# FAULT DETECTION COMBINING INTERACTING MULTIPLE MODEL AND MULTIPLE SOLUTION SEPARATION FOR AVIATION SATELLITE NAVIGATION SYSTEM

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## ABSTRACT

In civil aviation applications, satellite failures yield unacceptable positioning errors when using the Global Positioning System (GPS). To ensure the user security, the navigation system has to fulfill stringent performance requirements. Thus, detecting and excluding the faulty GPS measurements is necessary prior to estimating the mobile location. Classical fault detection algorithms based on Kalman filters (KF) are sensitive to the choice of an appropriate motion model for the mobile. To overcome this difficulty, we propose in this paper a new fault detection algorithm wherein the KF are replaced by multiple model algorithms. In this way, both the false alarm rate and the position mean square error are shown to be decreased.

*Index Terms*— Fault Diagnosis, Global Positioning System, Navigation, Recursive Estimation, Multiple Model.

#### 1. INTRODUCTION

In civil aviation, the navigation system has to fulfill stringent performance requirements in terms of reliability and accuracy to ensure the user safety. In addition to estimating the aircraft motion, the navigation system has to provide alerts to the user when the positioning error exceeds bounds defined by the International Civil Aviation Organization (ICAO). When using the GPS, these so-called positioning failures are due to satellite dysfunctioning. GPS integrity monitoring is then concerned with detecting and excluding faulty satellite measurements. For this purpose, several algorithms have been proposed, including the well-known Multiple Solution Separation algorithm (MSS) [1]. It consists of a bank of estimators determining the aircraft position, based on a main estimator using all the available GPS measurements and sub-estimators using different subsets of the GPS measurements. Then, the faulty measurement detection and exclusion is performed by comparing the main solution with the different sub-solutions. First designed to detect a single failure at a time, the MSS has been modified to accommodate multiple failures in [2]. In a standalone GPS context, the MSS is applied with least square (LS) estimators. A Kalman Filter (KF) based implementation is expected to improve detection performance. However, in this case, the main problem is to choose an appropriate state model for the kinetic parameters. Indeed, abrupt changes in the aircraft dynamics are misinterpreted by the MSS algorithm as measurement failures resulting in false alarms<sup>1</sup>. To overcome this difficulty, we propose to use Multiple Model (MM) algorithms, and more precisely the Interacting Multiple Model (IMM) introduced by Bar-Shalom [4], as an alternative to KF in the MSS filter hierarchy.

The remainder of the paper is organized as follows. In section 2, we formulate the GPS navigation problem in the presence of satellite failures. In section 3, we present the classical MSS algorithm. Section 4 is dedicated to the proposed algorithm. Simulation results are provided in section 5. Finally, conclusions and perspectives are drawn in Section 6.

#### 2. THE GPS NAVIGATION PROBLEM

Aircrafts are equipped with GPS receivers which determine their own positions by measuring the propagation delays of signals broadcast by in-view GPS satellites (SV). By denoting N the number of SV, N distance measurements are obtained at each time step by multiplying the propagation delays by the speed of light. The receiver position in 3 dimensions has to be estimated, hence 3 measurements are in theory sufficient. However, due to the synchronization issues, the receiver clock offset with respect to the GPS reference time has to be estimated as well. Consequently, at least 4 measurements, namely 4 SV, are required to solve the positioning problem.

Let Y(k) be the observation vector of size N at time<sup>2</sup> k, which is composed of the N measurements associated to the SV tracked by the receiver. Its *i*<sup>th</sup> component, for  $i \in [1, N]$ , satisfies:

$$Y_{i}(k) = \|p(k) - p_{i}(k)\| + b(k) + \sigma_{i}(k)\varepsilon_{i}(k),$$
(1)

where  $p(k) = [x(k), y(k), z(k)]^T$  represents the position coordinate vector of the receiver in the system of coordinates chosen as a reference for the motion<sup>3</sup>,  $p_i(k) = [x_i(k), y_i(k), z_i(k)]^T$  is the position coordinate vector of the *i*<sup>th</sup> satellite, b(k) is the GPS receiver clock offset with respect to the GPS reference time,  $\|.\|$  denotes the euclidian distance,  $\varepsilon_i(k)$  is a white Gaussian random variable of standard deviation (std) 1 and  $\sigma_i$  is the std of the *i*<sup>th</sup> measurement noise.

However, satellite failures result in measurement errors larger than the standard GPS measurement noise. Assuming the  $j^{\text{th}}$  measurement is faulty, the  $j^{\text{th}}$  component of the measurement vector Y(k) is also corrupted by a bias  $\mu_j(k)$ :

$$Y_j(k) = \|p(k) - p_j(k)\| + b(k) + \sigma_j(k)\varepsilon_j(k) + \mu_j(k).$$
 (2)

As integrity monitoring algorithms are usually based on the reasonable assumption of a single failure<sup>4</sup> at a time to prevent a prohibitive computational complexity, the problem at hand is to detect the presence of the bias  $\mu_j(k)$  on the j<sup>th</sup> SV while solving the positioning problem.

<sup>&</sup>lt;sup>1</sup>It should be noted that a MSS based on a hierarchy of KF has already been applied to integrate GPS with Inertial Navigation Systems (INS) [3]. In this case, the INS provides a reference model for the aircraft motion. Nevertheless, this approach is costly in terms of material resources.

 $<sup>^2 {\</sup>rm For}$  the sake of simplicity, k stands for  $k \Delta T$  , with  $\Delta T$  the time interval between 2 GPS measurements.

<sup>&</sup>lt;sup>3</sup>In the following, we consider that the aircraft motion is defined in the local frame. The local frame is centered at the aircraft position, its axes pointing to the local vertical, the North and the East direction.

 $<sup>^{4}</sup>$ The satellite failure probability is  $10^{-4}$ /h for GPS.

#### 3. MULTIPLE SOLUTION SEPARATION ALGORITHM

In this section, we focus on the KF-based implementation of the MSS. After a brief description of the state space representation of the system, we present the structure of the MSS algorithm. The reader can refer to [1] for more insight on the classical MSS method.

#### 3.1. Motion and measurement model

In addition to a measurement model, KF requires a state model describing the a priori dynamics of the unknown parameters. In this study, we consider a 2<sup>nd</sup> order model which consists of jointly estimating the mobile position and velocity as well as the GPS receiver clock offset and its derivative. In this way, the aircraft is assumed to have a nearly uniform motion with occasional bursts of acceleration. The overall state vector is defined as follows:

$$X(k) = \left\lfloor x(k), \dot{x}(k), y(k), \dot{y}(k), z(k), \dot{z}(k), b(k), \dot{b}(k) \right\rfloor^{T}$$

Then, the state vector evolution is expressed as:

$$X(k) = \Phi(k)X(k-1) + G(k)w(k),$$
(3)

where w(k) is the state noise vector ( $w(k) \sim \mathcal{N}(0, I_{4\times 4})$ ). The matrices  $\Phi(k)$  and G(k) are defined as follows:

$$\Phi(k) = diag\left([A(k), B(k)]\right),$$

$$G(k) = diag([E(k), E(k), E(k), \Sigma(k)]),$$

where diag([U, V]) is a block diagonal matrix with matrices U and V composing the main diagonal. The sub-block matrices are defined below:

$$\begin{split} A(k) &= diag\left([B(k), B(k), B(k)]\right), \ B(k) = \begin{pmatrix} 1 & \Delta T \\ 0 & 1 \end{pmatrix}, \\ E(k) &= \sigma_{acc} \left(\begin{array}{c} \frac{\Delta T^2}{2} \\ \Delta T \end{array}\right) \text{and} \ \Sigma(k) = \left(\begin{array}{c} \xi_1 \\ \xi_2 \end{array}\right), \end{split}$$

with  $\xi_i$  the state model components for the clock offset drift [5] and  $\sigma_{acc}$  the std of the acceleration.

The observation model is provided by equation (1). For the overall observation vector, we denote:

$$Y(k) = h(X(k);k) + R(k)\varepsilon(k).$$
(4)

where, at time k, one has  $\varepsilon(k) = [\varepsilon_1(k), \ldots, \varepsilon_N(k)]^T$  and  $R(k) = diag([\sigma_1, \ldots, \sigma_N])$ . The relationship between the state vector and the observation vector is expressed by a non linear function h(.;k): its components are the geometric SV to receiver distances. Therefore, we use an Extended Kalman Filter (EKF) to perform the state estimation. In the next section, we denote H(k) the so-called observation matrix, which is composed of the partial derivatives of h(.;k) with respect to the state vector components evaluated at the EKF prediction step.

#### 3.2. MSS fault detection step

In this section, we present the principle of the MSS algorithms applied to GPS integrity monitoring. A primary state estimate referred<sup>5</sup> to as  $\hat{X}_{00}^+(k)$  is computed by a main filter  $F_{00}$ , here an EKF, which uses the N available SV measurements at time k. In parallel, subfilter state estimates, denoted  $\hat{X}_{0n}^+(k)$ , are maintained by N EKF,

denoted  $\{F_{0n}\}_{n \in [1,N]}$ . Filter  $F_{0n}$  incorporates all available measurements except the  $n^{\text{th}}$  one. If  $n^{\text{th}}$  satellite fails, it will drive the solution computed by filter  $F_{0n}$  away from the primary solution. Fault detection (FD) is thus performed by applying an hypothesis test consisting of comparing pairwise the main filter and the sub-filter estimates. The test statistic is taken as the separation of the primary solution and the sub-filter solutions, denoted  $\{dX_{0n}^+(k)\}_{n \in [1,N]}$ :

$$dX_{0n}^{+}(k) = \widehat{X}_{00}^{+}(k) - \widehat{X}_{0n}^{+}(k).$$
(5)

Classically in aviation applications, the horizontal and vertical errors are processed separately. We focus herein on the horizontal (x, y coordinates) position separation, denoted  $dX^+_{H0n}(k)$  for the filter  $F_{0n}$ . Its  $2 \times 2$  covariance matrix is  $dP^+_{H0n}(k)$ .

A failure is detected by applying the following hypothesis test:

 $\begin{cases} H_0 \text{ no failure:} & \forall n \in \{1, N\}, \|dX^+_{H0n}(k)\| < D_{0n}, \\ H_1 \text{ failure detection:} & \exists n \in \{1, N\}, \|dX^+_{H0n}(k)\| \ge D_{0n}, \end{cases}$ 

where  $D_{0n}$  is a decision threshold which is computed for N measurements as a function of the false alarm probability  $P_{fa}$ . Vector  $dX_{H0n}^+(k)$  is Gaussian distributed, therefore the test statistic, like its amplitude, follows a Rayleigh distribution. However, in the Solution Separation patent description [6], only the principal component of the separation is considered. Thus,  $D_{0n}$  satisfies:

$$D_{0n} = \sqrt{\lambda^{dP}} Q\left(\frac{1}{2}\left(1 - \frac{P_{fa}}{N}\right)\right),\tag{6}$$

where Q(u) is the error function<sup>6</sup>,  $\lambda^{dP}$  is the maximal eigenvalue of  $dP_{H0n}^+(k)$  which is a sub matrix of the covariance matrix of  $dX_{0m}^+(k)$ , denoted  $dP_{0m}^+(k)$ . This matrix is expressed as:

$$dP_{0n}^{+}(k) = \mathbf{E} \left[ dX_{0n}^{+}(k) dX_{0n}^{+}(k)^{T} \right],$$
  
=  $P_{00}^{+}(k) + P_{0n}^{+}(k) - \Gamma_{0n}^{+}(k) - \left(\Gamma_{0n}^{+}(k)\right)^{T},$  (7)

with  $P_{00}^+(k)$  and  $P_{0n}^+(k)$  the covariance matrices of the estimation error for the primary EKF and the sub-EKF excluding the  $n^{\text{th}}$  measurement, respectively. In addition,  $\Gamma_{0n}^+(k)$  is the cross covariance matrix between the primary EKF and the  $F_{0n}$  sub-EKF estimation error which is sequentially updated, as described in [1].

#### 4. THE PROPOSED IMM-MSS ALGORITHM

This section describes the proposed FD algorithm which consists of replacing the EKF by MM algorithms to avoid false alarms due to abrupt changes in the aircraft motion. We first present the MM approach and more particularly the IMM. Then, we focus on the FD step of our new algorithm.

#### 4.1. The Interacting Multiple Model algorithm

MM algorithms aim at jointly estimating the state vector and deciding which model best describes its evolution. They consider a set of M possible state space representations. The equations describing the state space model  $\{m^{(i)}\}_{i \in [1,M]}$  are:

$$X(k) = \Phi^{(i)}(k)X(k-1) + G^{(i)}(k)w^{(i)}(k), \qquad (8)$$

$$f(k) = h(X(k);k) + R(k)\varepsilon(k).$$
(9)

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 $<sup>^{5}</sup>$ In the following, the superscripts + and - refer to the a posteriori estimation step and a priori estimation step, respectively.

<sup>&</sup>lt;sup>6</sup>The error function is defined as  $Q(u) = \frac{2}{\sqrt{\pi}} \int_0^u \exp{(-a^2)} da$ .

It should be noted that in the general case the observation model matrices can also vary with i.

The model switching process considered is of the Markov type. This process is specified by a transition matrix. Its components are expressed as, for  $(i, j) \in [1, M] \times [1, M]$ :

$$\pi_{ji} = P_r\left[m_{k+1}^{(i)} | m_k^{(j)}\right], \,\forall \, m^{(i)}, m^{(j)}, k, \tag{10}$$

where  $m_k^{(i)}$  means that  $m^{(i)}$  is used to describe the state vector evolution at time k.

The IMM consists of running as many EKF as possible model sequences up to the current time. Then, the overall state estimate is obtained by combining their estimates. Nevertheless, the number of sequences exponentially grows with time, amounting to  $M^{k+1}$  at time k. To keep a constant number of EKF, the IMM applies a merging strategy described hereafter in Figure 1.



Fig. 1. Structure of MM algorithm (with 2 model-based filters)

Let us assume, at time k - 1, the overall state estimate satisfies:

$$\widehat{X}^{+}(k-1) = \sum_{j=1}^{M} \alpha^{(j+)}(k-1)\widehat{X}^{(j+)}(k-1), \qquad (11)$$

where, at time k - 1,  $\widehat{X}^{(j+)}(k - 1)$  is the a posteriori state estimate based on model  $m^{(j)}$  and  $\alpha^{(j+)}(k - 1) = P_r[m_{k-1}^{(j)}|Y_{1:k-1}]$ .

The IMM error covariance matrix satisfies:

$$P^{+}(k-1) = \sum_{j=1}^{M} \alpha^{(j+)}(k-1) \left[ P^{(j+)}(k-1) + \Delta_{k-1}^{(j)} \left( \Delta_{k-1}^{(j)} \right)^{T} \right],$$
(12)

with  $P^{(j+)}(k-1)$  the error covariance matrix for  $\widehat{X}^{(j+)}(k-1)$  and  $\Delta_{k-1}^{(j)} = \widehat{X}^+(k-1) - \widehat{X}^{(j+)}(k-1)$ , at time k-1. At that stage, each of the M parallel EKF estimates

At that stage, each of the M parallel EKF estimates  $\{\hat{X}^{(j+)}(k-1)\}_{j\in[1,M]}$  can evolve according to any of the state models  $\{m^{(i)}\}_{i\in[1,M]}$ , leading to  $M \times M$  possible predicted state vectors:

$$\widehat{X}^{(ji-)}(k) = \Phi^{(i)}(k)\widehat{X}^{(j+)}(k-1).$$
(13)

To avoid an exponential increase of the computational complexity, the IMM merges the state predictions based on the same model at time k. In this way, the number of necessary EKF is decreased from  $M \times M$  to M. The predicted state vector based on model  $\{m^{(i)}\}_{i \in [1,M]}$  and its associated covariance matrix become:

$$\widehat{X}^{(i-)}(k) = \sum_{j=1}^{M} \alpha^{(ji-)}(k) \widehat{X}^{(ji-)}(k), \qquad (14)$$

$$P^{(i-)}(k) = \sum_{j=1}^{M} \left( \alpha^{(ji-)}(k) \left[ P^{(j+)}(k-1) + \Delta^{(ij)}_{k-1} \left( \Delta^{(ij)}_{k-1} \right)^{T} \right] \right) + G^{(i)}(k) \left( G^{(i)}(k) \right)^{T},$$
(15)

where  $\Delta_{k-1}^{(ij)} = \hat{X}^{(i-)}(k) - \hat{X}^{(j+)}(k-1)$  and the mixing probabilities for  $\{m^{(i)}, m^{(j)}\}_{(i,j) \in [1,M] \times [1,M]}$  satisfy:

$$\alpha^{(ji-)}(k) = \frac{1}{\alpha^{(i-)}(k)} \pi_{ji} \alpha^{(j+)}(k-1),$$
(16)

with  $\alpha^{(i-)}(k) = \sum_{j=1}^{M} \pi_{ji} \alpha^{(j+)}(k-1)$  the normalizing factor.  $\alpha^{(ji-)}(k)$  is the probability the sequence composed by the model  $m^{(j)}$  at time k and the model  $m^{(i)}$  at time k+1 best describes the mobile motion.

Then, the standard KF equations are used to update the state estimates given by (14):

$$\{\widehat{X}^{(i-)}(k), P^{(i-)}(k)\}_{i=1}^{M} \to \{\widehat{X}^{(i+)}(k), P^{(i+)}(k)\}_{i=1}^{M}.$$
 (17)

Finally, the IMM iteration is completed by computing the model probabilities required to merge the M EKF estimates and obtain the overall state vector estimation from equations (11-12) at time k. Note that:

$$\alpha^{(i+)}(k) = \frac{\alpha^{(i-)}(k-1)L_k^{(i)}}{\sum_{j=1}^M \alpha^{(j-)}(k-1)L_k^{(i)}},$$
(18)

with  $L_k^{(i)} = P_r \left[ Y(k) | m_k^{(i)}, Y(1:k-1) \right]$  the likelihood function of model  $m^{(i)}$  at time k.

After this brief presentation of the IMM algorithm, we detail the proposed IMM-MSS algorithm by focusing on the FD step in the next section.

## 4.2. IMM-MSS fault detection step

When replacing the EKF by IMM, we suggest using as test statistic the separation between the primary IMM and the IMM sub-filters using all but one measurement. For the sake of simplicity, we use the same notations as for the classical MSS, i.e.  $\hat{X}_{00}(k)$  the estimation computed by the main filter, etc. Equations (5-7) still hold. The main difference lies in the computation of the cross covariance matrix  $\Gamma_{0n}^+(k)$ . Indeed, since each IMM of the filter hierarchy uses MEKF, cross correlations between EKF estimation errors based on different models have to be taken into account when updating  $\Gamma_{0n}^+(k)$ :

$$\Gamma_{0n}^{+}(k) = \sum_{r=1}^{M} \sum_{l=1}^{M} \alpha_{00}^{(r+)}(k) \alpha_{0n}^{(l+)}(k) \Gamma_{0n}^{(rl+)}(k), \quad (19)$$
  
with  $\Gamma_{0n}^{(rl+)}(k) = E\left[ \left( X - \widehat{X}_{00}^{(r+)} \right) \left( X - \widehat{X}_{0n}^{(l+)} \right)^{T} \right],$ 

where the subscript 00 and 0n are associated to the mixing probabilities for the primary IMM and for the IMM excluding the measurement n, respectively. We propose to compute these matrices sequentially in 2 steps:

Step 1. Prediction step.

$$\Gamma_{0n}^{(rl-)}(k) = \sum_{q=1}^{M} \sum_{p=1}^{M} \alpha_{00}^{(qr-)}(k) \alpha_{0n}^{(pl-)}(k) \Phi^{(r)}(k) \Gamma_{0n}^{(qp+)}(k-1) \left( \Phi^{(l)}(k) \right)^{T} + G^{(r)}(k) \left( G^{(l)}(k) \right)^{T} \delta(r-l), \quad (20)$$

where  $\delta(.)$  is the dirac impulse.

Step 2. Estimation step.

$$\Gamma_{0n}^{(rl+)}(k) = \left(\mathbf{I} - K_{00}^{(r)}H_{00}^{(r)}(k)\right)\Gamma_{0n}^{(rl-)}(k) \times$$

$$\left(\mathbf{I} - K_{0n}^{(l)}H_{0n}^{(l)}(k)\right)^{T} + K_{00}^{(r)}R(k)\left(K_{0n}^{(l)}\right)^{T},$$
(21)



Fig. 2. MSE for a nearly uniform motion - Phase 1

ramp error size (m/s)	0.05	0.25	0.75	2.5	5
MSS <sup>(2)</sup> delay (s)	262.0	48.4	16.9	6.2	3.6
IMM-MSS delay (s)	151.2	30.8	10.8	4.9	3.5

Table 1. Mean delay of ramp failure detection

where  $K_{00}^{(r)}$  is the gain of the primary EKF using model  $m^{(r)}$  and  $H_{0n}^{(r)}(k)$  is the observation matrix of the EKF  $F_{0n}$  using the model  $m^{(r)}$ .

The performance of this algorithm is studied through simulation results in the next section.

#### 5. SIMULATIONS AND RESULTS

Several simulations have been conducted to illustrate the performance of the proposed algorithm. We have considered a real aircraft trajectory consisting of 2 phases: phase 1, with a nearly uniform motion and phase 2, with an accelerated motion. The navigation data have been generated by using GPS almanac files that provide the trajectories of the GPS satellites. As for the measurement noise statistics, they have been set in accordance with RTCA recommendations [7]. Satellite failures have been simulated by introducing ramp errors of standard values given by ICAO. Finally, we adjust the hypothesis test parameters in accordance with a non precise approach flight phase<sup>7</sup>, i.e. a false alarm rate  $P_{fa} = 2.778.10^{-9}$ /test.

In this study, we suggest using different values for the covariance matrix of the process noise corresponding to low and high values of the mobile acceleration<sup>8</sup>. The IMM-MSS is compared with the classical MSS by considering two  $2^{nd}$  order state models:

•  $m^{(1)}$  with  $\sigma_{acc} = 0.1 \text{ m/s}^2$ ,

•  $m^{(2)}$  with  $\sigma_{acc} = 15 \text{ m/s}^2$ .

In the sequel, we denote  $MSS^{(1)}$ , respectively  $MSS^{(2)}$ , the FD algorithm based on model  $m^{(1)}$ , respectively model  $m^{(2)}$ . The IMM-MSS uses a combination of both models.

As for the IMM model switching process, the probability for the dynamics of the aircraft to change between 2 consecutive time steps is assumed to be low. Thus, the transition probability is set to  $\pi_{ij} = 0.9$  for i = j.

The performance parameters considered in this paper are the mean detection delays (MDD) and the position mean square error (MSE). They have been computed by averaging the results obtained for 50 different realizations of the measurement noise.

Table 1 shows the MDD for different ramp error sizes. The MDD of the  $MSS^{(1)}$  are not represented because model  $m^{(1)}$  yields numerous false detections. Indeed, we observe a false alarm rate of



Fig. 3. MSE for an accelerated motion - Phase 2

 $7.5 \times 10^{-3}$ /test during phase 2 of the trajectory. More precisely, due to the small value of  $\sigma_{acc}$ , model  $m^{(1)}$  is not appropriate when the aircraft undergoes manoeuvers or high accelerations. Furthermore, the IMM-MSS outperforms the MSS<sup>(2)</sup>, particularly for small ramp error sizes. For instance, the MDD is decreased by more than 100 s for ramps of 0.05 m/s. Figures 2 and 3 show the position MSE for both phases of the aircraft trajectory. During phase 1, the IMM-MSS outperforms the MSS<sup>(2)</sup> and exhibits the same MSE as the MSS<sup>(1)</sup>. During phase 2, model  $m^{(2)}$  is the most appropriate model. As a result, the  $MSS^{(2)}$  has a slightly smaller MSE than the IMM-MSS which makes the compromise between model  $m^{(1)}$ and  $m^{(2)}$ . It should be noted that the MSS<sup>(1)</sup> MSE increases up to 15000 m<sup>2</sup> during this phase and therefore is not represented for the sake of clarity. As a conclusion, the proposed algorithm on average yields lower MSE by selecting at each step the most convenient motion model.

### 6. CONCLUSION

In this paper, we have presented a new approach to improve faulty GPS measurement detection in civil aviation. Our new algorithm is based on the FD MSS algorithm and a position estimation performed by a IMM algorithm. The IMM-MSS has a higher computational complexity than classical FD algorithms since the complexity is directly linked to the number of the filters operating in parallel. However, the MSE and the detection delays of the proposed algorithm are on average lower than the classical MSS ones. We are currently working on IMM-MSS based on more elaborated models including turn models for instance.

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<sup>&</sup>lt;sup>7</sup>It should be noted that the RTCA performance recommandations are different according to the flight phases.

<sup>&</sup>lt;sup>8</sup>Only the matrix G(k) varies with i in the state space representation.