OPTIMAL AND SUBOPTIMAL MICRO-DOPPLER ESTIMATION SCHEMES USING CARRIER DIVERSE DOPPLER RADARS

Pawan Setlur, Moeness Amin, and Fauzia Ahmad
Radar Imaging Lab, Center for Advanced Communications
Villanova University, Villanova, PA 19085, USA
E-mail: {pawan.setlur, moeness.amin, fauzia.ahmad}@villanova.edu

ABSTRACT

Carrier diverse radars employing two different frequencies, termed as dual-frequency radars, prove effective in determining the target range in urban sensing and through-the-wall applications. In this paper, we derive the maximum likelihood (ML) estimator for the dual frequency radar returns for a micro-Doppler motion profile, which is commonly exhibited by indoor moving targets. Unlike linear models, the respective ML estimator does not have a closed form. We solve the ML estimator for dual frequency radar operations, using iterative reweighted least squares (IRLS). The ML-IRLS algorithm is applied to experimental radar returns for estimating the motion parameters of indoor targets.

***Index Terms*-** Iterative maximum likelihood estimation, Doppler radar, Urban sensing, Micro-Doppler.

1. INTRODUCTION

The problem of urban sensing and through-the-wall imaging addresses the desire to obtain an electromagnetic blueprint of the scene under consideration along with possible knowledge of the type and location of the animate and inanimate targets, whether they are moving or stationary [1]. For such applications, we tackle the problem of moving target range estimation using carrier diverse Doppler radar, which is also known as dual frequency radar. The carrier diversity is induced by using two different carrier frequencies to satisfy the maximum range ambiguity condition based on the *a priori* knowledge (possible through aerial mapping or ground access) of the spatial extent of the urban structure under surveillance. The technique of using two frequencies to estimate range is not new and has been used in automotive and other radar applications [2]. Without a doubt, there exists many other techniques to estimate range of a moving target, for example, linear frequency modulated radar and pulse Doppler radar [3]. Such radar systems are wideband and employ some form of frequency modulation to obtain the range. The major advantages of Doppler radars vs. wideband conventional radars for the urban sensing/through-the-wall applications can be summarized as follows. 1) Much of the RF spectrum may be jammed or taken by other emitters. 2) RF penetration through the walls follows a lowpass filtering model with typical cutoff in the low GHz range. 3) Many modes of urban operations require mobility and simple portable radar platforms rather than complex surveillance systems.

In this paper, we deal with a single moving target whose motion profile can be modeled by a finite number of parameters. In particular, we consider the micro-Doppler (MD) which gives rise to sinusoidal FM type radar returns [4]. Maximum likelihood (ML) technique for motion parameter estimation is then formulated and solved using step-wise concentration to obtain an iterative weighted least squares algorithm. The ML estimator is initialized using suboptimal estimates. Section 2 describes the signal model. In Section 3, we discuss the ML and suboptimal schemes for the MD motion profile. Section 4 contains results based on simulations and experiments, followed by conclusions in Section 5.

2. SIGNAL MODEL

The signal returns for the dual frequency Doppler radar after down conversion to baseband can be written as

$$\begin{aligned} &x_i(n) = s_i(n) + v_i(n), n = 0,12...N-1 \\ &s_i(n) = \rho \exp(j4\pi f_i R(n;\boldsymbol{\psi})/c + j\gamma) \\ &E\{v_1(n)v_2^*(k)\} = 0, \forall n,k \;\; E\{v_i(n)v_i^*(k)\} = 0, \forall n \neq k, \\ &E\{v_i(n)v_i^*(k)\} = \sigma_i^2, \forall n = k, \forall \mathrm{i} = 1,2 \end{aligned} \quad (1)$$

In eq. (1), the target range, $R(n;\boldsymbol{\psi})$, is parameterized by a vector of desired parameters $\boldsymbol{\psi} \in \Re^{P\times 1}$, $\rho \exp(j\gamma)$ is the complex reflectivity of the target, $f_i, \forall i = 1,2$ are the carrier frequencies, and c is the speed of light. The noise at the two carrier frequencies are assumed to be complex AWGN, and uncorrelated. Further, the noise sequences are i.i.d for each carrier frequency. The returns in eq. (1) can be statistically characterized by a multivariate complex Gaussian probability density function,

$$p(\mathbf{x};\mathbf{s}) = \frac{1}{\pi^{2N}\det(\mathbf{C})} \exp(-(\mathbf{x}-\boldsymbol{\mu})^H \mathbf{C}^{-1}(\mathbf{x}-\boldsymbol{\mu})) \quad (2)$$

where the received signals at the two frequencies are appended to form a long vector,

$$\begin{aligned} &\mathbf{x} = \mathbf{s} + \mathbf{v} = [\mathbf{x_1} \quad \mathbf{x_2}]^T, \mathbf{s} = [\mathbf{s_1} \quad \mathbf{s_2}]^T, \mathbf{v} = [\mathbf{v_1} \quad \mathbf{v_2}]^T \\ &\mathbf{x} = [x_1(0), x_1(1),..x_1(N-1), x_2(0), x_2(1)..x_2(N-1)]^T \\ &\mathbf{s} = [s_1(0), s_1(1),..s_1(N-1), s_2(0), s_2(1)..s_2(N-1)]^T \\ &\mathbf{v} = [v_1(0), v_1(1),..v_1(N-1), v_2(0), v_2(1)..v_2(N-1)]^T \end{aligned} \quad (3)$$

The covariance matrix is Hermitian with the following diagonal structure

$$E\{\mathbf{x}\} = \boldsymbol{\mu} = [\mathbf{s_1} \quad \mathbf{s_2}]^T$$
$$E\{(\mathbf{x}-\boldsymbol{\mu})(\mathbf{x}-\boldsymbol{\mu})^H\} = \mathbf{C} = \begin{bmatrix} \sigma_1^2\mathbf{I} & \mathbf{0}_{N\times N} \\ \mathbf{0}_{N\times N} & \sigma_2^2\mathbf{I} \end{bmatrix} \qquad (4)$$

3. MAXIMUM LIKELIHOOD AND SUBOPTIMAL ESTIMATORS

In this section, we enforce $\mathbf{s} = \mathbf{s}(\boldsymbol{\psi})$, i.e., the noise free returns are completely parameterized by a vector of p parameters, $\boldsymbol{\psi} = [\psi_1, \psi_2, \psi_3 .....\psi_p]^T$. For the problem at hand, the ML estimator of the complete parameter vector, $\boldsymbol{\theta} = [\boldsymbol{\psi}^T, \sigma_1^2, \sigma_2^2]^T$ can be readily derived as

$$\hat{\boldsymbol{\psi}} = \arg\min_{\boldsymbol{\psi}} \ln\left(\prod_{i=1}^{2} \frac{\|\mathbf{x}_i - \mathbf{s}_i(\boldsymbol{\psi})\|^2}{N}\right),$$
$$\hat{\sigma}_i^2 = \frac{\|\mathbf{x}_i - \mathbf{s}_i(\hat{\boldsymbol{\psi}})\|^2}{N}, \forall i = 1,2 \qquad (5)$$

Equation (5) constitutes the ML estimator for the dual frequency radar, and does not have a closed form solution. We, therefore, employ the idea of step-wise concentration. It is noted that when the covariance matrix is known, the ML estimator for $\boldsymbol{\psi}$ takes the form

$$\hat{\boldsymbol{\psi}} = \arg\min_{\boldsymbol{\psi}} (\mathbf{x} - \mathbf{s}(\boldsymbol{\psi}))^H \mathbf{C}^{-1} (\mathbf{x} - \mathbf{s}(\boldsymbol{\psi}))$$
$$= \arg\min_{\boldsymbol{\psi}} \left(\frac{\|\mathbf{x}_1 - \mathbf{s}_1(\boldsymbol{\psi})\|^2}{\sigma_1^2} + \frac{\|\mathbf{x}_2 - \mathbf{s}_2(\boldsymbol{\psi})\|^2}{\sigma_2^2} \right) \qquad (6)$$

Equation (6) calls for an iterative solution to eq. (5). The iterative ML algorithm is formulated as follows.

1) Initialize with $\mathbf{C} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix of the same dimension as $\mathbf{C}$.
2) Using eq. (6), obtain the estimates $\hat{\boldsymbol{\psi}}_1$ of $\boldsymbol{\psi}$.
3) Use $\hat{\boldsymbol{\psi}} = \hat{\boldsymbol{\psi}}_1$ in eq. (5) to obtain noise variance estimates, $\hat{\sigma}_{i1}^2$.
4) Construct the estimated covariance matrix
$$\mathbf{C}_1 = Diag\left[\hat{\sigma}_{11}^2 \times \mathbf{1}, \hat{\sigma}_{21}^2 \times \mathbf{1}\right], \quad \mathbf{1} = [1,1,1..1]_{1\times N}$$
5) Recursively solve at the k^{th} iteration,
$$\hat{\boldsymbol{\psi}}_k = \arg\min_{\boldsymbol{\psi}} (\mathbf{x} - s(\boldsymbol{\psi}))^H \mathbf{C}_k^{-1} (\mathbf{x} - s(\boldsymbol{\psi})),$$
$$\text{initialized with } \hat{\boldsymbol{\psi}}_{k-1}$$
$$\hat{\sigma}_{i(k+1)}^2 = \|\mathbf{x}_i - \mathbf{s}_i(\hat{\boldsymbol{\psi}}_k)\|^2 / N, \forall i = 1,2 \qquad (7)$$
$$\mathbf{C}_{k+1} = Diag\left[\hat{\sigma}_{1(k+1)}^2 \times \mathbf{1}, \hat{\sigma}_{2(k+1)}^2 \times \mathbf{1}\right]$$
6) Stop after convergence or when an appropriate stopping criterion is satisfied.

The covariance matrix in step 4 and eq. (7) are not represented as estimates to stress the fact that step-wise concentration is employed. In other words, for every iteration, a quasi-ML objective is optimized. The iterative step-wise algorithm is definitely not new, and has been used in generalized linear models in statistical literature, and is often described as the iterative reweighted least squares (IRLS) [5].

5.1. Micro-Doppler

The MD arises due to vibrations of scatterers on the target or of the target itself, for example, a target undergoing simple harmonic motion. The vibrational MD is characterized by sinusoidal instantaneous frequency, and hence phase, and can, therefore, be parameterized by $\boldsymbol{\psi} = [R_o, d, \omega_o, \phi_o]$, where R_o is the initial range of the target, d is the maximum displacement, ω_o is the vibrational frequency, and ϕ_o is its associated phase. The time-varying range profile for this motion is given by [6]

$$R(n;\boldsymbol{\psi}) = R_o + d\cos(\omega_o n - \phi_o), n = 0,1..N-1 \qquad (8)$$

Substituting eq. (8) in eq. (1), we obtain the signal returns

$$x_i(n) = \rho e^{\left(j\frac{4\pi f_i (R_o + d\cos(\omega_o n - \phi_o))}{c} + j\gamma\right)} + v_i(n), \forall i = 1,2 \quad (9)$$

The returns in eq. (9) represent the classical sinusoidal FM signal. Using these signals in eq. (7) yields the IRLS-ML estimator for the MD motion profile. However, given that the signal vector $\mathbf{s}$ can be decomposed into two terms, one containing R_o and the other being a function of the remaining parameters in $\boldsymbol{\psi}$, we obtain the IRLS-iterations as

$$\boldsymbol{\psi}_k = \arg\min_{\psi_k} (\mathbf{x} - \mathbf{A}_k\mathbf{b}_k)^H \mathbf{C}_k^{-1} (\mathbf{x} - \mathbf{A}_k\mathbf{b}_k), \; k > 1$$
$$\sigma_{i(k+1)}^2 = \left\|\mathbf{x}_i - \hat{\mathbf{a}}_i \hat{b}_{ik}\right\|^2 / N, \forall \text{i} = 1,2$$
$$\mathbf{A}_k = \begin{bmatrix} \mathbf{a}_{1k} & \mathbf{0} \\ \mathbf{0} & \mathbf{a}_{2k} \end{bmatrix}, \mathbf{b}_k = [b_{1k}, b_{2k}]^T, \mathbf{0} = [0,0,..0]_{N\times 1}^T$$
$$z_k(n) := d_k \cos(\omega_{o,k} n - \phi_{o,k}) \qquad (10)$$
$$\mathbf{a}_{ik} := [1, e^{j4\pi f_i z_k(1)/c}, e^{j4\pi f_i z_k(2)/c}, ..e^{j4\pi f_i z_k(N-1)/c}]^T$$
$$b_{ik} := \rho \exp(j4\pi f_i R_o / c + j\gamma), \forall i = 1,2$$

Equation (10) can be simplified further by minimizing with respect to R_o in which case one obtains the well known weighted least squares solution,

$$\hat{\mathbf{b}}_k = (\hat{\mathbf{A}}_k^H \mathbf{C}_k^{-1} \hat{\mathbf{A}})^{-1} \hat{\mathbf{A}}_k^H \mathbf{C}_k^{-1} \mathbf{x}, k > 1$$
$$\hat{R}_{ok} = \frac{\angle(b_{2k} \times b_{1k}^*)}{4\pi(f_2 - f_1)} c, \hat{\rho}_k = \frac{1}{2}\sum_{i=1}^{2} |b_{ik}| \qquad (11)$$
$$\hat{\gamma}_k = \frac{1}{2}\sum_{i=1}^{2} \angle\hat{b}_{ik} \exp(-j4\pi f_i \hat{R}_{ok})$$

The IRLS-ML estimate for the remaining parameters is obtained by minimizing the expression

$$\left((\mathbf{x} - \mathbf{P}_k\mathbf{x})^H \mathbf{C}_k^{-1} (\mathbf{x} - \mathbf{P}_k\mathbf{x})\right)$$
$$\mathbf{P}_k := \mathbf{A}_k (\mathbf{A}_k^H \mathbf{C}_k^{-1} \mathbf{A}_k)^{-1} \mathbf{A}_k^H \mathbf{C}_k^{-1} \qquad (12)$$

Since the projection matrix satisfies $\mathbf{P}_k = \mathbf{P}_k^2 = \mathbf{P}_k^H$, one can simplify eq. (12) to obtain

$$(\hat{d}_k, \hat{\omega}_{o,k}, \hat{\phi}_{o,k}) = \arg\max_{d,\omega_o,\phi_o,k>1} \left(\sum_{i=1}^{2} \left(\frac{\left| \sum_{n=0}^{N-1} x_1(n) e^{(-j4\pi f_1 d \cos(\omega_o n - \phi_o)/c)} \right|^2}{\sigma_{ik}^2} \right) \right) \quad (13)$$

Equations (11) and (13) constitute the IRLS-ML estimates at the k^{th} iteration. The noise variance estimates at the $k+1^{th}$ iteration are then obtained using eq. (10). The estimate of R_o is dependent on the estimates of the other parameters, as evident from eqs. (11)-(13). The cost function is highly non-linear and requires good initial estimates in order to guarantee convergence. This necessitates discussing suboptimal estimators for MD.

We note that the Fourier spectrum of the return, $x_i(n), \forall i = 1,2$ in eq. (9) is not analytic, and consists of infinitely many harmonics weighted by Bessel functions of the first kind

$$X_i(\omega) = \rho e^{(-j4\pi f_i R_o/c + j\gamma)} \times \sum_{m=-\infty}^{m=\infty} j^m J_m(4\pi f_i d/c)\delta(\omega - m\omega_o)e^{-jm\phi_o}, \forall i = 1,2 \quad (14)$$

where $J_m(\cdot)$ is the m^{th} order Bessel function. Since the Bessel functions rapidly decrease in magnitude for increasing m, the Fourier transform in eq. (14) has at the most $m = M$ significant harmonics. With this in mind, we describe below the suboptimal estimation procedure. Since the noise variances are neither required for estimating the parameters suboptimally nor to start the IRLS iterations, their suboptimal estimates are omitted.

To obtain initial estimate of ω_o, we choose $m = K \le M$ peaks of $X_i(\omega), \forall i = 1,2$ and form the vectors,

$$\begin{aligned} \mathbf{y}_i &= [X_i(\omega_{-K}), X_i(\omega_{-K+1}),.0., X_i(\omega_{K-1}), X_i(\omega_{K+1})]^T \\ \boldsymbol{\omega}_i &= [\omega_{-K}, \omega_{-K+1},..0.., \omega_{K-1}, \omega_{K+1}]^T \text{ obtained from } \mathbf{y}_i \end{aligned} \quad (15)$$

In general, $\boldsymbol{\omega}_1 \ne \boldsymbol{\omega}_2$ since the noise in the signal returns is different for each carrier frequency. The suboptimal estimates $\omega_{oi,subopt}, \forall i = 1,2$ for ω_o can be obtained as

$$\hat{\omega}_{oi,subopt} = \frac{\mathbf{K}^T \boldsymbol{\omega}_i}{\left\| \mathbf{K}^T \mathbf{K} \right\|}, \forall i = 1,2, \mathbf{K} := [-K, -K+1,..0,1..K]^T \quad (16)$$

The suboptimal estimate of ρ is provided by employing the higher order statistics based technique of [7].

$$\hat{\rho}_{i,subopt} = \sqrt[4]{2\left(\frac{1}{N}\sum_{n=0}^{N-1}|x_i(n)|^2\right)^2 - \frac{1}{N}\sum_{n=0}^{N-1}|x_i(n)|^4} \quad (17)$$

Assuming the signal returns are wideband, the suboptimal estimate for $d_i, \forall i = 1,2$ is reached using the estimator proposed in [8] as

$$\hat{d}_{i,subopt} = \sqrt{\frac{2(\mathbf{y}_i \circ \mathbf{y}_i^*)^T (\mathbf{K} \circ \mathbf{K})}{\rho_{i,subopt}^2 (\mathbf{y}_i^H \mathbf{y}_i)}}, \forall i = 1,2 \quad (18)$$

where '$\circ$' denotes the Hadamard product. The estimates for R_o and γ are given by

$$\begin{aligned} \hat{R}_{o,subopt} &= \frac{\angle X_2(0) X_1^*(0)}{4\pi(f_2 - f_1)} c, \\ \hat{\gamma}_{subopt} &= \frac{1}{2}\sum_{i=1}^{2} \angle(X_i(0) e^{-\frac{j4\pi f_i \hat{R}_{o,subopt}}{c}} \end{aligned} \quad (19)$$

For ϕ_o, we use the LS estimator proposed in [7] after appropriately demodulating the estimates for R_o and γ.

$$\hat{\phi}_{oi,subopt} = \frac{-\mathbf{K}^T \arg(\mathbf{y}_i e^{-j4\pi f_i \hat{R}_{o,subopt}/c} e^{-j\gamma})}{\|\mathbf{K}\|^2}, \forall i = 1,2 \quad (20)$$

In eq. (20), $\arg(\cdot)$ is the unwrapped phase operator, and operates element-wise on a vector. We note that the suboptimal techniques for vibrational MD rely heavily on the peak picking in the Fourier transform and could suffer considerably for low signal-to-noise ratios (SNRs). This is further discussed in Section 4.

It is important to note that we have two sets of suboptimal estimates for $\omega_o, d,$ and ϕ_o corresponding to the two carrier frequencies, whereas only one suboptimal estimate for R_o. In step 2 of the IRLS, we only need a single suboptimal estimate of ω_o for initialization. In the absence of any *a priori* information on the operating conditions, for example, the SNR at the two carriers, we can simply average the suboptimal estimates to obtain a single value. Likewise for d and ϕ_o. It is further noted that the suboptimal estimates for R_o, ρ, and γ are not required to launch the IRLS, as these parameters are not involved in step 2 maximizations.

4. SIMULATIONS AND EXPERIMENTAL RESULTS

4.1. Simulations

The carrier frequencies are set to $f_1 = 903$ MHz and $f_2 = 921$ MHz. The SNRs at the radar are defined as $SNR_i = \rho^2/\sigma_i^2, \forall i = 1,2.$ We fix SNR_1 =10dB and vary SNR_2 from -10dB onwards, in 5dB increments. For simplicity, the phase parameter $\gamma = 0$ is known, which makes the CRBs of range and velocity, derived in [8], decoupled from those of the other nuisance parameters.

Figures 1(a)-(d) demonstrate the MSE for the IRLS, NLS, and the suboptimal estimation schemes. The number of Monte Carlo trials was 250, with $k_{\max} = 10$. The number of data sample $N = 1024$, and the parameters of the MD signal used were $\boldsymbol{\psi} = [R_o = 1.3, d = 0.07, \omega_o = 0.123\pi, \phi_o = \pi/3]^T$. The signal is wideband for the aforementioned carriers. In the suboptimal estimation scheme, we forced

$m = K = 1$ which makes $\mathbf{K} = [-1,0,1]^T$, in eqs. (16), (18), and (20). This choice of K amounts to using only the first harmonic and DC for estimation. The corresponding CRBs for both single frequency and dual frequency operations are also shown. The estimates are clearly below the dual frequency CRBs. The IRLS offers superior performance than the NLS for all parameters except R_o for which both the NLS and IRLS give identical MSE.

4.2. Experimental results

The data used corresponds to a 12inch conducting sphere tied to the ceiling, and excited to oscillate in a simple harmonic fashion. Only the first seven seconds of data, comprising of 700 samples, is used in order to avoid damped oscillations. The carrier frequencies were 906.3 MHz, and 919.6 MHz. Figure 2 shows the magnified versions of the spectrogram of the data overlaid on the IRLS, NLS, and suboptimal IF trajectories. Clearly, the IRLS yields better estimates compared to the NLS, and agrees with the IF of the data at both carriers.

5. CONCLUSIONS

In this paper, we considered a dual frequency Doppler radar for range estimation of moving targets with application to urban sensing. The ML estimator was derived for the micro-Doppler motion profile. It was shown that the ML estimator, although not solvable in closed form, can be provided using a step-wise iterative algorithm termed as the IRLS. For simulated data, the algorithm was shown to be superior in terms of the MSE when compared to suboptimal estimators. The iterative ML was also applied to real data generated from simple harmonic motion and corresponding to indoor oscillating targets.

6. ACKNOWLEDGMENT

This work was supported by ONR under grant N00014-07-1-0043.

7. REFERENCES

[1] *Proceedings of the 2008 IEEE International Conference on Acoustics, Speech, and Signal Processing*, Special Session on 'Through-the-Wall Radar Imaging,' April 2008.

[2] L. N. Ridenour, *Radar system engineering,* Vol. 1 of the MIT Radiation Laboratory Series, McGraw-Hill, NY, 1947.

[3] M. I. Skolnik, *Introduction to Radar Systems,* third edition, McGraw-Hill, NY, 2002.

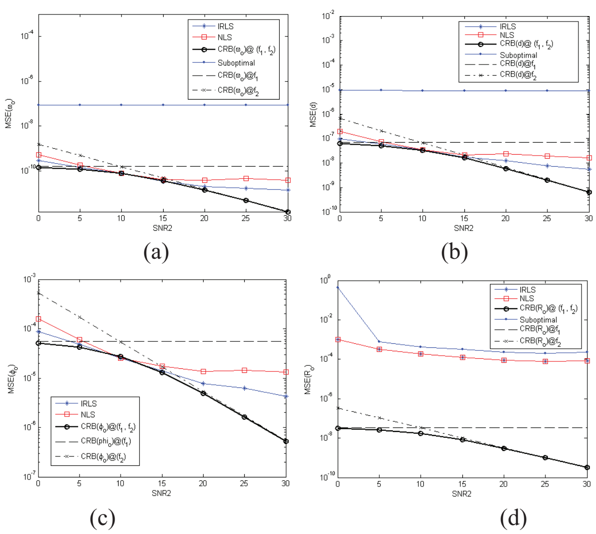
[4] V. C. Chen, F. Li, S. –S Ho, and H. Wechsler, "Analysis of micro-Doppler signatures," *IEE* Proc.*, Radar Sonar and Navig.,* vol. 150, no. 4, pp. 271-276, 2003.

[5] P.J. Green, "Iteratively reweighted least squares for maximum likelihood estimation, and some robust and resistant alternatives," *Journal of Royal Statistical Soc.,* vol. 46, no. 2, pp. 149-192, 1984.

[6] P Setlur, M. Amin, and F. Ahmad, "Cramér-Rao bounds for range and motion parameter estimations using dual frequency radars," *Proc. Int. Conf. Acoust., Speech Signal Process.,* Honolulu, HI, April 2007.

[7] F. Gini, and G. B. Giannakis, "Hybrid FM-polynomial phase signal modeling: parameter estimation and Cramér-Rao bounds," *IEEE Tran. Signal Process.,* vol. 47, no.2, pp.363-377, Feb. 1999.

[8] S.–R. Huang, R. M. Lerner, and K. J. Parker, "On estimating the amplitude of harmonic vibration from the Doppler spectrum of reflected signals," *J. Acou. Soc. Amer.,* vol. 88, pp.2702-2712, 1990.



(a) (b) (c) (d)

Fig. 1.MSE for MD parameters (a) ω_o, (b) d, (c) ϕ_o, (d) R_o.

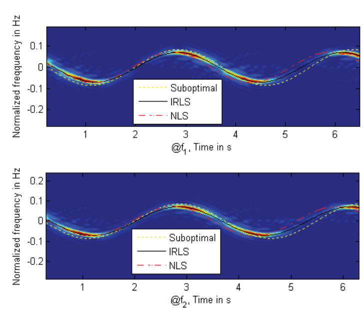


Fig. 2. IRLS, NLS, and suboptimal comparison