PHASE-BASED ALIGNMENT OF TWO SIGNALS HAVING PARTIALLY OVERLAPPED SPECTRA

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ABSTRACT

This paper proposes a novel method for aligning two signals using the information contained in the overlapped band. In particular the proposed method aligns two signals by compensating both time-delay and phase-offset in the second signal using the estimated gradient of phase difference in the overlapped band. Compared with other conventional methods, this method can align two signals without requiring a pilot tone or additional hardware. The proposed method was experimentally validated using RF pulses.

Index Terms— phase estimation, signal reconstruction, microwave frequency conversion, analog-digital conversion

1. INTRODUCTION

In this paper, we address the issue of estimating linear phase misalignment between two signals, which consists of time-delay and phase-offset as illustrated in Figure 1. This kind of problem is commonly found in telecommunications (e.g. link aggregation), high bandwidth signal measurement [1] and other applications where multiple channels are used for transmitting signals. In such multi-channel system applications, delay can be due to difference in signal path lengths and phase-offset is introduced by frequency shifting operations. These differences must be compensated for prior to further processing.

One conventional method for solving this problem requires two steps to accomplish [1]. The first step compensates for the delay due to electrical path length differences, and the second step removes the phase-offset at a particular frequency. This type of solution requires additional hardware, and therefore can be expensive. Moreover such an approach introduces a non-linear group delay issue into the system, which can severely alter the original input signal.

In this paper, we introduce a much simpler approach which is able to simultaneously perform time-delay and phase-offset removal. This approach does not require additional hardware. Furthermore, it can be applied to signals with partially overlapped frequencies in order to detect linear phase misalignment.

The rest of this paper describes how linear phase alignment can be realized between two signals that have spectra with overlapping frequency bands. In Section 2, the conventional method and its limitations are discussed. Section 3 covers the theory behind linear phase misalignment estimation method from partially overlapped spectra, and also provides a phase alignment algorithm. Section 4 verifies the proposed algorithm using a commercial ADC evaluation board and an RF pulse.

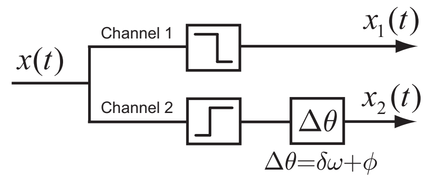


Fig. 1. Problem of estimating linear phase alignment (time-delay δ and phase-offset ϕ) from a system that splits a wideband input signal into two band-limited channels.

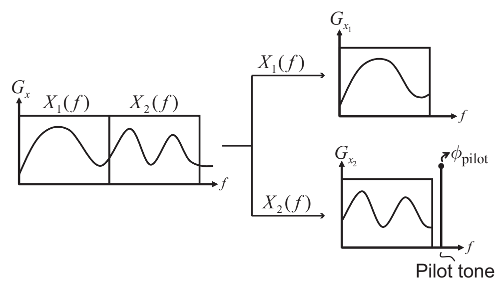


Fig. 2. Illustration of the conventional method that estimates phase-offset ϕ using pilot tone [1]. Time-delay needs to be estimated separately.

2. CONVENTIONAL METHOD AND ITS LIMITATIONS

As mentioned earlier, the conventional method separately measures time-delay and phase-offset [1]. Time-delay is measured by using a specific input to the system, and then compensated for by using a series of digital filters. Secondly, the phase-offset is detected from the phase of a known pilot tone. The phase of the pilot tone itself can be recovered using the Goertzel Algorithm [1] or discrete Fourier transform (DFT). Figure 2 illustrates the phase offset estimation in a two-channel system using the conventional method.

Several important issues must be considered with implementation of the conventional method. First, since most multi-channel system applications involve time-domain measurements, the pilot tone has to be inserted at a frequency which is outside of the measurement band [1]. Hence, a considerable amount of additional hardware, including a power splitter, a power combiner, attenuators, band-pass filter, and a frequency divider, is required for insertion and detection

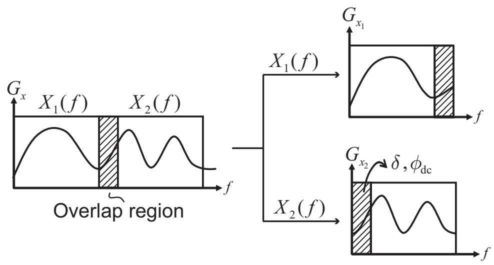


Fig. 3. Illustration of proposed method that estimates delay (δ) and phase-offset (ϕ) using partially overlapped spectra (shaded region).

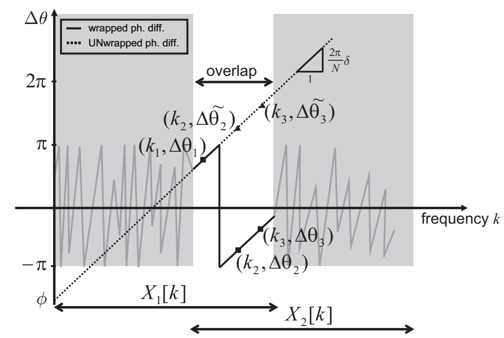


Fig. 4. Illustration of phase difference $\Delta\theta$ between two signals in the overlapped frequency range. Time-delay is computed from the slope of the unwrapped phase, while phase-offset is calculated from the $\Delta\theta$-axis intercept.

of the pilot tone. Of course this hardware creates additional cost and maintenance issues.

Also, the time-delay compensation, which is performed separately through hardware calibration using an integer and fractional digital filter, is inefficient since two distinct compensations are required for each measurement. Most importantly, a separate delay and phase-offset compensation approach might introduce errors since these two components of misalignment might be related to each other.

3. PHASE MISALIGNMENT

3.1. Basic Principles

In this section we describe the basic principles of our new phase alignment method which estimates linear phase misalignment from two overlapped frequency band, as illustrated in Figure 3. If the frequency spectra of two channels are overlapped, then time-delay (δ) and phase-offset (ϕ) can both be estimated from the gradient and the intercept of the phase difference in the overlapped band respectively. Hence, the simultaneous and consistent estimation of both δ and ϕ is possible with this approach.

To describe our method, we model the outputs of the first and second channels of a two-channel system as

$$x_1[n] = x_1(nT_s) + n_1[n], \quad (1a)$$
$$x_2[n] = x_2(nT_s - \delta) * e^{-j\phi} + n_2[n] \quad (1b)$$

where δ and ϕ represent the time-delay and phase-offset respectively. $1/T_s$ denotes the sampling frequency, which is assumed to be higher than the Nyquist rate of the measured signals. Moreover, the noise portions of the two signals, n_1 and n_2, are assumed to be uncorrelated with each other. And the operator $*$ denotes convolution.

The cross-spectrum $G_{x_1x_2}[k] = X_1[k]X_2^*[k]$ between $x_1[n]$ and $x_2[n]$ in the overlapped frequency band can be described as

$$G_{x_1x_2}[k] = G_{xx}[k]e^{j\frac{2\pi}{N}k\delta + j\phi} + G_{n_1n_2}[k] \quad (2)$$

where $G_{xx}[k]$ represents autospectrum in the overlapped frequency band, and $G_{n_1n_2}[k]$ represents the cross-spectrum between noise n_1 and n_2, which is zero in case of uncorrelated noise. The cross-spectrum between each input signal and its noise are assumed to be uncorrelated. The factor $e^{j\frac{2\pi}{N}k\delta + j\phi}$ contains both the unknown time-delay and phase-offset which need to be estimated.

From Eq. (2) the phase difference $\Delta\theta$ is computed as follows

$$\Delta\theta = \arg\left[\frac{G_{x_1x_2}[k]}{|G_{x_1x_2}[k]|}\right] = \frac{2\pi}{N}k\delta + \phi. \quad (3)$$

Here we note that the phase difference $\Delta\theta$ is a linear function of the k-th spectral line (or bin number k) with gradient (or slope) $\frac{2\pi}{N}\delta$. Eq. (3) clearly shows that the time-delay δ can be estimated from the slope of the phase difference data, while the phase-offset ϕ is estimated from the intercept of the $\Delta\theta$-axis, as shown in Figure 4.

Because of the periodicity of the tangent function, the phase difference is wrapped around the range $-\pi$ to π as illustrated in Figure 4. Therefore, before calculating the gradient we need to unwrap the phase difference by adding multiples of 2π to the original phase difference data. This can be written as

$$\Delta\tilde{\theta}_i = \Delta\theta_i + m_i 2\pi \qquad \text{where } i = 1, ..., L \text{ and } m_i \in \mathbf{N}, \quad (4)$$

where tilde denotes the unwrapped phase difference, and variable L represents the number of spectral lines contained in the overlapped band.

To estimate the slope, at least three pairs ($L = 3$) of frequency bins and phase difference are required. They are denoted as $(k_1, \Delta\theta_1)$, $(k_2, \Delta\theta_2)$ and $(k_3, \Delta\theta_3)$. Using the first two values the gradient is given by

$$\frac{2\pi}{N}\delta = \frac{\Delta\theta_2 - \Delta\theta_1 + m_2 2\pi}{k_2 - k_1} \quad (5)$$

where we assume that no multiples of 2π need to be added to the first phase difference, i.e. $m_1 = 0$, as we only need to calculate the gradient and not the exact unwrapped phase difference. Since $(k_1, \Delta\theta_1)$ and $(k_2, \Delta\theta_2)$ are readily available from measurement, the only remaining variable to estimate is m_2 which is computed from the third pair using the slope equation of the unwrapped phase difference

$$\frac{\Delta\tilde{\theta}_3 - \Delta\tilde{\theta}_1}{k_3 - k_1} = \frac{\Delta\tilde{\theta}_2 - \Delta\tilde{\theta}_1}{k_2 - k_1}. \quad (6)$$

By inserting Eq. (4) into Eq. (6) and rearranging the variables we obtain

$$(k_3 - k_1)m_2 = C + (k_2 - k_1)m_3 \quad (7)$$

where

$$C = \frac{1}{2\pi}((k_2 - k_1)(\Delta\theta_3 - \Delta\theta_1) - (k_3 - k_1)(\Delta\theta_2 - \Delta\theta_1)).$$

From Eq. (7) it can be deduced that $(k_3 - k_1)m_2 - C$ needs to be multiples of $k_2 - k_1$, and Eq. (7) can be rewritten as

$$(k_3 - k_1)m_2 \equiv C \pmod{k_2 - k_1} \quad (8)$$

which we need to solve for finding the value of m_2. Eq. (8) can be solved using a linear modular equation solver [2] which is based on the extended Euclidean algorithm.

The linear modular equation solver finds $d = \gcd((k_3 - k_1), (k_2 - k_1))$ number of solutions of Eq. (8). If $(k_3 - k_1)$ and $(k_2 - k_1)$ is co-prime, i.e. $d = 1$, then the solver will find a single solution in modulo $(k_2 - k_1)$. In general the solution has the form of $m_2 = m_{2,0} + i(k_2 - k_1)/d$ for $i = 0, 1, ..., d - 1$, where $m_{2,0} = a(C/d) \pmod{k_2 - k_1}$ and the integer a is the integer coefficient generated using the extended Euclidean algorithm such that $d = a(k_3 - k_1) + b(k_2 - k_1)$.

3.2. Alignment Algorithm

In this subsection, a four step algorithm is described for removing misalignment from two channels based on the method introduced in Section 3.1.

Before proceeding further, we will explain how our method can be applied to three strongly-correlated line spectra that have high signal-to-noise (SNR) ratios. Assuming that the random noise in the first and second channels has identical distributions, the SNR in each frequency bin can be formulated as

$$\mathrm{SNR}[k] = \frac{|\gamma_{x_1 x_2}[k]|}{1 - |\gamma_{x_1 x_2}[k]|} \tag{9}$$

where $|\gamma_{x_1 x_2}[k]|$ is the complex coherence function and is given by

$$|\gamma_{x_1 x_2}[k]| = \frac{|\hat{G}_{x_1 x_2}[k]|}{\sqrt{\hat{G}_{x_1 x_1}[k]\hat{G}_{x_2 x_2}[k]}}. \tag{10}$$

The hat symbol ($\wedge$) denote the power spectrum estimate using Welch's modified periodogram [3].

Using Eq. (9) and the method developed in Section 3.1, we describe the algorithm for recovering the original signal from two channels. The proposed algorithm consists of four steps which are described as follows:

Step 1. Find three strongly correlated line spectra with high SNR using Eq. (9). Then compute the corresponding phase difference which are denoted as $(k_1, \Delta\theta_1)$, $(k_2, \Delta\theta_2)$ and $(k_3, \Delta\theta_3)$.

Step 2. Compute the time-delay (δ) estimate using

$$\delta = \frac{N}{2\pi} \frac{\Delta\theta_2 - \Delta\theta_1 + m_2 2\pi}{k_2 - k_1} \tag{11}$$

where m_2 is computed from Eq. (8). From Eq. (3) and Eq. (11), the phase-offset (ϕ) estimate is computed as follows

$$\phi = \Delta\theta_1 - \frac{2\pi}{N}\delta \times k_1. \tag{12}$$

Step 3. Compensate the misalignment in the second channel with respect to the first channel in the frequency domain using the δ and ϕ values estimated in **Step 2**

$$X_{2c}[k] = e^{+j\frac{2\pi}{N}k\delta + j\phi} X_2[k]. \tag{13}$$

Here $X_{2c}[k]$ and $X_2[k]$ denote the compensated and uncompensated Fourier spectra (or Fourier transform) of the signal in the second channel, respectively.

Step 4. Merge together the spectra of the first channel and the compensated version of the second channel. The reconstructed time domain signal is obtained by taking the inverse DFT of the merged spectra.

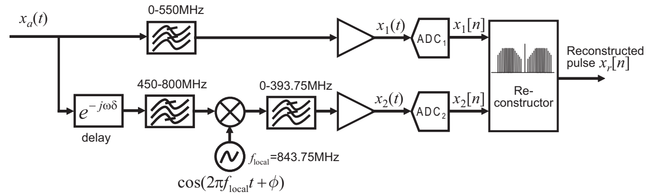
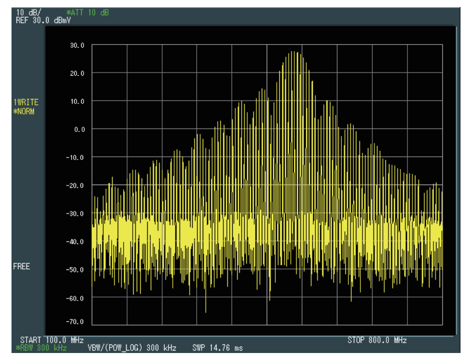
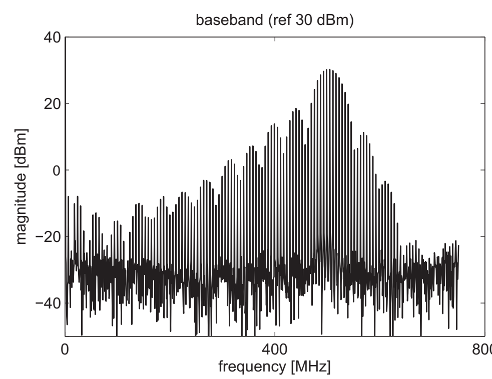


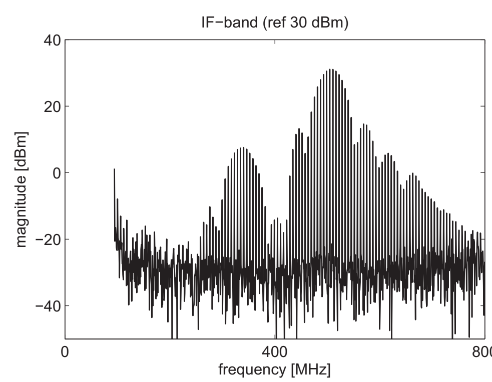
Fig. 5. Block diagram of the two-channel system being evaluated in this paper.



(a) RF pulse spectrum measured using spectrum analyzer

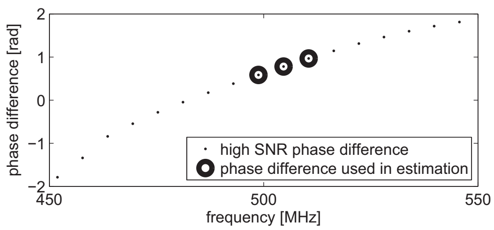


(b) Baseband signal (Channel-1)

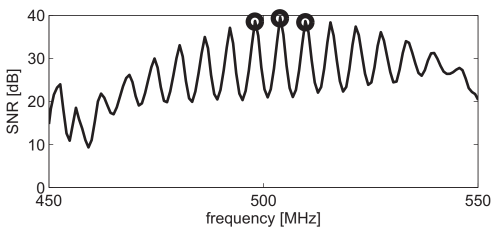


(c) IF-band signal (Channel-2)

Fig. 6. Spectrum of RF pulse with frequency 503.90625 MHz, pulse rate 5.859375 MHz, and pulse width 30 ns: (a) full spectrum, (b) baseband spectrum, (c) IF-band spectrum.



(a) Phase difference



(b) SNR

Fig. 7. Phase difference and SNR in the overlapped frequency range.

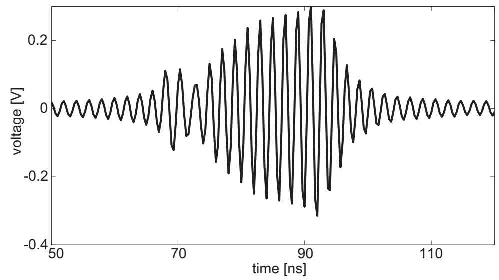
4. EXPERIMENTAL RESULTS

We evaluate the performance of the proposed method by reconstructing an RF pulse that was captured using a prototype of two-channel system as shown in Figure 5. In this figure, the hypothetical block $e^{-j\omega\delta}$ represents the relative time-delay between the first and second channels if all of the filters are assumed ideal and have zero group delays. The local oscillator's phase is represented by ϕ. In this experiment we do not have control over δ nor ϕ.

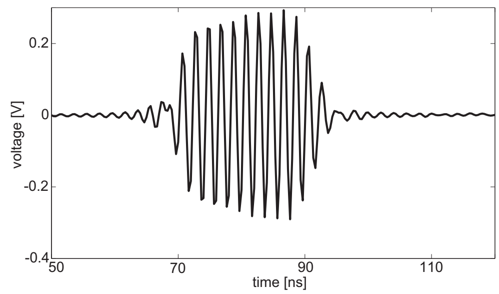
First, the input pulse is split into two pulses using a power splitter, and then fed into two channels. The first channel captures the signal in the baseband (0, 550MHz) using an analog low pass filter (LPF). The second channel is passed through an analog band-pass filter (BPF) with frequency band (450 MHz, 800 MHz) to capture the intermediate frequency (IF) signal. A local oscillator with a frequency of 843.75 MHz downconverts the filtered IF-band signal to a baseband signal. Finally an analog LPF filters out the unwanted images above 393.75 MHz. An ADC evaluation board (National Semiconductor ADC08D1500 evaluation board [4] which contains dual 8-bit 1.5-GSps ADCs) was used to digitize the pulses from each input channels.

The RF pulse was generated by gating a 503.90625 MHz sinusoidal wave with a switching rate of 5.859375 MHz and a pulse width of 30 ns. The resulting pulse is a wideband signal consisting of distinct line spectra as shown in Figure 6(a). Using the prototype board with the previously described setup, 2048 samples of each ADC channel were captured. The spectra of the digitized signal are shown in Figure 6(b) & 6(c). These sampled signals were used to reconstruct an RF pulse using Matlab.

Figure 7(a) shows the phase difference of the high SNR cross-spectrum. As expected from Eq. (3), the phase difference points form a straight line which enables us to estimate δ and ϕ values from the phase difference. Using the algorithm developed in Section 3.2 first we chose three peaks with high SNR; the SNR in the overlapped band is shown in Figure 7(b). These three highest SNR were found in frequencies: 498.7793MHz, 504.6387MHz, and 510.4980 MHz for which the phase difference were computed. The slope is 0.0246 and the intercept is 16.16. Thus the time-delay is $\delta = 8.02$ samples $=$ 5.35 ns and the phase-offset is $\phi = -16.16$ rad.



(a) Uncorrected



(b) Delay and phase corrected

Fig. 8. RF pulse reconstruction results using the proposed method.

Finally the second channel was compensated using the calculated δ and ϕ. The reconstruction results are shown in Figure 8. Figure 8(a) shows that reconstruction from an uncompensated signal does not produce a satisfactory result due to phase misalignment between the two channels, as can be observed in Figure 7(a). When both the time-delay and the phase-offset are correctly compensated, an accurate reconstructed pulse is obtained, as shown by Figure 8(b).

5. CONCLUSION

A new phase alignment method for a two-channel system with partially overlapped spectra was introduced and demonstrated. The method requires only three line spectra in the overlap spectra, and does not require a pilot tone or any additional hardware in order to estimate linear phase misalignment. Experiment results using an RF pulse validated our proposed method. The experimental results show that by using high SNR spectral lines of data, both time-delay and phase-offset can be simultaneously estimated, and used to align two signals from different channels.

6. REFERENCES

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