# RECURSIVE ERRORS-IN-VARIABLES APPROACH FOR AR PARAMETER ESTIMATION FROM NOISY OBSERVATIONS. APPLICATION TO RADAR SEA CLUTTER REJECTION

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## ABSTRACT

AR modeling is used in a wide range of applications from speech processing to Rayleigh fading channel simulation. When the observations are disturbed by an additive white noise, the standard Least Squares estimation of the AR parameters is biased. Some authors of this paper recently reformulated this problem as an errors-in-variables (EIV) issue and proposed an off-line solution, which outperforms other existing methods. Nevertheless, its computational cost may be high. In this paper, we present a blind recursive EIV method that can be implemented for real-time applications. It has the advantage of converging faster than the noise-compensated LMS based solutions. In addition, unlike EKF or Sigma Point Kalman filter, it does not require a priori knowledge such as the variances of the driving process and the additive noise. The approach is first tested with synthetic data; then, its relevance is illustrated in the field of radar sea clutter rejection.

***Index Terms—*** Autoregressive processes, recursive estimation, radar clutter, Kalman filtering.

## 1. INTRODUCTION

Linear-model based approaches are very popular in various applications such as speech processing and biomedical. When dealing with Autoregressive (AR) processes, the key issue is usually the estimation of the AR parameters from noisy observations. Indeed, to reduce the bias on the AR parameter estimation due to the additive measurement noise, one solution consists in using instrumental variable techniques such as the modified Yule-Walker (MYW) equations or mutually-interactive optimal filter based solutions [5]. Another approach relies on the "noise-compensated" Yule-Walker equations, which however require the estimation of the additive-noise variance [4]. To solve this dual estimation problem, several off-line approaches have been proposed, see, e.g., [3] and [12]. Nevertheless, this latter may sometimes diverge.

For what concerns on-line methods, one can use noise-compensated variants of the least mean square (LMS) algorithms such as the γ-LMS [7], ρ-LMS [10] and β-LMS [11]. However, these methods require a large number of samples to converge (e.g. a few thousands). Kalman algorithms can be also considered. Since the process and its parameters have to be jointly estimated, the resulting state space model is non-linear and the Extended Kalman Filter (EKF) or the Sigma-Point Kalman Filter [8], namely the Unscented Kalman filter (UKF) and the Central Difference Kalman filter (CDKF), are used. Nevertheless, the variances of the additive noise and the driving process must be a priori known.

AR parameter estimation can be viewed as an errors-in-variables (EIV) issue. In that case, the purpose is to study the semi-definite positiveness of specific observation correlation matrices by using the so-called Frisch scheme [1]. We have analyzed the relevance of the method for optimal filter-based speech enhancement using a single microphone [2] and for Rayleigh fading channel estimation [6]. In theory, this solution has the advantage of blindly providing the AR parameters, the model order, the variances of the driving process and the additive noise. However, its computational cost may be high.

Therefore, in this paper, we propose a recursive version of the blind identification algorithm. In addition, we suggest using this approach in the field of radar sea clutter rejection, where a real-time implementation is crucial.

The paper is organized as follows. In section 2, the estimation approach is detailed. In section 3, a comparative study with existing on-line noise-compensated Least Squares (LS) method is carried out and illustrates the relevance of our approaches with synthetic data. In section 4, radar sea clutter rejection based on this recursive EIV (REIV) algorithm is presented.

## 2. RECURSIVE EIV APPROACH

### 2.1. Problem statement

Let the $p^{\text{th}}$ order AR process $x(n)$ be defined as follows:

$$x(n) = -\sum_{l=1}^{p} a_l x(n-l) + u(n) \tag{1}$$

where $\left\{a_l\right\}_{l=1,\dots,p}$ are the AR parameters and $u(n)$ is a zero-mean white noise with variance $\sigma_u^2$.

This process is then disturbed by an additive zero-mean white noise $b(n)$ with variance $\sigma_b^2$:

$$y(n) = x(n) + b(n) \tag{2}$$

In the following, let us define the regressor vectors:

$$\overline{\varphi}_x(n) = \left[ x(n)\ x(n-1) \cdots x(n-p) \right]^T = \left[ x(n)\ \varphi_x^T(n) \right]^T$$

$$\overline{\varphi}_y(n) = \left[ y(n)\ y(n-1) \cdots y(n-p) \right]^T = \left[ y(n)\ \varphi_y^T(n) \right]^T$$

$$\overline{\varphi}_b(n) = \left[ b(n)\ b(n-1) \cdots b(n-p) \right]^T = \left[ b(n)\ \varphi_b^T(n) \right]^T$$

and $\overline{\varphi}_u(n) = \left[ u(n)\ \ \underbrace{0\ \cdots\ 0}_{p} \right]^T$.

Then, let us introduce the extended AR parameter vector:

$$\overline{\theta} = \left[ 1\ a_1\ \cdots\ a_p \right]^T = \left[ 1\ \theta^T \right] \quad (3)$$

The model (1)-(2) can be expressed in the matrix form:

$$\begin{cases} \left( \overline{\varphi}_x^T(n) - \overline{\varphi}_u^T(n) \right) \overline{\theta} = 0 \\ \overline{\varphi}_y(n) = \overline{\varphi}_x(n) + \overline{\varphi}_b(n) \end{cases} \quad (4)$$

Pre-multiplying (4) by $[\overline{\varphi}_x(n) - \overline{\varphi}_u(n)]^*$ and taking the expectation $E[.]$ leads to:

$$\overline{R}_{x,u}^* \overline{\theta} = \left( \overline{R}_x^* - diag\left[ \sigma_u^2\ \underbrace{0 \cdots 0}_{p} \right] \right) \overline{\theta} = \underline{0} \quad (5)$$

with $\overline{R}_x = E\left[ \overline{\varphi}_x(n) \overline{\varphi}_x^{T*}(n) \right]$.

Due to (5), $\overline{R}_{x,u}^*$ is a positive semi-definite matrix and the AR parameters span the kernel of $\overline{R}_{x,u}^*$. However, in all practical cases, $\overline{R}_{x,u}^*$ is not directly available and only the positive definite autocorrelation matrix of the noisy observations $\overline{R}_y$ can be considered. It satisfies the relation:

$$\overline{R}_y = \overline{R}_x + \sigma_b^2 I_{p+1} \quad (6)$$

where $I_{p+1}$ denotes the identity matrix of size $p+1$.
Given (6), (5) leads to the following relation:

$$\left( \overline{R}_y^* - diag\left[ \sigma_u^2 + \sigma_b^2\ \ \sigma_b^2\ \cdots\ \sigma_b^2 \right] \right) \overline{\theta} = \underline{0} \quad (7)$$

The admissible variances $\sigma_u^2$ and $\sigma_b^2$ are the scalars that make $\overline{R}_y^* - diag\left[ \sigma_u^2 + \sigma_b^2\ \ \sigma_b^2\ \cdots\ \sigma_b^2 \right]$ positive semi-definite. The AR parameters can thus be estimated by solving a set of noise-compensated Yule-Walker equations.
It should be noted that, by partitioning the observation autocorrelation matrix $\overline{R}_y^*$ as follows:

$$\overline{R}_y^* = \left[ \begin{array}{c|c} \sigma_y^2 & r^{T*} \\ \hline r & R_y^* \end{array} \right] \begin{array}{l} \} 1 \\ \} p \end{array} \quad (8)$$

and by taking into account (3), (7) can be split into the two following equalities:

$$\begin{cases} \sigma_y^2 - \sigma_u^2 - \sigma_b^2 + r^{T*}\theta = 0 & (9) \\ r + (R_y^* - \sigma_b^2 I_p)\theta = \underline{0} & (10) \end{cases}$$

If $\sigma_b^2$ is known, $\theta$ can be deduced from (10) and hence can be denoted $\theta(\sigma_b^2)$. Then, $\sigma_u^2$ can be obtained[1] from (9). So, in the following, we propose an on-line algorithm based on high-order Yule-Walker equations to estimate $\sigma_b^2$. For this purpose, let us define the following two $q \times 1$ vectors where $q \geq p$:

$$\varphi_x^h(n) = \left[ x(n-p-1)\ x(n-p-2)\ \cdots\ x(n-p-q) \right]^T$$

$$\varphi_y^h(n) = \left[ y(n-p-1)\ y(n-p-2)\ \cdots\ y(n-p-q) \right]^T$$

and the $q \times (p+1)$ correlation matrices:

$$R_y^h = E\left[ \varphi_y^h(n) \overline{\varphi}_y^{T*}(n) \right] = R_x^h = E\left[ \varphi_x^h(n) \overline{\varphi}_x^{T*}(n) \right]$$

Given (1), one can easily obtain a set of $q$ high-order Yule-Walker equations:

$$(R_y^h)^* \overline{\theta} = \underline{0} \quad (11)$$

Using relation (11), $\sigma_b^2$ can be estimated by minimizing the cost function $J(\sigma_b^2)$ defined by:

$$J(\sigma_b^2) = \left\| (R_y^h)^* \overline{\theta} \right\|_2^2 = \overline{\theta}(\sigma_b^2)^{T*} (R_y^h)^T (R_y^h)^* \overline{\theta}(\sigma_b^2) \quad (12)$$

with $0 \leq \sigma_b^2 \leq \sigma_{b,\max}^2$, $\overline{\theta}(\sigma_b^2) = \left[ 1\ \ \theta^T(\sigma_b^2) \right]^T$ and $\sigma_{b,\max}^2$ is the lowest eigenvalue of $\overline{R}_y$.

**2.2. Recursive estimation of the additive-noise variance**

In [1] and [2], an off-line solution was considered to obtain the noise variances. Here, we suggest using the Newton-Raphson method to derive an on-line algorithm. For this purpose, let us define the matrix:

$$\Sigma^h = (R_y^h)^T (R_y^h)^* = \left[ \begin{array}{c|c} \sigma & \rho^{T*} \\ \hline \rho & \Sigma \end{array} \right] \begin{array}{l} \} 1 \\ \} p \end{array} \quad (13)$$

Given (13) and (3), the criterion defined in (12) can be expressed as follows:

$$\begin{aligned} J(\sigma_b^2) &= \theta^{T*}(\sigma_b^2) \Sigma \theta(\sigma_b^2) + \rho^{T*}\theta(\sigma_b^2) + \theta^{T*}(\sigma_b^2)\rho + \sigma \\ &= f(\theta) = f(g(\sigma_b^2)) \end{aligned}$$

where $f(\theta) = \theta^{T*}\Sigma\theta + \rho^{T*}\theta + \theta^{T*}\rho + \sigma$

and $g(\sigma_b^2) = -(R_y^* - \sigma_b^2 I_p)^{-1} r$.

Therefore, by denoting $\dfrac{\partial^2 J(\sigma_b^2)}{\partial(\sigma_b^2)^2}$ as $J''(\sigma_b^2(n))$, the Newton-Raphson method can be implemented as follows:

$$\hat{\sigma}_b^2(n+1) = \hat{\sigma}_b^2(n) - \frac{J'(\hat{\sigma}_b^2(n))}{J''(\hat{\sigma}_b^2(n))}$$

where $\hat{\ }$ denotes the estimation.
Thus, one has:

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[1] Note that the maximum value of $\sigma_u^2$ is obtained when $\sigma_b^2$ is equal to 0. Hence, $\sigma_{u,\max}^2 = \sigma_y^2 + r^{T*}\theta = \sigma_y^2 - r^{T*}(R_y^*)^{-1} r$.

$$\hat{\sigma}_b^2(n+1) = \hat{\sigma}_b^2(n) - \frac{g'(\hat{\sigma}_b^2(n))^T f'(\hat{\theta}(n))}{g'(\hat{\sigma}_b^2(n))^T f''(\hat{\theta}(n)) g'(\hat{\sigma}_b^2(n))} \quad (14)$$

where $g'(\hat{\sigma}_b^2) = \frac{\partial g}{\partial \hat{\sigma}_b^2} = (\hat{R}_y^* - \hat{\sigma}_b^2 I_p)^{-1}\hat{\theta} = (\hat{R}_x^*)^{-1}\hat{\theta}$ (15)

$$f'(\hat{\theta}) = \frac{\partial f}{\partial \hat{\theta}} = 2(\hat{\Sigma}\hat{\theta} + \hat{\rho}) \quad (16)$$

$$f''(\hat{\theta}) = \frac{\partial^2 f}{\partial \hat{\theta}^2} = 2\hat{\Sigma} \quad (17)$$

By replacing (15), (16) and (17) in (14), one obtains:

$$\hat{\sigma}_b^2(n+1) = \hat{\sigma}_b^2(n) - \frac{((\hat{R}_x^*)^{-1}(n)\hat{\theta}(n))^T (\hat{\Sigma}(n)\hat{\theta}(n) + \hat{\rho}(n))}{((\hat{R}_x^*)^{-1}(n)\hat{\theta}(n))^T \hat{\Sigma}(n)(\hat{R}_x^*)^{-1}(n)\hat{\theta}(n)} \quad (18)$$

The recursive algorithm proposed here is thus the following:

1. Update $\hat{\sigma}_b^2(n)$ by using (18).
2. Update $\hat{R}_y(n)$ and $\hat{r}(n)$:

$$\hat{R}_y(n+1) = \frac{n-p}{n-p+1}\hat{R}_y(n) + \frac{\varphi_y(n+1)\varphi_y^{T*}(n+1)}{n-p+1}$$

$$\hat{r}(n+1) = \frac{n-p}{n-p+1}\hat{r}(n) + \frac{y(n+1)\varphi_y^*(n+1)}{n-p+1}$$

3. Compute $(\hat{R}_x^*(n+1))^{-1} = (\hat{R}_y^*(n+1) - \hat{\sigma}_b^2(n+1)I_p)^{-1}$.
4. Compute $\hat{\theta}(n+1) = -(\hat{R}_x^*(n+1))^{-1}\hat{r}(n+1)$
5. Update $\hat{\sigma}_y^2(n)$:

$$\hat{\sigma}_y^2(n+1) = \frac{n-p}{n-p+1}\hat{\sigma}_y^2(n) + \frac{|y(n+1)|^2}{n-p+1}$$

6. Update $\hat{\sigma}_u^2(n)$:

$$\hat{\sigma}_u^2(n+1) = \hat{\sigma}_y^2(n+1) - \hat{\sigma}_b^2(n+1) + \hat{r}^{T*}(n+1)\hat{\theta}(n+1)$$

7. Update $\hat{R}_y^h(n)$:

$$\hat{R}_y^h(n+1) = \frac{n-p-q}{n-p-q+1}\hat{R}_y^h(n) + \frac{\varphi_y^h(n+1)\overline{\varphi}_y^{T*}(n+1)}{n-p-q+1}$$

Compute $\Sigma^h(n+1)$ and extract from it $\hat{\Sigma}(n+1)$ and $\hat{\rho}(n+1)$.

8. Return to step 1.

*Remark*: a variant of the above algorithm can be considered to avoid any divergence in the on–line procedure by replacing equation (18) with:

$$\hat{\sigma}_b^2(n+1) = \hat{\sigma}_b^2(n) - \mu(n+1)\frac{((\hat{R}_x^*)^{-1}(n)\hat{\theta}(n))^T (\hat{\Sigma}(n)\hat{\theta}(n) + \hat{\rho}(n))}{((\hat{R}_x^*)^{-1}(n)\hat{\theta}(n))^T \hat{\Sigma}(n)(\hat{R}_x^*)^{-1}(n)\hat{\theta}(n)}$$

where $\mu(n+1)$ is first set to 1.

If $\hat{\sigma}_b^2(n+1) \leq 0$ or $\hat{\sigma}_u^2(n+1) \leq 0$, one can choose $0 < \mu(n+1) < 1$ to obtain $0 \leq \hat{\sigma}_b^2(n+1) \leq \hat{\sigma}_{b,\max}^2(n+1)$.

## 3. SIMULATION ON SYNTHETIC DATA

We have carried out various simulations with synthetic data. Here, we consider a $6^{th}$ order AR process defined by its poles[2] $p_{1,2} = 0.75e^{\pm 2\pi \times 0.2}$, $p_{3,4} = 0.8e^{\pm 2\pi \times 0.4}$ and $p_{5,6} = 0.85e^{\pm 2\pi \times 0.7}$.

The purpose of this first analysis is to evaluate the influence of the signal-to-noise ratio (SNR) and of the number of available samples. For this reason, the SNR first varies from 5 to 40 dB with a number $M$ of samples set to 1024. Then, $M$ has been varied from 64 to 1024 with a SNR equal to 10dB. The results are reported in Tables 1 and 2 and are based on the Itakura metric $I$ defined by:

$$I = \log_{10}\left(\frac{\hat{\theta}^T R_x \hat{\theta}}{\theta^T R_x \theta}\right)$$

| SNR (dB) | 5 | 10 | 20 | 40 |
|---|---|---|---|---|
| $\gamma$-LMS [7] | 2.309 | 2.254 | 2.241 | 2.254 |
| $\rho$-LMS [10] | 2.538 | 1.956 | 1.397 | 1.203 |
| $\beta$-LMS [11] | 3.371 | 2.330 | 1.259 | 1.063 |
| EKF [8] | 0.523 | 0.117 | 0.013 | 0.0001 |
| UKF [8] | 0.498 | 0.110 | 0.013 | 0.0001 |
| REIV | 1.154 | 0.149 | 0.057 | 0.008 |

Table 1 : influence of SNR in Itakura metrics ($10^{-1}$) with $M$=1024.

| $M$ | 64 | 128 | 256 | 512 | 1024 |
|---|---|---|---|---|---|
| $\gamma$-LMS | 5.333 | 4.557 | 3.637 | 2.915 | 2.262 |
| $\rho$-LMS | 6.235 | 5.492 | 3.732 | 2.652 | 1.871 |
| $\beta$-LMS | 3.924 | 3.187 | 2.657 | 2.414 | 2.302 |
| EKF | 1.289 | 0.825 | 0.470 | 0.291 | 0.109 |
| UKF | 1.269 | 0.796 | 0.452 | 0.275 | 0.104 |
| REIV | 1.384 | 0.780 | 0.323 | 0.292 | 0.230 |

Table 2 : influence of samples in Itakura metrics ($10^{-1}$) with SNR = 10 dB

The REIV outperforms $\gamma$-LMS [7], $\rho$-LMS [10] and $\beta$-LMS. The methods based on Kalman filtering give slightly lower Itakura metric than REIV. However, REIV is totally blind whereas EKF and SPKF require the a priori knowledge of the noise and driving-process variances.

The purpose of the next section is to study EIV methods in radar processing.

## 4. REIV FOR RADAR SEA CLUTTER REJECTION

When dealing with one airborne radar antenna in sea surveillance mission, the received signal is composed of the target signal, the thermal noise due to receiver system and the sea clutter due to the environment returns. The purpose

[2] The poles satisfy $A(z) = 1 + \sum_{i=1}^{p} a_i z^{-i} = \prod_{i=1}^{p}(1 - p_i z^{-1})$

is then to get rid off the influence of the noise and the clutter in order to detect the target.

In [9], Wensink proposes firstly to model the sea clutter by an AR process and to estimate the corresponding parameters by using Burg's method for *M* samples at the same range (See figure 1). They constitute the Cell Under test (CUT). For this purpose, the sea clutter is assumed to be stationary in the neighborhood composed of *2N* adjacent cells. The AR parameters are estimated on each adjacent cell and averaged. Then, the author deduces the FIR "inverse filter" to reject the clutter in CUT. To prevent the AR parameter estimation from being disturbed by the target itself, one guard cell on both sides of the CUT is not taken into account.

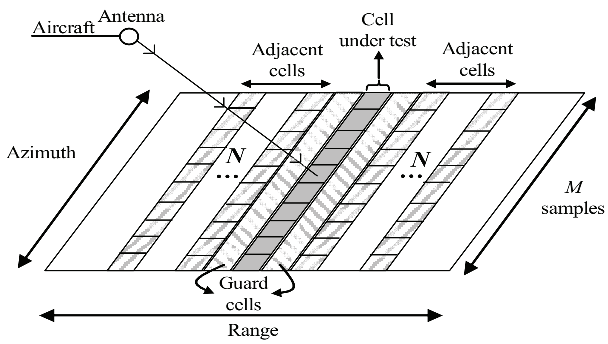
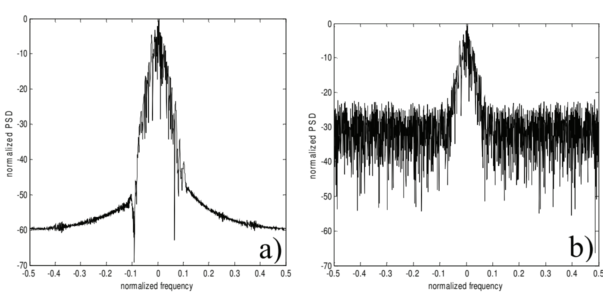
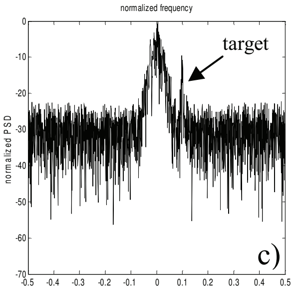



Figure 1: airborne surveillance mission configuration.




a) PSD of clutter.
b) PSD of clutter disturbed by noise, with a clutter to noise ratio (CNR) equal to 10 dB.
c) PSD of target plus clutter and noise with a signal to clutter plus noise ratio (SCNR) equal to -10dB.

Figure 2: Normalized power spectrum density of CUT with *M* samples.

However, when dealing with a long-range cell (i.e. cell far from the aircraft), the additive thermal noise is no longer negligible and clutter-to-noise ratio (CNR) becomes low. In that case, using standard LS estimation leads to biased AR parameter estimations. The resulting filter is no longer selective enough and the false alarms rate increases.

To avoid this problem and compensate for the influence of such an additive white noise, EIV approaches can be used. In the simulation tests we present here, the CNR is varying from 5 dB to 20dB. To study the relevance of our approach, the following detection error rate (DER) is defined:

$$r_{DE} = \frac{\text{detection of error samples}}{\text{number of samples under test}} \tag{19}$$

| CNR (dB) | Detection Error Rate $r_{DE}$ | | |
|---|---|---|---|
| | Wensink method | EIV method [2]/[6] | REIV method |
| 20 | 0 | 0 | 0 |
| 15 | $8.2\ 10^{-5}$ | 0 | 0 |
| 10 | $1.066\ 10^{-3}$ | 0 | $8.2\ 10^{-5}$ |
| 8 | $6.400\ 10^{-3}$ | $6.4\ 10^{-5}$ | $6.03\ 10^{-4}$ |
| 5 | $2.872\ 10^{-3}$ | $8.2\ 10^{-5}$ | $9.34\ 10^{-4}$ |

Table 3: detection error rate

The DER is estimated for each method on synthetic data composed of a K-distributed clutter and Gaussian noise. As shown in table 3, The off-line EIV provides lower DER than the recursive one, but its computational cost is much higher. In any case, EIV [2] and REIV methods outperform Wensink's method.

We are currently working on an extension of this recursive EIV in the multichannel case for a subsequent use in phased array antenna in radar processing.

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