# CUBIC HIGHER-ORDER CRITERION AND ALGORITHM FOR BLIND EXTRACTION OF A SOURCE SIGNAL

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## ABSTRACT

The paper deals with the problem of the blind extraction of a source signal after a MIMO convolutive mixture. The extraction is performed using a MISO equalizer. A contrast function based on high order statistics is first proposed. It is more general than the existing contrast in the same context and exhibits a cubic dependence w.r.t. the unknown equalizer parameters. This allows us to propose a new extraction algorithm based on a third order tensor decomposition. Computer simulations illustrate the good behavior and the usefulness of our algorithm.

***Index Terms*****—** Contrast Functions, Blind Source Extraction, Higher Order Statistics, Tensor Decomposition

## 1. INTRODUCTION

We consider the blind source equalization in a MIMO context. In this case non observable source signals are mixed through an unknown multidimensional convolutive channel. The goal of extraction consists of recovering one source signal. This is referred to as MISO equalization.

A rather classical way to solve the equalization problem consists of optimizing criteria based on high order statistics, see e.g. [1, 2, 3, 4]. One (potential) drawback in considering high order statistics is the high order dependence on the parameters involved in the criteria. Generally, this leads to "time consuming" optimization schemes. Recently solutions were proposed that still consider high order statistics but exhibit a quadratic dependence [5, 6]. It is realized through the use of so-called reference signals. However it was made possible by imposing constraints on the criterion that becomes less general.

In this paper, first we propose a new criterion based on high order statistics but showing a cubic dependence on parameters. The advantage of the proposed "cubic" criterion is twofold. It does not require additional contraints as "quadratic" criterion developped in [5] and shows a low order (lower than the considered cumulant order) parameters dependence. Then, we establish a link between the proposed criterion and the problem of the best rank one approximation of third order tensors. Finally, this allows us to suggest a useful algorithm based on tensor decompositions.

## 2. MODEL AND PROBLEM FORMULATION

We consider the following noise free convolutive mixing model

$$\mathbf{x}(n) = \sum_{k\in\mathbb{Z}} \mathbf{M}(k)\mathbf{s}(n-k) \triangleq \{\mathbf{M}(z)\}\mathbf{s}(n). \qquad (1)$$

where $\mathbf{x}(n)$ is the $(N,1)$ observation vector ($N \in \mathbb{N}, N \geqslant 2$), $\mathbf{s}(n)$ is the $(K,1)$ source vector ($K \in \mathbb{N}^*$), and $\mathbf{M}(n)$ is the $(N,K)$ matrix corresponding to the impulse response of the convolutive mixing system, whose transfer function is denoted by $\mathbf{M}(z) = \sum_{n\in\mathbb{Z}} \mathbf{M}(n)z^{-n}$. Moreover $n$ stands for any generic integer ($n \in \mathbb{Z}$) representing the discrete time index. For simplicity, the notation $\{\mathbf{M}(z)\}$ stands for the multivariate filtering operator.

Since the vector of source signals is assumed unknown and unobservable, the goal of blind extraction is to estimate one source from the only use of the observations. To achieve this aim, the following assumptions are required about the mixing system:

- A0. The LTI mixing system is stable i.e. for all $(i,j)$, the $(i,j)$-th element $M_{ij}(n)$ of the matrix $\mathbf{M}(n)$ forms an absolutely summable sequence: $\sum_{n\in\mathbb{Z}} |M_{ij}(n)| < \infty$ $\forall (i,j)$.
- A0'. The LTI mixing system is left invertible, that is there exists a stable LTI system $\{\mathbf{W}(z)\}$ such that the global LTI system with impulse response $\mathbf{G}(n) \triangleq \sum_{k\in\mathbb{Z}} \mathbf{W}(k)\mathbf{M}(n-k)$ corresponds to an identity LTI multichannel system.
- A0". The polynomial matrix z-transform $\mathbf{M}(z)$ is irreducible.

One should notice that A0' and A0" can be satisfied only if there are strictly more observed signals than sources ($N > K$). In the paper, we only consider the real case for clarity.

We consider the problem of the extraction of a single source signal. Notice that in this context, the extraction of all sources can be realized through the use of a deflation procedure [7, 1]. In this MISO context, the aim is to estimate one row of the separating matrix $\mathbf{W}(z)$, that is a $(1, N)$ vector filter $\{\mathbf{w}(z)\}$, called an equalizer, with impulse response $\mathbf{w}(n)$, such that the scalar signal

$$y(n) = \sum_{k\in\mathbb{Z}} \mathbf{w}(k)\mathbf{x}(n-k) \tag{2}$$

restores one of the components $s_i(n)$, $i \in \{1, \dots, K\}$, of the source vector. In this context it is classical to define the corresponding $(1, K)$ global vector filter $\{\mathbf{g}(z)\}$ by its impulse response $\mathbf{g}(n) \triangleq \sum_{k\in\mathbb{Z}} \mathbf{w}(k)\mathbf{M}(n-k)$. Hence we have

$$y(n) = \sum_{k\in\mathbb{Z}} \mathbf{g}(n-k)\mathbf{s}(k) \triangleq \{\mathbf{g}(z)\}\mathbf{s}(n)\ . \tag{3}$$

To be able to carry out the estimation, assumptions about the source signals are also required. In this paper, we assume that

A1. The source signals $s_i(n)$, $i \in \{1, \dots, K\}$ are zero-mean, unit-norm, i.i.d. random signals. Moreover they are statistically mutually independent (at least up to the order of the considered cumulants).

As the source signals are unobservable, there exist some inherent undetermined factors in their estimation. The extraction is done when the global filter reads

$$\exists i_0 \in \{1, \dots, K\}, \exists l \in \mathbb{Z} \quad g_i(n) \triangleq (\mathbf{g}(n))_i = \alpha\delta_{n-l}\delta_{i-i_0} \tag{4}$$

where $\alpha \in \mathbb{R}, \alpha \neq 0$.
The above relation is called the "equalization condition" and expresses the fact that $y(n)$ is equal to one source signal, $s_{i_0}(n-l)$ up to a delay.

Since the sources are unit power, one can restrict the multiplicative factor in (4) to $|\alpha| = 1$ by imposing the constraint $\mathrm{E}\{|y(n)|^2\} = 1$. For i.i.d. sources, this constraint is equivalent to:

$$\sum_{i=1}^{K}\sum_{k\in\mathbb{Z}} |g_i(k)|^2 = 1. \tag{5}$$

It is useful to introduce the following notations. The source signals set satisfying assumptions A1. will be denoted by $\mathcal{S}$. The set of unit norm vector filters will be denoted by $\mathcal{G}_1$ and the subset of filters in $\mathcal{G}_1$ satisfying the equalization condition (4) by $\mathcal{G}_{1e}^{i_0}$. Finally, we denote by $\mathcal{Y}$ the set of the ouput MISO equalizer $y(n)$ when the input source signals belongs to $\mathcal{S}$ and the global system belongs to $\mathcal{G}_1$.

## 3. NEW CONTRAST FUNCTION

We consider the definition of contrasts within the classical context of i.i.d. source signals given in [5].

One of the main goals of the paper is to propose an interesting contrast function which is cubic w.r.t. the searched parameters. For that, we consider the following fourth-order cross-cumulant

$$\kappa_{3,4,\mathbf{z}}\{y(n)\} \triangleq \mathrm{Cum}\{y(n), y(n), y(n), z(n)\} \tag{6}$$

where $z(n)$ is a given reference signal which is assumed to be obtained by a stable filtering of the sources [5], *i.e.* it depends on source signals linearly. For example, under the assumption A0', the reference signal can be chosen as the first observation signal. We now define the following function:

$$\mathcal{C}_{3,4,\mathbf{z}}\{y(n)\} \triangleq |\kappa_{3,4,\mathbf{z}}\{y(n)\}|\ . \tag{7}$$

We also define the following supremum,

$$\kappa_{3,4}^{\max} = \max_{j=1}^{N} \sup_{k\in\mathbb{Z}} |\kappa_{3,4,\mathbf{z}}\{s_j(n-k)\}|. \tag{8}$$

which is assumed to satisfy

A2. $\exists(j_0, k_0)$ such that:

$$\kappa_{3,4}^{\max} = |\kappa_{3,4,\mathbf{z}}\{s_{j_0}(n-k_0)\}| < \infty. \tag{9}$$

We can now propose the following proposition:

**Proposition 3.1.** *In the case of i.i.d. source signals, under assumption A2., the function $\mathcal{C}_{3,4,\mathbf{z}}$ is a contrast over the set $\mathcal{G}_1$.*

*Proof. (Sketch of)* Using the multilinearity property of cumulants and the i.i.d. assumption of source signals, we have

$$\kappa_{3,4,\mathbf{z}}\{y(n)\} = \sum_{j,k} g_j^3(k)\kappa_{3,4,\mathbf{z}}\{s_j(n-k)\}\ . \tag{10}$$

Hence

$$\mathcal{C}_{3,4,\mathbf{z}}\{y(n)\} = \left|\sum_{j,k} g_j^3(k)\kappa_{3,4,\mathbf{z}}\{s_j(n-k)\}\right|\ , \tag{11}$$

and thus

$$\mathcal{C}_{3,4,\mathbf{z}}\{y(n)\} \leqslant \sum_{j,k} |g_j(k)|^3\, |\kappa_{3,4,\mathbf{z}}\{s_j(n-k)\}|\ . \tag{12}$$

As $y(n)$ is unit power, we have $\sum_{j,k} |g_j(k)|^2 = 1$. Hence $|g_j(k)|^2 \leqslant 1$ and thus $|g_j(k)|^3 \leqslant |g_j(k)|^2$ for all j and k. Using this result, we have

$$\mathcal{C}_{3,4,\mathbf{z}}\{y(n)\} \leqslant \sum_{j,k} |g_j(k)|^2\, |\kappa_{3,4,\mathbf{z}}\{s_j(n-k)\}|\ . \tag{13}$$

Now in considering $\kappa_{3,4}^{\max}$ defined in (8), we have

$$\mathcal{C}_{3,4,\mathbf{z}}\{y(n)\} \leqslant \kappa_{3,4}^{\max} \sum_{j,k} |g_j(k)|^2 = \kappa_{3,4}^{\max} . \tag{14}$$

Finally considering the equality, we have $|g_j(k)|^3 = |g_j(k)|^2$ only if it exists a given couple $(j_0, k_0)$ such that

$$\begin{cases} g_{j_0}(k_0) & = 1 \\ g_j(k) & = 0 \ \forall (j,k) \neq (j_0, k_0) . \end{cases} \tag{15}$$

That corresponds to the equalization condition. □

## 4. NEW ALGORITHM

We present an algorithm for the optimization of the proposed contrast function. We assume that the mixing filter admits a MIMO-FIR left inverse filter of length $D$, which is causal because of the delay ambiguity. The row vectors which define the impulse response can be stacked in the following $(1, QD)$ row vector:

$$\underline{\mathbf{w}} \triangleq (\mathbf{w}(0) \ldots \mathbf{w}(D-1)). \tag{16}$$

We also define the $(QD, 1)$ column vector

$$\underline{\mathbf{x}}(n) \triangleq (\mathbf{x}(n)^T \mathbf{x}(n-1)^T \ldots \mathbf{x}(n-D-1)^T)^T . \tag{17}$$

It is then easily seen that

$$y(n) = \underline{\mathbf{w}}\,\underline{\mathbf{x}}(n). \tag{18}$$

In considering the covariance matrix $\mathbf{R} = \mathrm{E}\{\underline{\mathbf{x}}(n)\underline{\mathbf{x}}(n)^T\}$, we have $\mathrm{E}\{|y(n)|^2\} = \underline{\mathbf{w}}\mathbf{R}\underline{\mathbf{w}}^T$.

Now using the multilinearity property of cumulants and (18), we have

$$\kappa_{3,4,\mathbf{z}}\{y(n)\} = \sum_{i,j,k} \underline{w}_i \underline{w}_j \underline{w}_k \mathrm{Cum}\{\underline{x}_i(n), \underline{x}_j(n), \underline{x}_k(n), z(n)\}. \tag{19}$$

Thus this relation can be written as a third order tensor decomposition ([8])

$$\kappa_{3,4,\mathbf{z}}\{y(n)\} = \boldsymbol{\mathcal{C}} \times_1 \underline{\mathbf{w}} \times_2 \underline{\mathbf{w}} \times_3 \underline{\mathbf{w}} \tag{20}$$

where the tensor $\boldsymbol{\mathcal{C}}$ is defined component wise as

$$(\boldsymbol{\mathcal{C}})_{i,j,k} = \mathrm{Cum}\{\underline{x}_i(n), \underline{x}_j(n), \underline{x}_k(n), z(n)\} . \tag{21}$$

Hence the optimization of the contrast function in (7) under the unit power constraint reads

$$\max |\boldsymbol{\mathcal{C}} \times_1 \underline{\mathbf{w}} \times_2 \underline{\mathbf{w}} \times_3 \underline{\mathbf{w}}| \quad \text{with} \quad \underline{\mathbf{w}}\mathbf{R}\underline{\mathbf{w}}^T = 1 . \tag{22}$$

It is important to notice that for any row vector such that $\underline{\mathbf{w}}_0^T \in \ker \mathbf{R}$ we have $\underline{\mathbf{w}}_0 \mathbf{R} \underline{\mathbf{w}}_0^T = \mathrm{E}\{\underline{\mathbf{w}}_0 \mathbf{x}(n)\mathbf{x}(n)^T \underline{\mathbf{w}}_0^T\} = 0$ and hence the signal $\underline{\mathbf{w}}_0 \mathbf{x}(n)$ vanished identically. It follows that we may impose in addition $\underline{\mathbf{w}}^T \in (\ker \mathbf{R})^\perp$ to the optimization problem given by (22). By projection onto the signal subspace, we obtain $\tilde{\boldsymbol{\mathcal{C}}}$ and $\tilde{\mathbf{w}}$ and finally reduces the problem to the following one:

$$\max |\tilde{\boldsymbol{\mathcal{C}}} \times_1 \tilde{\mathbf{w}} \times_2 \tilde{\mathbf{w}} \times_3 \tilde{\mathbf{w}}| \quad \text{with} \quad \tilde{\mathbf{w}}\tilde{\mathbf{w}}^T = 1. \tag{23}$$

A way to realize the above maximization consists in searching for the best rank-1 approximation of tensor $\tilde{\boldsymbol{\mathcal{C}}}$. This can be done in using the algorithm proposed in [8] that can be seen as a third order extension of the power method for best rank-1 approximation of matrix.

## 5. SIMULATION RESULTS

We now propose computer simulations to illustrate the usefulness of the algorithm. We consider real-valued binary source signals (they take their values in {-1,1 } with equal probabilities) and real-valued mixing systems. We compare the cubic and the quadratic higher-order criteria [5]. All the results presented below ensue from Monte-Carlo simulations involving 100 realizations. At each run, the mixing system and the $N_e$ sources samples have been drawn randomly (according to a normal distribution). The quality of extraction is measured thanks to a performance index derived from [9] and defined by:

$$\mathrm{ind}(\mathbf{g}) \triangleq \sum_{i \in \{1,\ldots,N\}} \left( \frac{\sum_{k\in\mathbb{Z}} |g_i(k)|^2}{\max_{i \in \{1,\ldots,N\}} \sum_{k\in\mathbb{Z}} |g_i(k)|^2} \right) - 1. \tag{24}$$

We consider here a mixture of $K = 3$ source signals. The length of the mixing filter is $L = 3$ and the number of observations is $N = 6$. In Figure 1, we give the average performance index versus the number of sources samples $N_e$ for quadratic and cubic criteria. The validity of the method is confirmed and the cubic criteria have better results than the quadratic one for the number of samples tested. In presence of additive white gaussian noise (figure 2), the performances are quite similar for both methods, but cubic criterion performs better above 6 dB.

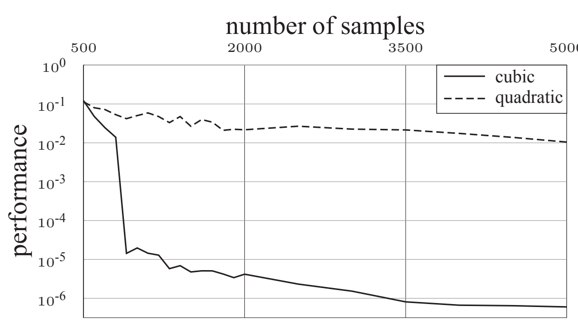


**Fig. 1**. Performance versus number of samples without noise.

We consider the case of $K = 2$ source signals, $N = 5$ obervation signals and a mixing filter of length $L = 3$. To

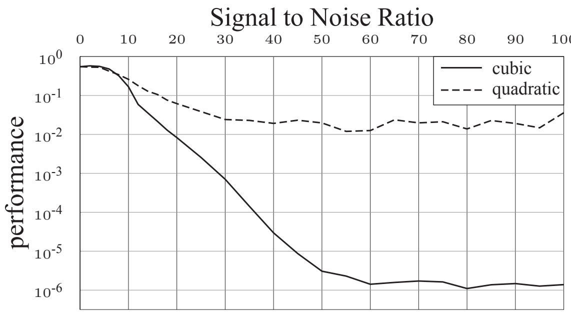


**Fig. 2**. Performance versus SNR for $N_e = 3000$.

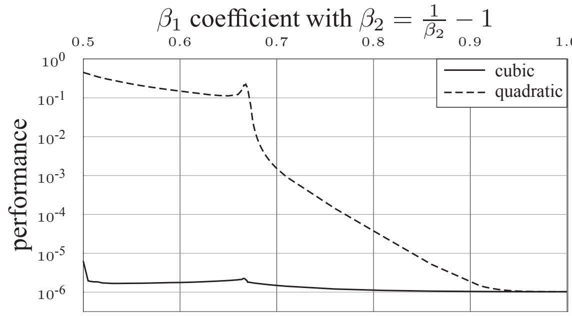


**Fig. 4**. Performance versus coefficient $\beta_1$ for three sources.

evaluate the influence of the reference signal, we choose it as $z(n) = \beta s_1(n) + (1 - \beta)s_2(n)$ with $\beta \in [0, 1]$.
In Figure 3, for $N_e = 3000$, the index performance is plotted versus $\beta$ for cubic and quadratic criteria. Better results are obtained for values of $\beta$ near 0 or 1, when the reference signal is closer to one of the sources. The performance of the quadratic criteria is still very sensible to this point while the cubic's performance remains similar.

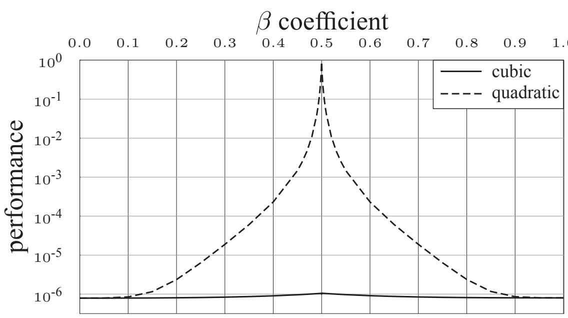


**Fig. 3**. Performance versus coefficient $\beta$ for two sources.

To evaluate the influence of the reference signal with $K = 3$ source signals, $N = 6$ observations and a mixing filter length $L = 3$, we choose it as $z(n) = \beta_1\beta_2 s_1(n) + \beta_1(1 - \beta_2)s_2(n) + (1 - \beta_1)s_3(n)$ with $\beta_1 \in [0, 1]$ and $\beta_2 \in [0, 1]$. We choose for the sake of simplicity to consider the "worst" case $\beta_2 = \frac{1}{\beta_1} - 1$. In Figure 4, for $N_e = 3000$, the index performance is plotted versus $\beta_1$ for cubic and quadratic criteria. As expected the change of reference signal have almost no influence on performances of the cubic criteria whereas the opposite holds for the quadratic one.

## 6. CONCLUSION

We have proposed a new contrast function and a new algorithm for the blind extraction problem. They realize a good compromise between generality and implementation simplicity. Computer simulations illustrate interesting features and performances in comparison with a quadratic algorithm.

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