# OPTIMAL GENERALIZED DESIGN OF TRANSFORM-BASED BLOCK DIGITAL FILTERS

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**ABSTRACT**

Transform-based block implementation of digital filters is useful for high throughput filtering due to inherent parallelism and complexity reduction provided by using the fast transforms. In basic form, for example the overlap-save implementation, the block digital filter (BDF) is represented by a vector. In this paper, the basic form of block filtering and the optimal design of BDF are described. Therefore, we propose a generalization of the block digital filtering where the BDF is represented by a matrix. This generalized form and its corresponding optimal BDF design are developed. The generalized BDF allows reducing the global distortion of the block filtering.

***Index Terms***— Block digital filtering, filter design, overlap-save, filtering distortion.

## 1. INTRODUCTION

In digital filtering, filters characteristics are specified in the frequency domain in terms of desired amplitude and phase response. Conventional FIR and IIR filters synthesis techniques are found to better meet the specifications of the desired filter. Although the IIR filters are more efficient and require less operations and memory than the FIR filters, the FIR filters are more widely used because they are always stable with linear phase. For implementation on digital signal processors (DSP's), there are three ways:

- Direct method for direct linear convolution (multiply-add accumulate type operation).
- Indirect methods (overlap save/add) for block filtering. They use transforms (discrete Fourier transform : DFT, discrete cosinus transform : DCT,...) and benefit from fast algorithms (fast Fourier transform : FFT, ...).
- Mixed methods where indirect methods and direct linear convolution are mixed in the implementation .

The most appropriate implementation depends on the filters' order. For short-length FIR filters, most commercial DSP chips are optimized for direct implementation of linear convolution. For large-length FIR filters, transform-based block digital filtering is proposed to reduce the computational complexity of digital filtering systems and to increase the parallelism of computation.

In basic form, the block digital filter (BDF) is represented by a vector. The most widely spread methods to design the BDF, consist of constraining the BDF to be time-invariant [1], for example the overlap-save method ([2, p. 558]), and then using the conventional filter synthesis techniques. An optimal BDF design that do not restrict the BDF to be time-invariant has been developed in [3, 4]. Although the aliasing error in overlap-save method is null, the global distortion obtained by the optimal BDF design is lower than the global distortion obtained in overlap-save or other approaches.

In this paper, the basic form of transform-based block digital filtering and the criterion for optimal BDF design are presented. A generalized form and its corresponding optimal BDF design are proposed.

The rest of this paper is organized as follows. In Section II, the basic form of transform-based block digital filtering is presented. In Section III, our generalized BDF form is described. Experimental results, in terms of distortion and filtering cost, for basic and generalized forms, are shown in Section IV. Finally, a conclusion is drawn in Section V.

## 2. BASIC FORM OF TRANSFORM-BASED BLOCK DIGITAL FILTERING

The principle of transform-based block digital filtering in basic form is illustrated in Fig. 1. The input signal is divided into overlapping blocks of $M$ samples. The amount of overlapping is $2d$. Each block is then processed as follows:

- The block is transformed through an $M-$point transform.
- The transformed block is multiplied term by term by the BDF-vector $g$.

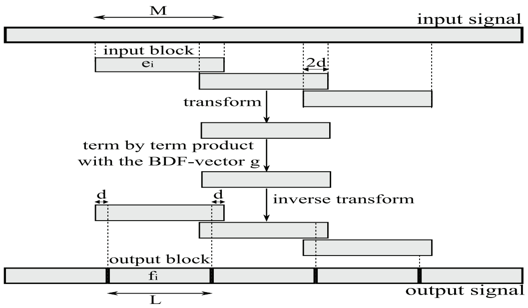


Fig. 1 Transform-based block digital filtering

- An inverse $M-$point transform is performed to obtain the filtered block.
- Only the $L$ central points of the transformed block are kept : $L = M - 2d$. The concatenation of the $L$ points kept in each filtered block forms the filtered signal.

A typical example of that is the time-invariant system of overlap-save implementation. For fixed $M$ and $L$, overlap-save compels the BDF to have an order less than or equal to $P = M - L$ and uses conventional filter synthesis techniques to design the BDF.

### 2.1. Optimal basic BDF design

In practice, the spectrum of the desired filter is always analyzed with a limited frequency resolution $\theta$. Hence, due to the properties of the DFT, the digital input signal can be considered as periodical of period $K = F_e/\theta$, where $F_e$ is the frequency sampling. $K$ is the number of samples considered on the desired frequency response. Therefore, choosing $K$ is equivalent to choosing the frequency resolution $\theta$. We consider that $K = bL$ with $b$ an integer.

In what follows, we will note:

- $e_i$ : the $M-$dimensional vector containing the samples of an input block;
- $f_i$ : the $L-$dimensional vector containing the samples of an output block;
- $\overline{x}$ : the DFT of the $K-$periodical input signal $x$;
- $F_N$ : the $N \times N$ matrix corresponding to an $N-$point DFT and $F_N^{-1}$ as its inverse;
- $S$ : the $L \times M$ selection matrix, a binary matrix which selects $L$ values out of $M$ from the middle;

The desired output signal $y_d$ and its DFT $\overline{y_d}$ are given by

$$\overline{y_d} = \overline{\overline{H_d}}\,\overline{x} \Longleftrightarrow y_d = H_d x \tag{1}$$

$\overline{\overline{H_d}}$ is a diagonal matrix, $\overline{\overline{H_d}}(k,k) = f\,(k)$ where $f\,(k)$ is the desired frequency response. $H_d$ is the $K \times K$ matrix defined by

$$H_d = F_K^{-1}\overline{\overline{H_d}}F_K \tag{2}$$

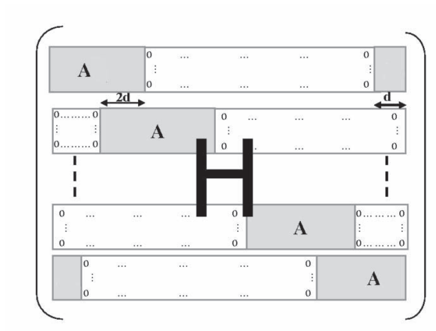


Fig. 2 Matrix $H$

Based on Fig. 1, the input block $e_i$ and the output block $f_i$ are related by

$$f_i = Ae_i \tag{3}$$

with $A$ the $L \times M$ matrix decomposed as follows

$$A = SF_M^{-1}GF_M \tag{4}$$

where $G$ is the diagonal matrix with $diag\,(G) = g$. The global output signal $y$ and its DFT $\overline{y}$ obtained by block filtering will be

$$y = Hx \Longleftrightarrow \overline{y} = \overline{\overline{H}}\,\overline{x} \tag{5}$$

where $H$ is a $K \times K$ matrix containing $b$ copies of matrix $A$ as shown in Fig. 2, and $\overline{\overline{H}}$ is the $K \times K$ matrix defined by

$$\overline{\overline{H}} = F_K H F_K^{-1} \tag{6}$$

Our criterion to design the optimal BDF is to minimize the quadratic error between the desired filtering and the block filtering [3]:

$$e = \left\|\overline{\overline{H}} - \overline{\overline{H_d}}\right\|^2 \tag{7}$$

where $\|.\|^2$ stands for the Frobenius norm. This criterion allows to obtain the optimal BDF which is not necessarily time-invariant. To obtain the optimal BDF for overlap-save implementation, the optimization problem is compelled by the constraint of time-invariant system.

### 2.2. Computational complexity of the filtering

Whatever the BDF is synthesized by the optimal approach or by any other conventional method, and whatever the BDF is compelled to be time-invariant or not, the cost of the block filtering remains the same. The differences are in the BDF design cost and the distortion errors. To approximate the computation complexity of the block filtering process, we have considered that the computation cost is the required number of multiplications. For each block, we need:

- $M-$point FFT for the input block. The required number of multiplications is approximately $Mlog_2M$ [5].

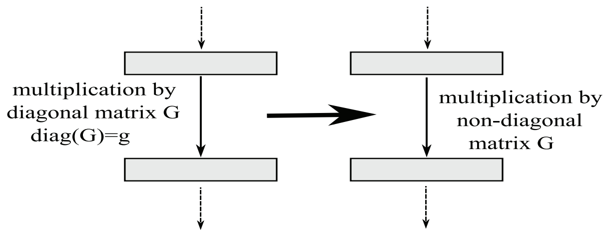


Fig. 3 Generalized transform-based block digital filtering

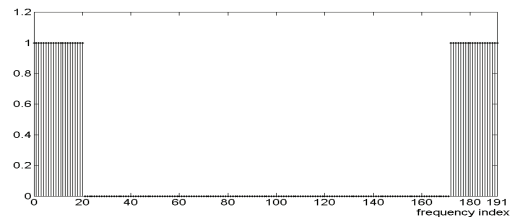


Fig. 4. Desired frequency response

- $M$ term by term product by the elements of the BDF vector.
- $M-$point inverse FFT for the result. This requires approximately $Mlog_2M$ multiplications.

The total filtering cost for each block will be approximately $M\,(2log_2M+1)$.

## 3. GENERALIZED FORM OF TRANSFORM-BASED BLOCK DIGITAL FILTERING

In the basic form of transform-based block digital filtering, the BDF has been characterized by the vector $g$. The term by term product of the input block transform and the vector $g$ is equivalent to multiplying the input block transform by a diagonal matrix $G$, with $diag\,(G) = g$. This is equivalent to representing the BDF by the diagonal matrix $G$. For generalization, we will consider that the BDF is represented by a non constrained matrix $G$ (non-diagonal matrix $G$ ) as shown in Fig. 3. In basic form, the free parameters of the matrix $G$ are only the diagonal elements while other elements are fixed to zero. In our proposed generalized form, all the parameters of the matrix $G$ are free and can be optimized.

### 3.1. Optimal generalized BDF design

To design the optimal generalized BDF, the parameters of the BDF represented by the matrix $G$ are optimized by minimizing the defined quadratic criterion (7). However, increasing the number of free parameters in matrix $G$ will increase the computation complexity of the block filtering.

### 3.2. Computational complexity of the filtering

Let us evaluate the computational complexity of the block filtering while the matrix $G$ has $\alpha$ nonzero parameters in each row. In this case, we always need an $M-$point FFT for the input block and $M-$point inverse FFT for the output block. However, the term by term product is replaced by the matrix-vector multiplication. As each row of the matrix $G$ contains $\alpha$ nonzero parameters, we need $\alpha M$ multiplications instead of $M$ multiplications in the basic form. The total filtering cost for each block will be approximately $M\,(2log_2M+\alpha)$.

**Table 1**. Global errors in basic form

| method | overlap-save | optimal |
|---|---|---|
| time-invariant error | 2.067 | 0.868 |
| aliasing | 0 | 0.212 |
| global distortion | 2.067 | 1.080 |

## 4. EXPERIMENTAL RESULTS

To make visualizations easier, we use relatively small block sizes: $M = 64$ and $L = 48$ with a desired frequency resolution $\theta = F_e/192$, ($F_e$ is the sampling frequency) hence $K = 192$. Fig. 4 shows the desired frequency response $f\,(k)$ $(k : 0 \rightarrow 191)$. The horizontal axis is the frequency index ($k$ represents the normalized frequency $\varpi = 2\pi k/K$). The desired filter is a lowpass real filter with cut-off frequency $F_c = 20F_e/192$. It is equal to 0 between indexes 21 and 171, and 1 elsewhere.

In comparison, the aliasing is defined as the off-diagonal quadratic error between the BDF frequency response matrix $\overline{\overline{H}}$ and the desired frequency response matrix $\overline{\overline{H_d}}$ while the time-invariant error is the diagonal quadratic error.

We compare the overlap-save (BDF-order $P = 16$) and the optimal BDF design in basic form. Table 1 shows the results of distortion errors obtained in these cases.

We can see that tolerating a small amount of aliasing allows to provide a large improvement on time-invariant error. The global distortion is reduced $2.067/1.080 = 1.914$ times. We remember that, in both cases, the cost of filtering is the same.

In generalized form, we will consider that each row of the matrix $G$ contains 3 nonzero parameters as shown below

$$\begin{bmatrix} g_{1,1} & g_{1,2} & 0 & \cdots & \cdots & 0 & g_{1,64} \\ g_{2,1} & g_{2,2} & g_{2,3} & 0 & \cdots & \cdots & 0 \\ 0 & g_{1,1} & g_{1,1} & g_{1,1} & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & g_{62,61} & g_{62,62} & g_{62,63} & 0 \\ 0 & \cdots & \cdots & 0 & g_{63,62} & g_{63,63} & g_{63,64} \\ g_{64,1} & 0 & \cdots & \cdots & 0 & g_{64,63} & g_{64,64} \end{bmatrix}$$

Optimal BDF in this particular generalized form is compared to the optimal BDF in basic form. Table 2 shows the results. The new generalized form (non-diagonal matrix $G$) provides a large improvement on time-invariant error while the aliasing

**Table 2**. Global errors for optimal BDF design

| design | basic | generalized |
|---|---|---|
| time-invariant error | 0.868 | 0.560 |
| aliasing | 0.212 | 0.284 |
| total | 1.080 | 0.844 |

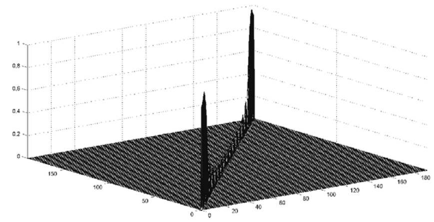

Fig. 5 Three-dimensional view of matrix $\overline{\overline{H}}$ for overlap-save approach

is slightly larger. We can see that the global distortion error is reduced.

Considering $\alpha$ parameters instead of one diagonal parameter in each row (in the previous example, $\alpha = 3$) increases the computational cost of the filtering by a factor of

$$r = \frac{2log_2M + \alpha}{2log_2M + 1} \quad (8)$$

Higher is $\alpha$, higher is the factor $r$ but lower is the distortion error. Then, a compromise between the distortion and the filtering complexity must be taken into account. Moreover, we note that the complexity of transform-based block filtering must not exceed the complexity of direct filtering or block filtering without the use of transforms. To obtain $L$ samples in the output signal, we need $LK$ multiplications in direct filtering and $LM$ multiplications in block filtering without the use of transforms. Then, by increasing $\alpha$, the interest of transform-based block filtering remains as soon as the following relationship exists

$$M\left(2log_2M + \alpha\right) \leq LM \Longleftrightarrow \alpha \leq L - 2log_2M \quad (9)$$

For applications where $\alpha \ll 2log_2M$, the distortion error can be reduced while the factor $r$ tends towards 1 ($r \rightarrow 1$). For example, for applications with $M = 1024$, considering two free parameters instead of one diagonal parameter in each row increases the computational complexity of filtering by a factor $r = 22/21 = 1.05$ only.

For illustration, Figures 5, 6 and 7 show the matrix $\overline{\overline{H}}$ (the frequency response matrix of the BDF) in the three cases: overlap-save, optimal design in basic form, and our generalized optimal design. As expected, overlap-save method provides a diagonal matrix (hence, there is no aliasing), while, in both basic and generalized forms, the optimal design provides a non-diagonal one (hence,aliasing is present).

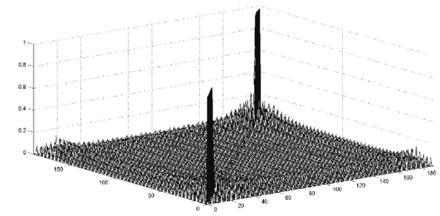

Fig. 6 Three-dimensional view of matrix $\overline{\overline{H}}$ for basic optimal BDF design

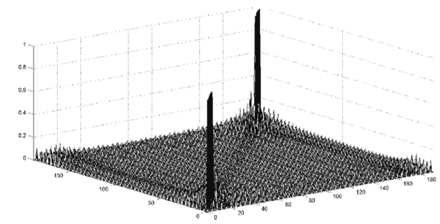

Fig. 7 Three-dimensional view of matrix $\overline{\overline{H}}$ for generalized optimal BDF design

## 5. CONCLUSION

In this paper, we have described the principle of transform-based block digital filtering in the basic form. We have compared the overlap-save approach with an optimal approach to design the block digital filter (BDF). We have proposed a new generalized form for the transform-based block digital filtering. We have shown that, for fixed transform size $M$, the new generalized form allows reducing the distortion error at the cost of only a small increase in the computational complexity of the filtering process.

## 6. REFERENCES


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