# ROBUST MOBILE TERMINAL TRACKING IN NLOS ENVIRONMENTS USING INTERACTING MULTIPLE MODEL ALGORITHM

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## **ABSTRACT**

An extended Kalman filter-based interacting multiple model algorithm (IMM-EKF) is proposed for mobile terminal tracking in cellular networks based on time of arrival estimates. The proposed IMM-EKF is able to cope with line-of-sight (LOS) and non-line-of-sight (NLOS) conditions modeled by a Markov chain, where the LOS and NLOS errors are described by different noise models. Road-constraints are included into the IMM-EKF to improve performance. Simulation results show that the IMM-EKF outperforms conventional methods. A comparison to the posterior Cramér-Rao lower bound is given to demonstrate the effectiveness of the IMM-EKF.

*Index Terms*— Interacting multiple model, extended Kalman filter, NLOS mitigation, robust tracking, positioning.

#### 1. INTRODUCTION

Network-based wireless location systems have become an important field for researchers and engineers in recent years. Applications arise in emergency services, fleet management, intelligent transportation systems and others where different positioning techniques can be adopted [1, 2].

Here, we consider tracking a mobile terminal (MT) where the positions of the base stations (BSs) are known and Time-of-Arrival (ToA) estimates between BS and MT are used to calculate the ranges and positions of the MT. The Federal Communications Commission (FCC) standard requires cellular network providers to achieve a certain position accuracy for mobile centric solutions [1] which can be achieved if a line-of-sight (LOS) channel between the MT and the BS is available. However, in urban areas, multiple reflections at buildings and other obstacles prevent the signal to arrive at the MT via the direct path which leads to biased ToA estimates. This bias in time due to non-line-of-sight (NLOS) propagation can severely effect the position estimates of tracking algorithms such as Kalman filters (KF). Thus, methods to cope with NLOS impairments are needed [3, 4].

In order to model NLOS and LOS, we distinguish between a noise model for LOS channels and a noise model with a pos-

itive bias for NLOS channels. One way to deal with different measurement noise models is to use the interacting multiple model (IMM) algorithm that is a powerful tool for target tracking in the context of different noise or motion models with a reasonable cost of computational complexity [5]. The IMM is a hybrid filter that distinguishes among a fixed number of discrete modes. In each mode, the continuous state vector is estimated and combined with the estimated state vectors of the other modes yielding the final state estimate. In [6], the IMM has been applied in the context of NLOS mitigation where one IMM with two KF at each BS is used to distinguish between the two noise models. The KF estimate in the NLOS mode, where the bias of the ToA estimates is subtracted, is combined with the KF estimate in LOS mode in order to determine the ranges between the MT and the BSs, irrespective which BS is in LOS or NLOS mode. The final

position is obtained from the smoothed range estimates by

using a geometric method [6].

However, one can do better if the position of the MT is estimated directly with one IMM using a bank of extended Kalman filters (EKFs) that deal with the non-linear relationship between the measured ranges and the positions. This approach, that outperforms the algorithm in [6] but suffers from higher computational complexity, is followed here, where the LOS/NLOS occurrences are modeled as a two-state Markov chain. We assume a fixed number of N BSs where each BS can either be in LOS or NLOS mode. Hence, we obtain  $2^N$  possible modes which makes our method predestined for small numbers of BSs to keep complexity low. The EKFs used in each mode substract the bias from the ToA estimates and calculate the state vector, consisting of position and velocity. This estimate is weighted with the probabilities of each mode to yield the final state estimate. Road constraints [7] are incorporated in the proposed IMM-EKF to improve performance.

## 2. PROBLEM STATEMENT

We assume that the states to be estimated are the two-dimensional position and velocity of the MT, i.e.,  $\mathbf{x} =$ 

 $[x_{MS}, \dot{x}_{MS}, y_{MS}, \dot{y}_{MS}]^{\mathsf{T}}$ . The evolution of the state vector is assumed to follow a nearly constant velocity (CV) model  $\mathbf{x}(k+1) = \mathbf{F}\mathbf{x}(k) + \mathbf{\Gamma}\mathbf{v}(k)$  [1], where

$$\mathbf{F} = \mathbf{I}_2 \otimes \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix}, \quad \mathbf{\Gamma} = \mathbf{I}_2 \otimes \begin{bmatrix} T_s^2/2 \\ T_s \end{bmatrix}, \quad (1)$$

 $\mathbf{I}_2$  is the identity matrix of size  $2, \otimes$  is the Kronecker product,  $T_s$  is the sampling time,  $\mathbf{v}(k) = [v_x, v_y]^\mathsf{T}$  is process driving noise with covariance matrix  $\mathbf{Q}(k) = \mathrm{diag}(\sigma_{a_x}^2, \sigma_{a_y}^2)$ , where  $\sigma_{a_x}^2$  and  $\sigma_{a_y}^2$  are the noise variances in x- and y-direction, respectively. Let  $\mathbf{z}(k)$  denote the vector of ToA estimates multiplied by the speed of light. Then, the measurement equation is [1]

$$\mathbf{z}(k) = \mathbf{h}(\mathbf{x}(k)) + \mathbf{w}(k), \tag{2}$$

where  $\mathbf{h}(\mathbf{x}(k)) = [h_1(\mathbf{x}(k)), \cdots, h_N(\mathbf{x}(k))]^\mathsf{T}$ , and  $h_n(\mathbf{x}(k))$  is the Euclidean distance between the MS and the n-th BS at time k. The random variable (rv)  $\mathbf{w}(k)$  describes measurement noise which can either be a zero-mean Gaussian rv modeling sensor noise in case of a LOS channel or a positive mean-shift Gaussian rv describing sensor noise and the bias which occurs due to NLOS propagation. Since we do not know when NLOS propagation occurs, subtracting the bias becomes intractable and using the EKF on the contaminated measurements leads to high position errors.

## 3. ROBUST IMM-EKF

## 3.1. Mode-dependent measurement noise

In order to cope with the changing LOS/NLOS conditions at the BSs, we introduce a mode-dependent measurement noise vector  $\mathbf{w}(k, M(k))$ , where M(k) denotes a discrete mode variable in effect at time k that models whether the BSs are in a LOS or NLOS condition. If N BSs are available, we have  $r = 2^N$  possible modes. The mode M(k) at time k is assumed to be among the r possible modes  $M(k) \in \{M_j\}_{j=1}^r$ . For example, if N=3, we obtain  $M_1=1,...,M_8=8$ , and the corresponding events {LOS, LOS, LOS}, ..., {NLOS, NLOS, NLOS}. The modedependent measurement noise vector is assumed to be Gaussian distributed with multivariate probability density function (pdf)  $p(\mathbf{w}(k, M(k))) = \mathcal{N}(\mathbf{w}(k); \mathbf{b}_i(k), \mathbf{R}_i(k))$ , where  $\mathbf{b}_{i}(k)$  and  $\mathbf{R}_{i}(k)$  denote the mean vector and covariance matrix of the Gaussian pdf for mode  $M_i$ . It is assumed that the mode dependent covariance matrix is given by  $\mathbf{R}_i(k) = \operatorname{diag}(\sigma_1^2, \cdots, \sigma_N^2), \text{ where each element } \sigma_n^2 \text{ can}$ be defined as

$$\sigma_n^2 = \begin{cases} \sigma_{LOS}^2 & \text{if } n\text{-th BS is in LOS} \\ \sigma_{LOS}^2 + \sigma_{NLOS}^2 & \text{if } n\text{-th BS is in NLOS,} \end{cases}$$
(3)

where  $\sigma_{LOS}^2$  and  $\sigma_{NLOS}^2$  denote the variances of the noise in LOS and NLOS situations, respectively. Equivalently, the

bias vector is given by  $\mathbf{b}_j(k) = [b_1, \dots, b_N]^\mathsf{T}$ , where each element  $b_n$  can be defined as

$$b_n = \begin{cases} 0 & \text{if } n\text{-th BS is in LOS} \\ m_{NLOS} & \text{if } n\text{-th BS is in NLOS,} \end{cases}$$
 (4)

where  $m_{NLOS}$  denotes a positive NLOS range offset. In order to account for the unknown transitions between LOS and NLOS conditions at each BS, the discrete mode variable M(k) is modeled by a time-homogeneous  $2^N$ -state first-order Markov chain with transition matrix  $\Pi$  and corresponding transition probabilities

$$\pi_{ij} = P\{M(k) = M_j | M(k-1) = M_i\} \quad (i, j \in \mathcal{S}), \quad (5)$$

where  $\mathcal{S} = \{1,...,2^N\}$ .  $\Pi$  is obtained, assuming that the two-state Markov chains with transition matrices  $\Pi_n$  modeling the LOS/NLOS transition at each BS are independent. Thus, we have

$$\Pi = \Pi_1 \otimes \Pi_2 \otimes \cdots \otimes \Pi_N. \tag{6}$$

As in [6], we assume  $\sigma_{LOS}^2$ ,  $\sigma_{NLOS}^2$  and  $m_{NLOS}$  to be known. If we knew M(k), we could substract the corresponding bias from the measurements and use an EKF for estimating the  $\mathbf{x}(k)$ . In practice, however, M(k) is not known and has to be estimated together with  $\mathbf{x}(k)$  using a hybrid estimator.

#### 3.2. MT Tracking Algorithm

The MT tracking problem is solved using an IMM estimator, which is a suitable tool for hybrid estimation and excellently trades off performance versus computational complexity [5]. In Fig. 1 and Table 1, a single cycle of the IMM algorithm is shown. In each cycle, the IMM algorithm consists of three major steps: interaction, filtering and combination.

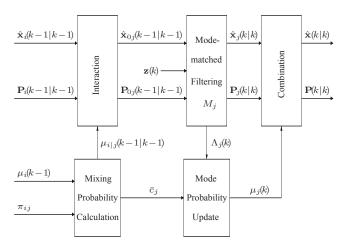


Fig. 1: One cycle of the IMM algorithm

Step 1: Interaction. In order to obtain the initial conditions  $\hat{\mathbf{x}}_{0j}(k-1|k-1)$  and  $\mathbf{P}_{0j}(k-1|k-1)$  for the mode-matched filters, the state vectors  $\hat{\mathbf{x}}_i(k-1|k-1)$  and covariance matrices  $\mathbf{P}_i(k-1|k-1)$  of all mode-matched filters of the previous

time step are weighted with their corresponding mixing probabilities  $\mu_{i|j}(k-1|k-1)$ .

Step 2: Mode-matched filtering. For each possible mode  $M_j$  a filtering step is performed in parallel. Due to the nonlinear relationship between the measurements and the state, cf. (2), EKFs are used here. Note that the mode-matched EKFs only differ in the evaluation of the residual error vector  $\mathbf{e}_j(k)$ , cf. (7) in Table 1, where the mode-dependent bias  $\mathbf{b}_j(k)$  is subtracted, and the residual covariance matrix  $\mathbf{S}_j(k)$ , cf. (8), where the mode-dependent measurement covariance matrix  $\mathbf{R}_j(k)$  is incorporated. In addition, each mode-matched EKF computes the likelihood  $\Lambda_j(k)$ , which indicates the probability of being in the mode  $M_j$  at the current time step.

Step 3: Combination. The state and covariance estimates  $\hat{\mathbf{x}}_j(k|k)$  and  $\mathbf{P}_j(k|k)$  at the output of the mode-matched filters are weighted with their corresponding mode probabilities  $\mu_j(k)$ , yielding the final state and covariance estimates  $\hat{\mathbf{x}}(k|k)$  and  $\mathbf{P}(k|k)$ .

## 3.3. Incorporation of road constraints

In many situations, additional information such as road maps are available. This a-priori knowledge can be used to restrict the movement of the MT to roads. Since each EKF of the proposed IMM algorithm estimates the state vector containting the MT  $x_{MS}$ - and  $y_{MS}$ -position, road-constraints on the MT position can be easily incorporated into the EKF, e.g., as additional pseudomeasurements [7].

#### 4. SIMULATION RESULTS

In the following, it is assumed that N=3 BSs are available with positions at  $(-3 \,\mathrm{km}, -2 \,\mathrm{km})$ ,  $(3 \,\mathrm{km}, 5 \,\mathrm{km})$ ,  $(6 \,\mathrm{km}, 2 \,\mathrm{km})$ . The MT is moving with a constant velocity of v = 70 km/h on a straight line, starting at (0 km, 0 km) and ending at approximately (2.75 km, 2.75 km). The sample length is 1000 and the sampling time is  $T_s = 0.2$  s. The LOS and NLOS ToA measurements are generated according to (2) with  $\sigma_{LOS} = 150$  m,  $\sigma_{NLOS} = 409 \,\mathrm{m}$  and  $m_{NLOS} = 513 \,\mathrm{m}$  [6]. The LOS/NLOS transitions at each BS are modeled by a Markov chain with transition matrix  $\Pi_1 = \Pi_2 = \Pi_3 = I_2 \cdot 0.995 + (I_2 - I_2) \cdot 0.005$ , where  $\mathbf{1}_2$  denotes a matrix of size 2 whose entries are all 1. In the following, the performance of five different estimators are compared: the standard EKF and road-constrained EKF (C-EKF) assuming LOS [7], the IMM-KF suggested in [6] and the proposed IMM-EKF without and with roadconstraints (IMM-C-EKF). All filters are initialized with the same state vector and covariance matrix. The initial MT position is obtained from the three ToA estimates at time k = 0using a least-squares approach [2]. The initial MT velocities are set to zero. The initial state covariance matrix is set to  $P(0) = diag((600m)^2, (600m)^2, (30m/s)^2, (30m/s)^2)$  and the variance of the driving noise is assumed to be  $\sigma_{a_x} = \sigma_{a_y} =$ 1 m/s<sup>2</sup>. For the IMM-based estimators the transition matrix,

1. Mixing Probability Calculation 
$$(i, j = 1, ..., 2^N)$$

$$\mu_{i|j}(k-1|k-1) = (1/\bar{c}_j)\pi_{ij}\mu_i(k-1)$$

$$\bar{c}_j = \sum_i \pi_{ij}\mu_i(k-1)$$
2. Interaction  $(j = 1, ..., 2^N)$ 

$$\hat{\mathbf{x}}_{0j}(k-1|k-1) = \sum_i \hat{\mathbf{x}}_i(k-1|k-1)\mu_{i|j}(k-1|k-1)$$

$$\hat{\mathbf{x}}_{ij}(k-1|k-1) = \hat{\mathbf{x}}_i(k-1|k-1) - \hat{\mathbf{x}}_{0j}(k-1|k-1)$$

$$\mathbf{P}_{0j}(k-1|k-1) = \sum_i \mu_{i|j}(k-1|k-1) \left\{ \mathbf{P}_i(k-1|k-1) + \hat{\mathbf{x}}_{ij}(k-1|k-1) \right\}$$
3. Mode-matched Extended Kalman Filtering  $(j = 1, ..., 2^N)$ 

$$\hat{\mathbf{x}}_j(k|k-1) = \mathbf{F}\hat{\mathbf{x}}_{0j}(k-1|k-1)$$

$$\mathbf{P}_j(k|k-1) = \mathbf{F}\mathbf{P}_{0j}(k-1|k-1)\mathbf{F}^\mathsf{T} + \mathbf{\Gamma}\mathbf{Q}(k)\mathbf{\Gamma}^\mathsf{T}$$

$$\mathbf{H}_j(k) = \frac{\partial \mathbf{h}(\mathbf{x}(\mathbf{k}))}{\partial \mathbf{x}(k)} \Big|_{\hat{\mathbf{x}}_j(k|k-1)}$$

$$\mathbf{e}_j(k) = \mathbf{z}(k) - \mathbf{b}_j(k) - h(\hat{\mathbf{x}}_j(k|k-1)) \qquad (7)$$

$$\mathbf{S}_j(k) = \mathbf{H}_j(k)\mathbf{P}_j(k|k-1)\mathbf{H}_j^\mathsf{T}(k) + \mathbf{R}_j(k) \qquad (8)$$

$$\mathbf{K}_j(k) = \mathbf{P}_j(k|k-1)\mathbf{H}_j^\mathsf{T}(k)\mathbf{S}_j^{-1}(k)$$

$$\hat{\mathbf{x}}_j(k|k) = \hat{\mathbf{x}}_j(k|k-1) + \mathbf{K}_j(k)\mathbf{e}_j(k)$$

$$\mathbf{P}_j(k|k) = [\mathbf{I}_4 - \mathbf{K}_j(k)\mathbf{H}_j(k)]\mathbf{P}_j(k|k-1)$$

$$\Lambda_j(k) = \mathcal{N}(\mathbf{e}_j(k); \mathbf{0}, \mathbf{S}_j(k))$$
4. Mode Probability Update  $(j = 1, ..., 2^N)$ 

$$\mu_j(k) = (1/c)\Lambda_j(k)\bar{c}_j, \quad c = \sum_j \Lambda_j(k)\bar{c}_j$$
5. Combination
$$\hat{\mathbf{x}}(k|k) = \sum_j \hat{\mathbf{x}}_j(k|k)\mu_j(k), \quad \tilde{\mathbf{x}}_j(k|k) = \hat{\mathbf{x}}_j(k|k) - \hat{\mathbf{x}}(k|k)$$

$$\mathbf{P}(k|k) = \sum_j \mu_j(k) \cdot \left\{\mathbf{P}_j(k|k) + \tilde{\mathbf{x}}_j(k|k) \cdot \tilde{\mathbf{x}}_j^\mathsf{T}(k|k)\right\}$$

Table 1: IMM-EKF algorithm

cf. (6), of the Markov chain is assumed to be known. The initial mode probabilities are set to  $\mu_i(0) = 1/8$  for i = 1, ..., 8. In Fig. 2, the cumulative distribution function (CDF) versus the localization error for 100 Monte Carlo runs is shown, where approximately 50% of the ToA estimates are contaminated by NLOS occurrences. For calculating the error CDF, the first 100 state estimates of each Monte Carlo run are neglected, as done in [6]. From Fig. 2, it can be observed that the EKF and C-EKF break down in such an environment. A significant performance improvement can be achieved by the IMM-KF and the proposed IMM-EKF. The difference in the performance between the IMM-KF and IMM-EKF comes from the fact that in the IMM-KF approach, the IMM algorithm is only applied to smooth the ToA estimates at each BS separately. The proposed IMM-EKF approach, however, exploits more efficiently the MT position information available from the ToA estimates, as it jointly smoothes the ToA estimates from all BSs and estimates the MT state vector with an EKF. Further performance improvements can be achieved using an IMM-C-EKF. In Table 2, simulation results for different degrees of NLOS occurrences are shown. Here, only

the percentiles (67%, 95%) of the localization error are presented and compared to the FCC target (50 m for 67% and  $150 \,\mathrm{m}$  for 95%). It can be observed that the proposed methods outperform conventional methods, whereas only the IMM-C-EKF achieves the FCC target even in environments that are highly contaminated by NLOS occurrences. Note that similar results have been obtained for other degrees of NLOS contaminations where the transition matrix, cf. (6) is not exactly known. In Fig. 3, simulation results are shown for a scenario with a predetermined mode sequence, where roughly 50% of the ToA estimates are contaminated by NLOS occurrences. For this scenario, with fixed LOS/NLOS transitions, 5000 Monte Carlo runs are performed and the position rootmean square error (RMSE) is calculated and compared to the posterior Cramér-Rao lower bound without road-constraints (PCRLB) and with road constraints (C-PCRLB). The PCRLB is calculated assuming zero process noise and a-priori known mode sequence as done in [8] which is an optimistic bound, because the mode history is not known in practice. Note that the PCRLBs are only valid for unbiased estimators. Although we have not proven that the above mentioned estimators are unbiased, the PCRLB is still used as a tool for benchmarking. Simulation results show that compared to the IMM-KF, a significant gain in performance can be achieved by the IMM-EKF. In particular, the IMM-EKF converges faster to the theoretical bound than the IMM-KF. Further improvement can be obtained using the IMM-C-EKF. Even though not presented here, simulation results have shown that the proposed methods are robust against imperfect knowledge of the NLOS bias and variance.

Method	NLOS contamination		
	0%	50%	100%
EKF	(30,58)	(579,1170)	(379,1320)
C-EKF [7]	(14,28)	(192,362)	(165,218)
IMM-KF [6]	(33,85)	(53,150)	(82,186)
IMM-EKF	(30,58)	(43,96)	(74,156)
IMM-C-EKF	(14,28)	(18,38)	(31,65)

**Table 2**: Results for different degrees of NLOS contamination. Number in parentheses gives the error in m of the 67- and 95- percentile.

# 5. CONCLUSION

A robust tracking algorithm based on ToA estimates in a cellular network is presented that copes with different measurement noise models due to NLOS impairments. It outperforms its competitors at the cost of additional computational effort. Road constraints are incorporated to further improve positioning accuracy. The major drawback of the proposed method is that it requires some knowledge of the noise statistic. Estimation of the noise covariance is considered in future work. Furthermore, different motion models may be incorporated in the IMM algorithm.

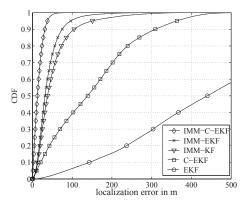


Fig. 2: CDFs for different methods.

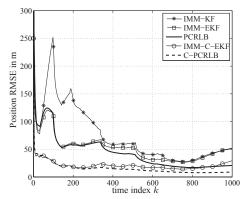


Fig. 3: RMSE for different methods compared to PCRLB.

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