \{1, 2, \ldots, N\}$) begins transmitting to the network. We next characterize next behavior of every relay terminal j and source terminal i within a dynamic Bayesian game framework.

3.1. Relay Behavior

We assume that relay terminals maintain private information pertaining to their behavior (i.e., to either cooperate or misbehave). Private information θ_j of every relay terminal j corresponds the notation of $type$ in Bayesian games. The set of $types$ available to relay terminal j constitutes relay terminal's $type$ $space$ defined as $\Theta_j = \{\theta_0 = Cooperate, \theta_1 = Misbehave\}$. Since every terminal j either conforms to cooperation or deviates from it, Θ_j is also the $global$ $type$ $space$ of the game, i.e., $\forall \theta_j \in \Theta_j$. Following the notation of Bayesian games, $type$ of player j is denoted by θ_j while other players' $type$ is denoted by θ_{-j}, where $\theta_j, \theta_{-j} \in \Theta_j$. We assume that $types$ associated with each relay terminal are independent. $Type$ $space$ of every relay terminal j maps to an $action$ $space$ A_j which defines a set of actions $a_j(t_k)$ available to player j of $type$ θ_j. We assume that $type$ of terminal j and the associated action $a(t_k)$ do not change within a stage game. Indeed, a relay that obeys rules of cooperation do not change its $type$ at each stage game. On the other hand, a misbehaving relay may strategically change its $type$ at the beginning of each stage game. In this paper, we assume that a misbehaving relay adopts $behavior$ $strategy$ wherein it randomly changes its behavior at each stage game. $Behavior$ $strategy$ σ_j assigns a conditional probability over A_j, i.e, $\sigma_j = p(a_j(t_k)|h^{t_k}, \theta_j)$.
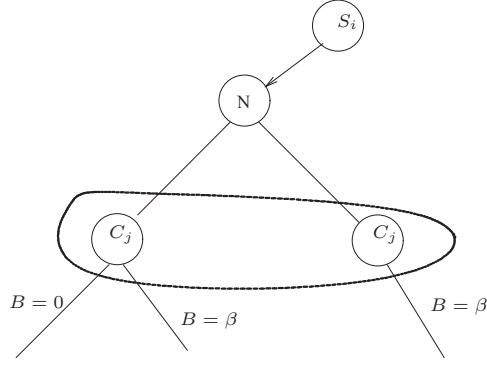
Fig. 2. Game tree that represents an instance of **AF** dynamic Bayesian game wherein relay adopts *behavior strategy*, i.e., it switches randomly between cooperation and misbehaving. Node **N** represents Nature which assigns *type* to each player. The behavior strategy of relay terminal is represented by two nodes at each time instant. The curve around C_j indicate that the relay terminal doesn't know which behavior (cooperate or misbehave) Nature will assign to it. In dynamic game terminology, the relay terminal has two nodes in its information set. On the other hand, S_i has a single *type* (its information set is singleton), i.e., it transmits to the network whenever it has information to send. For instance, a relay that conforms to cooperation also has an information set that is singleton.

Note that the conditioning is on history of the game and *type* of relay. We define history of the game at the beginning stage game t_k is $h^{t_k} = (a(t_0), a(t_1), \ldots, a(t_{k-1}))$. It is safe to assume that a relay which violates rules of cooperation may not need to observe history of the game when it chooses its actions.

We consider an example to give a significant insight into the notion of *type* and *action space* in cooperative **AF**. Define $B = a_j(t_k)\beta$ amplification at the relay where $a_j(t_k)$ captures action of relay j of type θ_j and β is amplification due to the channel and power constraint at the relay. We describe below various *types* of relay terminal j,

- relay terminal j obeys the rules of cooperation: in this case $\theta_j = 0$ and its action space is $A_j = \{1\}$. Terminal j would play pure strategy $a_j(t_k) = 1$ (i.e., $\sigma_j = 1$) assuming that the source terminal i is of *type* $\theta_i = 0$ where $h^{t_k} = (1, 1, \ldots, 1)$. The amplification B is then a function of channel dynamics and cooperator's power constraint, i.e., $B = \beta$.

- relay terminal j violates rules of cooperation in probabilistic manner: in this case *type* of relay jj is $\theta_j = 1$ and its *action space* is captured in random variable A_j where $A_j = \{0, 1\}$. *Note that a misbehaving relay will not follow the rules of the game, i.e., it will not take into account history of the game while making its decision.* The relay's action $a_j(t_k)$ is then a mapping from its *type* $\theta_j = 1$ to A_j, i.e, $a_j(t_k) = 1$ with σ_j and $a_j(t_k) = 0$ with $1 - \sigma_j$, where $\sigma_j = p(a_j(t_k)|\theta_j)$ is *behavior strategy*. The amplification is then $B \in \{0, \beta\}$ which indicates that a misbehaving terminal refuses to forward ($B = 0$) with probability $1 - \sigma_j$ or forwards information bits for source terminal ($B = \beta$) with probability σ_j. Note that a relay may always refuse to forward in which case $a_j(t_k) \in A_j = \{0\}$ with probability $\sigma_j = 1$, obviously $B = 0$.

- an intelligent (but selfish) relay j of *type* $\theta_j = 1$ attempts to cheat by lowering its power to a random level when forwarding signals for its partner. The goal of the relay is maximizing its utility by minimizing cost of cooperation. The *action space* of the selfish relay j is then $A_j = (0, \ldots, 1)$ which occurs with probability $\sigma_j = p(a_j(t_k) \in A_j | \theta_j = 0)$. This would result in an amplification $B < \beta$ since $0 < a_j(t_k) < 1$. Note that a terminal which exhibits such ambiguous behavior exploits dynamics of the channel to evade a detection mechanism (if there is any).

The utility derived by the various relay *types* considered above is shown in Fig.3.
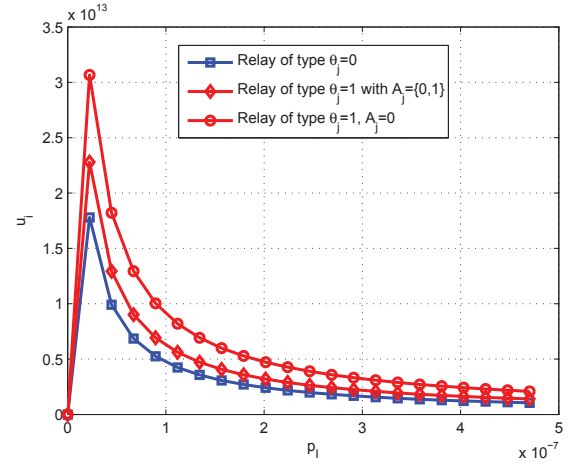
Fig. 3. Utility of relay terminal.

3.2. Behavior of Source Terminal

Even though each potential cooperator maintain a private information, source terminals have beliefs about the possible *types* of each relay terminal j. Source terminal i's belief $\mu_i^j(\theta_j|\theta_i, h^{t_k})$ defines the probability that relay terminal j's *type* is θ_j given source terminal i's *type* and history of the game. We assume that beliefs are independent across the network. and that every source terminal i assigns a strictly positive belief over the *types* θ_j of every relay terminal $j(j \neq i)$, i.e., $\mu_j^i(\theta_j|\theta_i, h^{t_k}) > 0$. This is intuitively valid in commercial wireless networks that are characterized by dynamic user population where it may be difficult to have definite prior knowledge about the behavior of every user. Relay terminals have knowledge about the belief structure of the game although belief of each individual source terminal is not known.

It is interesting to note that by maintaining beliefs, users deviate from the assumption, as in existing cooperation protocols, that their partners are always willing to cooperate. Thus, belief is a security parameter that determines the optimal strategy of source terminal i in the presence of misbehavior. A user's belief characterizes the level of trust its has about other users in the cooperative diversity system. For instance, a user may expect a high likelihood of cooperative behavior from trusted users.

3.3. Belief System

The belief system defines updating beliefs of source terminal i using Bayes' rule at the end of each stage game t_k. We assume that every
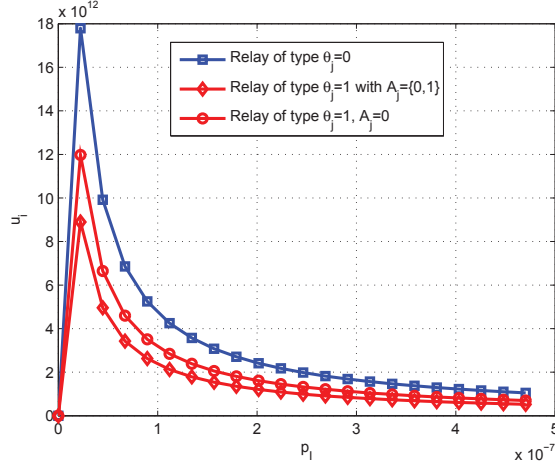
Fig. 4. Utility of source terminal in the presence of selfish relay.

source terminal i monitors cooperative interactions to learn about *type* of relay terminal j, using misbehavior detection techniques such as in [10]. The outcome of the detection mechanism is used to update source terminal i's belief at the end of stage game t_k. The posterior belief at the end of stage game t_k is then,

$$\mu_j^i(\theta_j|a_j(t_k), h^{t_k}) = \frac{\mu_j^i(\theta_j|h^{t_k})p(a_j(t_k)|h^{t_k}, \theta_j)}{\sum_{\tilde{\theta}_j}\mu_j(\tilde{\theta}_j|h^{t_k})p(a_j(t_k)|h^{t_k}, \tilde{\theta}_j)} \quad (4)$$

where $\mu_j^i(\tilde{\theta}_j|h^{t_k}) > 0$ and $p(a_j(t_k)|h^{t_k}, \theta_j) > 0$ [9]. Note that $p(a_j(t_k)|h^{t_k}, \theta_j)$ is probability that action $a_j(t_k)$ is detected at stage game t_k. The beliefs at the end of stage game t_k will be used as prior beliefs of the next stage game t_{k+1}.

3.4. Perfect Bayesian Equilibrium (PBE)

We show that the proposed dynamic Bayesian game model satisfies the requirements for the existence of PBE [11]-pp. 323,

1. at every information set the player with the move has some beliefs about which node in its information set has been reached,

2. given its belief a player must be sequentially rational, i.e., whenever it is its turn to move, it must choose an optimal strategy from that point on,

3. beliefs are determined using Bayes' rule

We intentionally left out a fourth requirement that deals with *unreationalizable* strategies which have no practical meaning in our setting since the *action space* of the game is concisely defined.

Requirement (1) is trivially satisfied since information sets of source terminals are singleton sets to (Fig. 2) which we can assign probability *one*. That is, whenever S_i has information to send, it transmits to the network as shown in Figure 2. Requirement (2) is met by the problem we set out to solve. Requirement (3) is satisfied by the belief system in (4). Thus, the proposed dynamic game model satisfies the conditions for the existence of PBE. It also allows sequential equilibrium since for every extensive game, there exists at least one sequential equilibrium [12]-Proposition 1.

The dynamic game formulation provides a framework based on which a mechanism can be developed to ensure that users derive a Pareto optimal utility function. For instance, a reputation system can be developed which allows users to exchange belief information. It can be shown using evolutionary game theory arguments that if a significant fraction of users adopt reputation based cooperation, an evolutionary stable cooperation is attainable.

4. SUMMARY

We present a dynamic Bayesian game formulation to capture interaction of users in cooperative **AF**. We show that the dynamic Bayesian game formulation captures well information and temporal structure of cooperative interactions.

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