# QUANTIZER NOISE BENEFITS IN NONLINEAR SIGNAL DETECTION WITH ALPHA-STABLE CHANNEL NOISE

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## ABSTRACT

Two new theorems show how deliberately adding quantizer noise can improve statistical signal detection in array-based nonlinear correlation detection even in the case of infinite-variance $\alpha$-stable channel noise. The first theorem gives a necessary and sufficient condition for such quantizer noise to increase the detection probability for a fixed false-alarm probability. The second theorem shows that the array must contain more than one quantizer for a stochastic-resonance noise benefit and that the noise benefit improves in the small-quantizer noise limit as the number of array quantizers increases. It further shows that symmetric uniform quantizer noise gives the optimal noise benefit among all symmetric scale-family noise types.

***Index Terms—*** optimal noise, stochastic resonance, nonlinear detection, neyman-pearson, array processing

## 1. NOISE BENEFITS IN NONLINEAR SIGNAL DETECTION

Some noise can improve nonlinear signal processing [9, 13, 15, 18, 20, 24]. The *stochastic resonance* (SR) effect in signal detection occurs when small amounts of noise improves detection performance while too much noise degrades it [5, 6, 11, 21, 22, 29]. Such SR noise benefits arise in many physical and biological signal systems from carbon nanotubes to neurons [7, 10, 14, 19, 23, 25, 27]. We focus here on the special case of SR in a quantizer-array-based nonlinear correlation detector that uses Neyman-Pearson or constant-false-alarm hypothesis testing to detect the presence of a constant (dc) signal in infinite-variance $\alpha$-stable channel noise. Such detection occurs in sonar, radar, and watermark detection [2, 3]. We show that injecting small amounts of quantizer noise in this type of nonlinear detector can increase the signal detection probability $P_D$ while the false-alarm probability $P_{FA}$ stays at a preset level. Figure 2 shows SR's characteristic nonmonotonic signature of noise-enhanced signal detection$-$here with more than 12$\%$ increase in the detection probability. This appears to be the first demonstration of the SR effect for Neyman-Pearson signal detection in $\alpha$-stable channel noise.

The detector consists of a nonlinear preprocessor that precedes a correlator and Neyman-Pearson likelihood-ratio test on the correlator's output. This nonlinear detector takes $K$ samples of a noise-corrupted signal and sends each sample to the nonlinear preprocessor array of $Q$ noisy quantizers connected in parallel. Each quantizer in the array adds its independent quantizer noise to the noisy input sample and then quantizes this doubly noisy data sample into a binary value. The quantizer array output for each sample is just the sum of all $Q$ quantizer outputs. The correlator then correlates these preprocessed $K$ samples with the signal. The detector's final stage applies the Neyman-Pearson likelihood-ratio test to the correlator's output so that the false-alarm probability $P_{FA}$ remains at a preset level $\tau$.

Stocks [24] first showed that adding quantizer noise in an array of parallel-connected quantizers improves the mutual information between the array's input and output. Then Rousseau and Chapeau-Blondeau [20, 21] used such a quantizer array for signal detection. They first showed the SR effect for Neyman-Pearson detection of time-varying signals and for Bayesian detection of both constant and time-varying signals in different types of non-Gaussian channel noise. But their noise always had finite variance. Their work also did not characterize their observed SR effects in terms of detector parameters such as the number of quantizers $Q$ or the type of quantizer noise.

We present two SR theorems that apply to broad classes of channel and quantizer noises for the above nonlinear detector. We show that adding small amounts of quantizer noise in the detector produces the SR effect for dc signal detection in $\alpha$-stable channel noise. Theorem 1 in Section 3 gives a necessary and sufficient condition for this type of SR noise benefit. This result applies to all types of symmetric channel noise and symmetric quantizer noise. It requires only that the number of data samples $K$ be large.

Theorem 2 gives three results based on Theorem 1. They require only that the quantizer noise come from a symmetric scale-family probability density function (pdf) with finite variance. These results involve an initial SR effect or increase in the detection probability $P_D$ for small amounts of noise. We define the SR effect as an *initial* SR effect if there exists some $b > 0$ such that $P_D(\sigma_N) > P_D(0)$ for all $\sigma_N \in (0, b)$. Here $P_D(\sigma_N)$ is the detection probability when the quantizer noise intensity is $\sigma_N$ and $P_D(0)$ is the detection probability in the absence of quantizer noise. The first result shows that $Q > 1$ is necessary for the initial SR effect. The second result is that the rate of the initial SR effect in the small quantizer noise limit ($\lim_{\sigma_N \to 0} \frac{dP_D}{d\sigma_N}$) improves if the number of quantizers $Q$ in the array increases. The final result is that symmetric uniform quantizer noise gives the optimal initial SR effect rate among all symmetric scale-family noise types. The next section presents the theorems' framework of signal detection in $\alpha$-stable channel noise.

## 2. BINARY SIGNAL DETECTION IN $\alpha$-STABLE NOISE

Consider detecting a known deterministic signal $s_k$ in additive white symmetric $\alpha$-stable ($S\alpha S$) channel noise $W_k$ given $K$ observed samples:

$$\begin{array}{lll} H_0: & X_k = W_k & k = 1, 2, ..., K, \\ H_1: & X_k = s_k + W_k & k = 1, 2, ..., K. \end{array} \tag{1}$$

The $W_k$ are independent and identically distributed (i.i.d.) zero-location $S\alpha S$ random variables. We consider only constant (dc) signals so that $s_k = A$ for all $k$. The characteristic function $\varphi$ of the $S\alpha S$ noise random variable $W_k$ has the exponential form [8, 16]

$$\varphi(\omega) \;=\; \exp(j\delta\omega - \gamma|\omega|^{\alpha}) \tag{2}$$

where finite $\delta$ is the location parameter, $\gamma > 0$ is the dispersion, and $\alpha \in (0, 2]$ is the characteristic exponent that controls the density's tail thickness. $S\alpha S$ pdfs can model heavy-tailed or impulsive noise in many applications such as underwater acoustic signals, telephone noise, clutter returns in radar, internet traffic, financial data, and transform domain image or audio signals [1, 3, 16]. The only closed-form $S\alpha S$ pdfs are for $\alpha$ = 1 and $\alpha$ = 2 for the respective Cauchy and Gaussian densities. The $r^{\text{th}}$ lower-order moments of an $\alpha$-stable pdf with $\alpha < 2$ exist if and only if $r < \alpha$. So the Gaussian density alone has finite variance and higher-order moments. The location parameter $\delta$ is the mean for $1 < \alpha \leq 2$ and serves only as the median for $0 < \alpha \leq 1$.

The uniformly most powerful detector for the hypotheses in (1) is a Neyman-Pearson log-likelihood ratio test [26, 28]:

$$\Lambda_{NP}(\mathbf{X}) \;=\; \sum_{k=1}^{K} \log(f_\alpha(X_k\text{-}s_k))\text{-}\log(f_\alpha(X_k)) \;\; \underset{H_0}{\overset{H_1}{\gtrless}} \;\; \lambda \tag{3}$$

since the observed $K$ samples $\mathbf{X} = \{X_1, ..., X_K\}$ are i.i.d. We choose $\lambda$ so that it has a preset false-alarm probability $P_{FA} = \tau$. This Neyman-Pearson detector is difficult to implement because again the $S\alpha S$ pdf $f_\alpha$ has no closed form except for $\alpha$ = 1 and $\alpha$ = 2. A Taylor expansion of $\Lambda_{NP}(\mathbf{X})$ around $X_k$ in the small-signal limit gives the locally optimal detector with a familiar correlation structure [16]:

$$\Lambda_{LO}(\mathbf{X}) \;=\; \sum_{k=1}^{K} s_k \frac{-f'_\alpha(X_k)}{f_\alpha(X_k)} \;=\; \sum_{k=1}^{K} s_k g_{LO}(X_k) \;\; \underset{H_0}{\overset{H_1}{\gtrless}} \;\; \tilde{\lambda} \tag{4}$$

where the score function $g_{LO}$ is nonlinear for $\alpha < 2$. This test also is not practical because it uses $f'_\alpha$ and $f_\alpha$. So researchers have suggested different suboptimal detectors that preserve the correlation structure but that replace $g_{LO}$ with other zero-memory nonlinear functions $g$ [4, 26, 28]. These nonlinearities range from simple soft-limiters ($g_{SL}(x) = ax$ for $|x| < c$ and $g_{SL}(x) = ac$ else) and hole-puncher functions ($g_{HP}(x) = ax$ for $|x| < c$ and $g_{HP}(x) = 0$ else) to more complex nonlinearities that better approximate $g_{LO}$. The next section presents the two SR theorems for the simple nonlinear correlation detector with a noisy quantizer-array-based nonlinearity $g_{NQ}$.

## 3. QUANTIZER NOISE BENEFITS IN NONLINEAR CORRELATION DETECTOR

Consider the nonlinear correlation detector

$$\Lambda_{NQ}(\mathbf{X}) \;=\; \sum_{k=1}^{K} s_k g_{NQ}(X_k) \;\; \underset{H_0}{\overset{H_1}{\gtrless}} \;\; \tilde{\lambda} \tag{5}$$

$$\text{where} \quad g_{NQ}(X_k) \;=\; \sum_{q=1}^{Q} sign(X_k + N_q - \theta). \tag{6}$$

Here $\theta$ is a quantization threshold, $N_q$ are independent symmetric scale-family quantizer noises with standard deviation $\sigma_N$, and $sign(X_k + N_q - \theta) = \pm 1$ for $q$ = 1, ..., $Q$. We choose $\theta = \frac{A}{2}$ because the channel noise $W_k$ and the quantizer noise $N_q$ are both symmetric noises. This detector is simple to implement and requires only one bit to represent each quantizer's output. These properties can help in applications such as sensor networks or distributed systems that have limited energy or involve limited data handling and storage [12, 17].

Define $\mu_i(\sigma_N)$ and $\sigma_i^2(\sigma_N)$ as the respective mean and variance of $\Lambda_{NQ}$ under the hypothesis $H_i$ when $\sigma_N$ is the quantizer noise intensity. Then $\mu_0(\sigma_N) = -\mu_1(\sigma_N)$ and $\sigma_0^2(\sigma_N) = \sigma_1^2(\sigma_N)$ for all $\sigma_N$ because both $W$ and $N$ are symmetric. The mean $\mu_i$ and variance $\sigma_i^2$ of the test statistic $\Lambda_{NQ}$ depend on both the additive channel noise $W$ and the quantizer noise $N$. So $\mu_i$ and $\sigma_i^2$ depend on the noise intensities $\sigma_\zeta$ and $\sigma_N$. But we write $\mu_i(\sigma_N)$ and $\sigma_i^2(\sigma_N)$ because we control only the quantizer noise intensity $\sigma_N$.

The pdf of $\Lambda_{NQ}$ is approximately Gaussian for either hypothesis because the central limit theorem applies to the sum (5) if the sample size $K$ is large. Then Theorem 1 gives a necessary and sufficient condition for the SR effect.

**Theorem 1:** Suppose that $\Lambda_{NQ}|H_0 \sim \text{N}(\mu_0(\sigma_N), \sigma_0^2(\sigma_N))$ and $\Lambda_{NQ}|H_1 \sim \text{N}(\mu_1(\sigma_N), \sigma_1^2(\sigma_N))$ where $\mu_0(\sigma_N) = -\mu_1(\sigma_N)$ and $\sigma_0^2(\sigma_N) = \sigma_1^2(\sigma_N)$. Then

$$\sigma_1(\sigma_N)\mu_1'(\sigma_N) \;>\; \mu_1(\sigma_N)\sigma_1'(\sigma_N) \tag{7}$$

is necessary and sufficient for the SR effect in Neyman-Pearson detection using the nonlinear test statistic $\Lambda_{NQ}$.

**Proof:** The Neyman-Pearson detection rule based on $\Lambda_{NQ}$ rejects $H_0$ if $\Lambda_{NQ} > \lambda_\tau$ where we choose $\lambda_\tau$ such that $P(\Lambda_{NQ} > \lambda_\tau | H_0) = \tau$. Then the detection threshold is $\lambda_\tau = z_\tau \sigma_0(\sigma_N) + \mu_0(\sigma_N)$ where $1 - \Phi(z_\tau) = \tau$ for the cumulative distribution function $\Phi$ of the standard normal random variable. This gives the detection probability

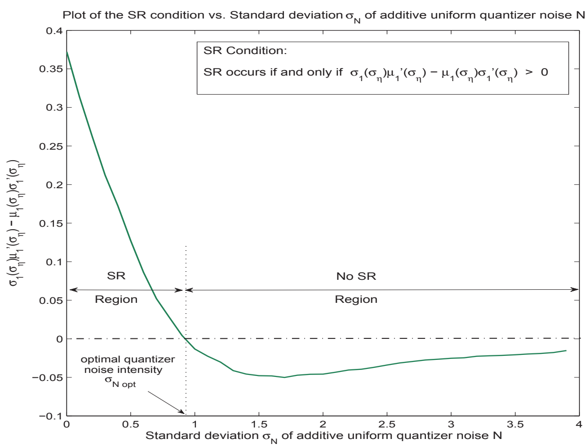
$$P_D = 1 - \Phi\left(z_\tau - \frac{2\mu_1(\sigma_N)}{\sigma_0(\sigma_N)}\right) \tag{8}$$

because $\mu_0(\sigma_N) = -\mu_1(\sigma_N)$ and $\sigma_0^2(\sigma_N) = \sigma_1^2(\sigma_N)$. Then

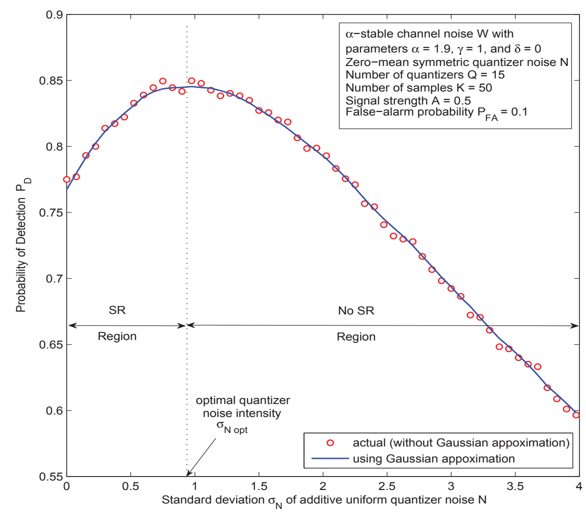
$$\frac{dP_D}{d\sigma_N} = 2\phi\left(\frac{\mu_1(\sigma_N)}{\sigma_1(\sigma_N)}\right)\frac{\sigma_1(\sigma_N)\mu_1'(\sigma_N) - \mu_1(\sigma_N)\sigma_1{}'(\sigma_N)}{\sigma_1^2(\sigma_N)} \tag{9}$$

with normal pdf $\phi = \Phi'$. So $\sigma_1(\sigma_N)\mu_1'(\sigma_N) > \mu_1(\sigma_N)\sigma_1'(\sigma_N)$ is necessary and sufficient for the SR effect ($\frac{dP_D}{d\sigma_N} > 0$) because $\phi > 0$. $\square$

Figure 1 shows a simulation instance of the SR condition in Theorem 1 for dc signal detection in impulsive infinite-variance channel noise. The dc signal has magnitude $A$ = 0.5 and we set the false-alarm probability to $P_{FA}$ = 0.1. The channel noise is $S\alpha S$ with parameters $\alpha$ = 1.9, $\gamma$ = 1, and $\delta$ = 0. The detector preprocesses each of the $K$ = 50 noisy samples $X_k$ with $Q$ = 15 quantizers in the array. Each quantizer has quantization threshold $\theta = A/2$ and adds an independent uniform quantizer noise $N$ to the noisy sample $X_k$ before quantization. Figure 1(a) shows the plot of $\sigma_1(\sigma_N)\mu_1'(\sigma_N)$ - $\mu_1(\sigma_N)\sigma_1'(\sigma_N)$ versus the standard deviation $\sigma_N$ of the additive uniform quantizer noise.




(a)




(b)

**Fig. 1**. Plots of inequality condition (7) for the predicted SR effect and the detection probabilities with and without the Gaussian approximation of $\Lambda_{NQ}$'s distribution for dc signal detection in $\alpha$-stable channel noise. (a) The plot of $\sigma_1(\sigma_N)\mu_1'(\sigma_N)$ - $\mu_1(\sigma_N)\sigma_1'(\sigma_N)$ versus the standard deviation $\sigma_N$ of additive uniform quantizer noise. The zero-crossing occurs at the quantizer noise standard deviation $\sigma_{N_{\mathrm{opt}}}$. (b) The solid line and square markers show the respective plots of the detection probabilities $P_D$ with and without the Gaussian approximation of $\Lambda_{NQ}$'s distribution. Adding small amounts of quantizer noise $N$ improves the detection probability $P_D$. This SR effect occurs until inequality (7) holds. So $\sigma_{N_{\mathrm{opt}}}$ maximizes the detection probability.

Adding small amounts of quantizer noise $N$ improves the detection probability $P_D$ in Figure 1(b). This SR effect occurs until the inequality (7) holds in Figure 1(a). Figure 1(b) also shows the accuracy of the Gaussian approximation of the detection statistic $\Lambda_{NQ}$'s distribution. The solid line shows the plot of the detection probability $P_D$ using the Gaussian approximation of $\Lambda_{NQ}$'s distribution. We used $10^5$ simulation trials to estimate $\mu_1(\sigma_N)$ and $\sigma_1(\sigma_N)$ and used them in (8) to obtain this plot. Circle markers show the actual detection probabilities.

Theorem 2 below states that more than one quantizer is necessary for the initial SR effect and that the rate of initial SR effect increases as the number of quantizer increases. It also states that uniform quantizer noise gives the maximal initial SR effect among all finite-variance symmetric scale-family quantizer noise types. Theorem 2 follows from Theorem 1 if we substitute the expressions for $\mu_1(\sigma_N)$, $\mu_1'(\sigma_N)$, $\sigma_1(\sigma_N)$, and $\sigma_1'(\sigma_N)$ and then pass to the limit $\sigma_N \rightarrow 0$. The proof is quite lengthy and we omit it for reasons of space.

**Theorem 2:**
(a) $Q > 1$ is necessary for the initial SR effect in the Neyman-Pearson detection of a dc signal in a channel with $S\alpha S$ noise $W$ using the nonlinear test statistic $\Lambda_{NQ}$ in (5).

(b) Suppose that the initial SR effect occurs with $Q_1$ quantizers and with some symmetric quantizer noise. Then the rate of the initial SR effect with $Q_2$ quantizers is larger than the rate of initial initial SR effect with $Q_1$ quantizers if $Q_2 > Q_1$.

(c) Zero-mean uniform noise is the optimal finite-variance symmetric scale-family quantizer noise in the sense that it maximizes the rate of the initial SR effect.

Figures 2 and 3 show simulation instances of Theorem 2. The dashed line in Figure 2 shows that the SR effect does not occur if $Q = 1$ as Theorem 2(a) predicts. The solid lines show that the initial SR effect increases as the number of quantizers $Q$ increases as Theorem 2(b) predicts. $Q = 31$ quantizers give a 0.865 maximal detection probability and thus a 12.7% improvement over the noiseless 0.767 detection probability. The horizontal lines $P_D(g_{SL})$ = 0.84 and $P_D(g_{HP})$ = 0.814 show the respective detection probabilities of the non-noisy correlation detector with

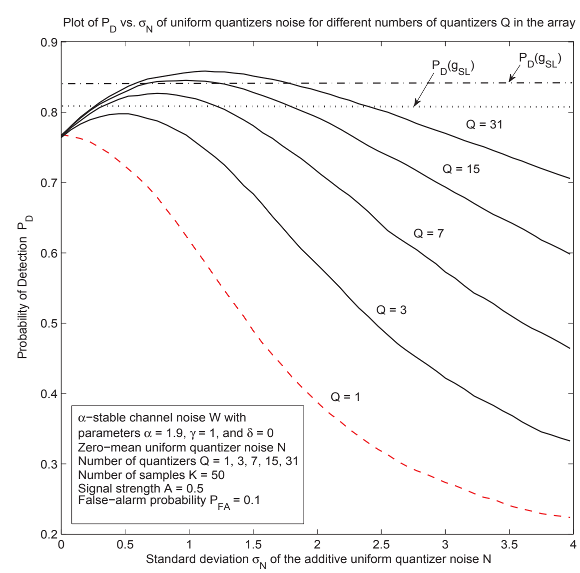



**Fig. 2**. Initial SR effects in a nonlinear correlation detector for dc signal detection in an impulsive $\alpha$-stable channel noise ($\alpha$ = 1.9). The solid lines show that the detection probability $P_D$ improves initially as the quantizer noise intensity $\sigma_N$ increases. The dashed line shows that the SR effect does not occur if $Q$ = 1 as Theorem 2(a) predicts. The solid lines also show that the rate of the initial SR effect increases as the number of quantizers $Q$ increases as Theorem 2(b) predicts. The horizontal lines $P_D(g_{SL})$ = 0.84 and $P_D(g_{HP})$ = 0.814 show the respective detection probabilities of the non-noisy correlation detector with soft-limiter ($g_{SL}$, $a$ = 2, $c$ = 1) and hole-puncher ($g_{HP}$, $a$ = 2, $c$ = 3) nonlinearities.

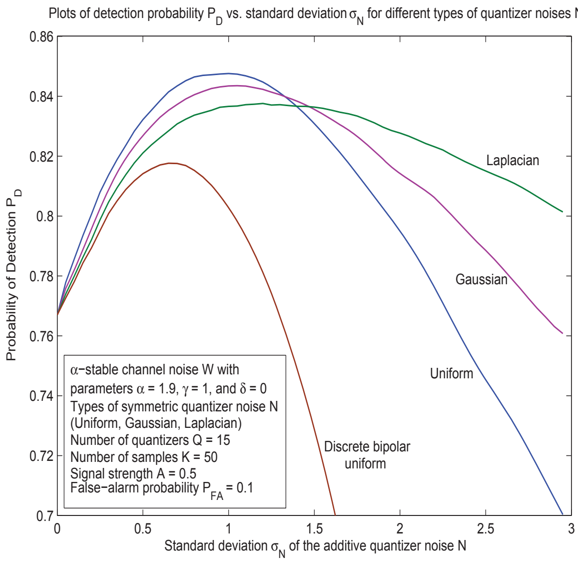



**Fig. 3**. Comparison of initial SR effects in the nonlinear correlation detector for dc signal detection in $\alpha$-stable channel noise ($\alpha$ = 1.9) for different types of symmetric quantizer noise. Symmetric uniform noise gives the maximal rate of the initial SR effect as Theorem 2(c) predicts whereas symmetric discrete bipolar noise gives the smallest SR effect and is the least robust. The SR effect is most robust against Laplacian quantizer noise.

soft-limiter ($g_{SL}$, $a$ = 2, $c$ = 1) and hole-puncher ($g_{HP}$, $a$ = 2, $c$ = 3) nonlinearities. Detectors with at least $Q$ = 15 quantizers have maximal detection probabilities that exceed both of these values. Figure 3 compares the initial SR effects for different types of simple zero-mean symmetric quantizer noises such as Laplacian, Gaussian, uniform, and discrete bipolar noise. Symmetric uniform noise gives the maximal rate of the initial SR effect as Theorem 2(c) predicts. Symmetric discrete bipolar noise gives the smallest SR effect and is the least robust. The SR effect is most robust against Laplacian quantizer noise.

An open question is whether some form of Theorem 2 holds for time-varying signals or for asymmetric $\alpha$-stable channel noise.

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