COMPRESSIVE SENSING FOR MIMO RADAR

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ABSTRACT

Multiple-input multiple-output (MIMO) radar systems have been shown to achieve superior resolution as compared to traditional radar systems with the same number of transmit and receive antennas. This paper considers a distributed MIMO radar scenario, in which each transmit element is a node in a wireless network, and investigates the use of compressive sampling for direction-of-arrival (DOA) estimation. According to the theory of compressive sampling, a signal that is sparse in some domain can be recovered based on far fewer samples than required by the Nyquist sampling theorem. The DOA of targets form a sparse vector in the angle space, and therefore, compressive sampling can be applied for DOA estimation. The proposed approach achieves the superior resolution of MIMO radar with far fewer samples than other approaches. This is particularly useful in a distributed scenario, in which the results at each receive node need to be transmitted to a fusion center for further processing.

Keywords: compressive sampling, compressive sensing, MIMO radar, DOA estimation

1. INTRODUCTION

Unlike a conventional transmit beamforming radar system that uses highly correlated waveforms, a multiple-input multipleoutput (MIMO) radar system transmits multiple independent waveforms via its antennas [1]-[5]. A MIMO radar system is advantageous in two different scenarios [5]. In the first one, the transmit antennas are located far apart from each other relative to their distance to the target. The MIMO radar system transmits independent probing signals from decorrelated transmitters through different paths, and thus each waveform carries independent information about the target. Therefore, the MIMO radar system can reduce the target radar cross sections (RCS) scintillations and provide spatial diversity. In the second scenario, a MIMO radar is equipped with M_t transmit and M_r receive antennas that are close to each other relative to the target, so that the RCS does not vary between the different paths. In this scenario, the phase differences induced by transmit and receive antennas can be exploited to form a long virtual array with $M_t M_r$ elements. This enables the MIMO

radar system to achieve superior spatial resolution as compared to a traditional radar system. In this paper we consider the second scenario.

Compressive sensing (CS) has received considerable attention recently, and has been applied successfully in diverse fields, e.g., image processing [6] and wireless communications [7]. The theory of CS states that a K-sparse signal x of length N can be recovered exactly from $\mathcal{O}(K \log N)$ measurements with high probability via linear programming. Let Ψ denote the basis matrix that spans this sparse space, and Φ a measurement matrix. The convex optimization problem arising from CS is formulated as follows:

$$\min \|\mathbf{s}\|_1, \text{ subject to } \mathbf{y} = \Phi \mathbf{x} = \Phi \Psi \mathbf{s}$$
(1)

where s is a sparse vector with K principal elements and the remaining elements can be ignored; Φ is an $M \times N$ matrix incoherent with Ψ and $M \ll N$.

The application of compressive sensing to a radar system was investigated in [8], [9] and [10]. In [8], it was demonstrated that the CS method can eliminate the need for match filtering at the receiver and has the potential to reduce the required sampling rate. In the context of Ground Penetrating Radar (GPR), [9] presented a CS data acquisition and imaging algorithm that by exploiting the sparsity of targets in the spatial space can generate sharper target space images with much less CS measurements than the standard backprojection methods. Also the sparsity of targets in the time-frequency plane was exploited for radar in [10]. In the context of communication, [11] proposed the direction of arrival estimation (DOA) estimation using CS. In [11], the basis matrix Ψ is formed by the discretization of the angle space. Since the signal sources were assumed to be unknown, the basis matrix was approximated based on the signal received by a reference vector. That signal would have to be sampled at a very high rate in order to construct a good basis matrix.

In this paper, we extend the idea of [8]-[11] to the problem of DOA estimation for MIMO radar. Since the number of targets is typically smaller than the number of snapshots that can be obtained, DOA estimation can be formulated as the recovery of a sparse vector using CS. Unlike the scenario considered in [11], in MIMO radar the transmitted waveforms are known at each receive antennas, so that each receive antenna can construct the basis matrix locally, without the knowledge

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of the received signal at other antennas. Further, radar systems often suffer from interference due to jammers. Jammer suppression is investigated here by exploiting the uncorrelatedness of the transmitted waveforms with the jammer signal in order to design the measurement matrix. We provide analytical expressions for the signal-to-jammer ratio (SJR) for the proposed approach. We also provide simulation results to show that the proposed approach can accomplish superresolution in MIMO radar systems by using far fewer samples than existing methods, such as Capon, amplitude and phase estimation (APES) and generalized likelihood ratio test (GLRT) [2]. This is very significant in a distributed scenario, in which the receive nodes would need to transmit the locally obtained information to a fusion center. For such systems, we show that the proposed approach can enable each node to obtain a good DOA estimate independently. Further, it requires much less information to be transmitted to a fusion center, thus enabling savings in terms of transmission energy.

2. SIGNAL MODEL FOR MIMO RADAR

We consider a MIMO radar system with M_t transmit antennas and M_r receive antennas. For simplicity, we assume that targets and antennas all lie in the same plane. Let us denote the locations in rectangular coordinates of the *i*-th transmit and receive antenna by (x_i^t, y_i^t) and (x_i^r, y_i^r) , respectively. We assume that all transmit and receive node locations relative to some reference point are known to each node in the network. In a clustered system this information pattern may be achieved via a beacon from the cluster-head [3]. The location of the *k*-th target is denoted by (d_k, θ_k) , where d_k is the distance between this target and the origin, and θ_k is the azimuthal angle, which is the unknown parameter to be estimated in this paper.

Under the far-field assumption $d_k \gg \sqrt{(x_i^t)^2 + (y_i^t)^2}$ and $d_k \gg \sqrt{(x_i^r)^2 + (y_i^r)^2}$, the distance between the *i*th transmit/receive antenna and the *k*-th target d_{ik}^t/d_{ik}^r can be approximated as $d_{ik}^{t/r} \approx d_k - \eta_i^{t/r}(\theta_k)$, where $\eta_i^{t/r}(\theta_k) = x_i^{t/r}\cos(\theta_k) + y_i^{t/r}\sin(\theta_k)$.

Let $x_i(n)$ denote the discrete-time waveform transmitted by the *i*-th transmit antenna. Assuming the transmitted waveforms are narrowband and the propagation is non-dispersive, the received baseband signal at the *k*-th target equals [4]

$$y_k(n) = \beta_k \sum_{i=1}^{M_t} x_i(n) e^{-j\frac{2\pi}{\lambda}d_{ik}^t} = \beta_k e^{-j\frac{2\pi}{\lambda}d_k} \mathbf{x}^T(n) \mathbf{v}(\theta_k)$$
(2)

for k = 1, ..., K, where $\mathbf{x}(n) = [x_1(n), ..., x_{M_t}(n)]^T$ and $\mathbf{v}(\theta_k) = [e^{j\frac{2\pi}{\lambda}\eta_1^t(\theta_k)}, ..., e^{j\frac{2\pi}{\lambda}\eta_{M_t}^t(\theta_k)}]^T$ is the steering vector. Due to reflection by the target, the *l*-th antenna element receives

$$z_l(n) = \sum_{k=1}^{K} e^{-j\frac{2\pi}{\lambda} d_{lk}^r} y_k(n) + \epsilon_l(n), \ l = 1, \dots, M_r \quad (3)$$

where $\epsilon_l(n)$ represents independent and identically distributed (i.i.d.) Gaussian noise with variance σ^2 .

On letting L denote the number of snapshots, and placing $z_l(n), n = 0, ..., L - 1$ in vector \mathbf{z}_l we have

$$\mathbf{z}_{l} = \sum_{k=1}^{K} e^{-j\frac{2\pi}{\lambda}(2d_{k} - \eta_{l}^{r}(\theta_{k}))} \beta_{k} \mathbf{X} \mathbf{v}(\theta_{k}) + \mathbf{e}_{l}$$
(4)

where $\mathbf{y}_k = [y_k(0), \dots, y_k(L-1)]^T$, $\mathbf{e}_l = [\epsilon_l(0), \dots, \epsilon_l(L-1)]^T$ and $\mathbf{X} = [\mathbf{x}(0), \dots, \mathbf{x}(L-1)]^T$.

By discretizing the angle space as $\mathbf{a} = [\alpha_1, \ldots, \alpha_N]$, we can rewrite (4) as $\mathbf{z}_l = \sum_{n=1}^N e^{j\frac{2\pi}{\lambda}\eta_l^r(\alpha_n)} s_n \mathbf{X} \mathbf{v}(\alpha_n) + \mathbf{e}_l$, where N >> L, and

$$s_n = \begin{cases} e^{-j\frac{4\pi}{\lambda}d_k}\beta_k & \text{if there is target at } \alpha_n \\ 0 & \text{otherwise} \end{cases}$$

3. COMPRESSIVE SENSING FOR MIMO RADAR

Assuming that there exists a small number of targets, the DOAs are sparse in the angle space, i.e., $\mathbf{s} = [s_1, \ldots, s_N]$ is a sparse vector. A non-zero element with index j in \mathbf{s} indicates that there is a target at the angle α_j . By CS theory, we can construct a basis matrix Ψ_l for the *l*-th antenna as

$$\Psi_l = \left[e^{j\frac{2\pi}{\lambda}\eta_l^r(\alpha_1)} \mathbf{X} \mathbf{v}(\alpha_1), \dots, e^{j\frac{2\pi}{\lambda}\eta_l^r(\alpha_N)} \mathbf{X} \mathbf{v}(\alpha_N)\right].$$
(5)

Ignoring the noise, we have $\mathbf{z}_l = \Psi_l \mathbf{s}$. Then we measure linear projections of the received signal at the *l*-th antenna as

$$\mathbf{r}_l = \Phi_l \mathbf{z}_l = \Phi_l \Psi_l \mathbf{s} \tag{6}$$

where Φ_l is an $M \times L$ random Gaussian matrix which has small correlation with Ψ_l . Placing the output of N_r receive antennas, i.e., $\mathbf{r}_1, ..., \mathbf{r}_{N_r}$, in vector \mathbf{r} we have

$$\mathbf{r} = \Theta \mathbf{s}, \quad 1 \le Nr \le M_r$$
. (7)

and the structure of Θ can be easily inferred based on (6). Therefore, we can recover s by applying the Dantzig selector to the convex problem in (7) as in [12]:

$$\hat{\mathbf{s}} = \min \|\mathbf{s}\|_1 \quad s.t. \|\Theta^H (\mathbf{r} - \Theta \mathbf{s})\|_{\infty} < \mu.$$
(8)

According to [12], we can recover the sparse vector s with very high probability to select $\mu = (1 + t^{-1})\sqrt{2 \log N \sigma^2}$, where t is a positive scalar.

4. PERFORMANCE ANALYSIS IN THE PRESENCE OF A JAMMER SIGNAL

In this section, we analyze the effects of a jammer signal on the performance of DOA estimation for MIMO radar using CS in terms of the SJR. In the presence of a jammer signal, the received signal of the l-th receive antenna is given by

$$\mathbf{r}_{l} = \Phi_{l} \sum_{k=1}^{K} e^{-j\frac{2\pi}{\lambda}(2d_{k}-\eta_{l}^{r}(\theta_{k}))} \beta_{k} \mathbf{X} \mathbf{v}(\theta_{k}) + \Phi_{l} e^{-j\frac{2\pi}{\lambda}(d-\eta_{l}^{r}(\theta))} \beta \mathbf{b} + \Phi_{l} \mathbf{e}_{l} .$$
(9)

The location of the jammer is denoted by (d, θ) , and β and b denote respectively the reflection amplitude and waveforms of this jammer. Since b is uncorrelated with the transmitted waveforms **X**, the effects of the jammer signal are similar to those of the addictive noise. Let $\mathbf{A}_l = \Phi_l^H \Phi_l$. Given the the transmit and receive node locations, the average power of the desirable signal $P_s(l)$ over the transmit waveforms is

$$P_{s}(l) = E\{\sum_{k,k'=1}^{K} \underbrace{e^{j\frac{2\pi}{\lambda}[2(d_{k}-d_{k'})-(\eta_{l}^{r}(\theta_{k})-\eta_{l}^{r}(\theta_{k'}))]}}_{\rho_{l}(k,k')}\beta_{k}^{*}\beta_{k'}$$

$$\times \underbrace{\mathbf{v}^{H}(\theta_{k})\mathbf{X}^{H}\mathbf{A}_{l}\mathbf{X}\mathbf{v}(\theta_{k'})}_{R_{kk'}}\} = \underbrace{E\{\sum_{k=1}^{K} |\beta_{k}|^{2}R_{kk}\}}_{C_{1}(l)}$$

$$+ \underbrace{E\{\sum_{k\neq k'} \rho_{l}(k,k')\beta_{k}^{*}\beta_{k'}R_{kk'}\}}_{C_{2}(l)}$$
(10)

where $C_1(l)$ is the dominant term and $C_2(l)$ can be ignored when the number of transmit antennas is sufficiently large. Therefore, $P_s(l)$ can be approximated by $C_1(l)$. To simplify the derivation of the SJR, we assume that the transmitted waveforms are independently generated, orthogonal quadrature phase shift keyed (QPSK) sequences with $\mathbf{X}^H \mathbf{X} = \mathbf{I}_{M_t}$, $\mathbf{b}^H \mathbf{b} = 1$ and $\Phi_l \Phi_l^H = \mathbf{I}_M$. Then $P_s(l)$ is approximated by

$$P_s(l) \approx \sum_{k=1}^{K} |\beta_k|^2 \operatorname{tr}(\mathbf{A}_l) M_t / L = \frac{MM_t}{L} \sum_{k=1}^{K} |\beta_k|^2 .$$
 (11)

Similarly, the average power of the jammer interference over the jammer waveforms is given by

$$P_{j}(l) = E\{(e^{-j\frac{2\pi}{\lambda}(d-\eta_{l}^{r}(\theta))}\beta)(e^{-j\frac{2\pi}{\lambda}(d-\eta_{l}^{r}(\theta))}\beta)^{*} \times \mathbf{b}^{H}\mathbf{A}_{l}\mathbf{b}\} = |\beta|^{2}M/L.$$
(12)

From these two expressions, the SJR becomes

$$SJR = \frac{P_s(l)}{P_j(l)} \approx \frac{M_t \sum_{k=1}^K |\beta_k|^2}{|\beta|^2} .$$
(13)

Since the jammer signal is uncorrelated with the transmitted signal, the SJR can be improved by correlating the jammer signal with the transmitted signal. Combining this with CS, the measurement matrix in (6) is modified as $\tilde{\Phi}_l = \Phi_l \mathbf{X}^H$. Moreover, since Φ_l is a Gaussian random matrix, $\tilde{\Phi}_l$ is still Gaussian; therefore it satisfies the restricted isometry property (RIP) and is incoherent with Ψ_l , thus guaranteeing a stable solution to (8). Based on $\tilde{\Phi}_l$, the SJR can be obtained as

$$SJR \approx \frac{L \sum_{k=1}^{K} |\beta_k|^2}{|\beta|^2} .$$
(14)

Generally, the SJR can be improved by a factor of L/M_t using $\tilde{\Phi}_l$ since $L \gg M_t$. (14) indicates that the increase in L will improve the DOA estimates. However, more calculations are required by ℓ_1 -norm minimization due to the increase in the size of the basis matrix, and the time duration of the radar pulse needs to be longer as well.

The proposed method is especially advantageous in a distributed MIMO radar system in which the receive elements are randomly distributed. In particular, many fewer measurements are required to be sent to the base station or fusion center (FC) in this situation than are need by conventional methods. As simulation results show (see Section 5), the proposed method can yield good performance even using a single receive antenna. With a good initial estimate of DOA, the receive nodes can adaptively refine their estimates by constructing a higher resolution basis matrix Ψ_l around that DOA. Restricting the candidate angle space, may reduce the samples in the angle space that are required for constructing the basis matrix, thus reducing the complexity of the ℓ_1 minimization step. On the other hand, the resolution of target detection can be improved by taking the denser samples of the angle space around the intimal DOA estimate.

5. SIMULATION RESULTS

In this section, we consider a MIMO radar system with the transmit/receive antennas randomly distributed within a small area on a two-dimensional (2-D) disk. $M_t = 30$ antennas transmit independent QPSK waveforms. The carrier frequency is 8.62 GHz. A maximum of L = 512 snapshots are considered at the receive node. The received signal is corrupted by zero mean Gaussian noise. The SNR is set to 20 dB. There are two targets located at $\theta_1 = -3^\circ$ and $\theta_1 = -2^\circ$, with reflection coefficients $\beta_k = 1, k = 1, 2$. A jammer is located at 0° with an unknown Gaussian random waveform and with amplitude 10, i.e., 20 dB above the target reflection coefficients β_k . We sample the angle space by increments of 0.2° from -5° to 5° , i.e., $\mathbf{a} = [-5^\circ, -4.8^\circ, \ldots, 4.8^\circ, 5^\circ]$. We compare the performance of the proposed method and three approaches [2], i.e., the Capon, APES and GLRT techniques.

Figs. 1 shows the moduli of the estimated reflection coefficients β_k , as functions of the azimuthal angle for (a) $M_r = 1$ and (b) 10 receive antennas, respectively. In both (a) and (b), the top three curves correspond to the azimuthal estimates obtained via Capon, APES and GLRT, using 512 snapshots. The bottom curve is the result of the proposed approach, obtained using 15 snapshots only. One can see that in the case of using only one receive node, the presence of the two targets is

clearly evident via the proposed method based on 15 snapshots only. The other methods produce spurious peaks away from the target locations. When the measurements of multiple receive nodes are used at a fusion center, the proposed approach can yield similar performance to the other three methods. However, the comparison methods would have to transmit to the fusion center 512 received samples each, while in the proposed approach, each node would need to transmit 15 samples each.

The threshold μ in (8) affects DOA estimation for the proposed method. The increase in μ within a range can reduce the ripples of DOA estimates at the non-target azimuth angles at the expense of the accuracy of the target-reflection-coefficient estimates. The increase in μ can also reduce the complexity of (8) because the constraint is looser than that of smaller μ . If μ is too large, however, the ℓ_1 -norm minimization does not work. In Fig.1, a relatively large threshold, i.e., $\mu = 3$, was used for the single receive node case. As a result, the CS method yielded less accurate estimates of the reflection coefficients magnitude than the Capon and GLRT, but with very few ripples.

Fig. 2 shows the effect of the number of snapshots L on the DOA estimates of the proposed method and the other methods. Here, we consider the case of one receive antenna only. In order to quantify the performances of DOA estimation, we define the ratio of the square amplitude of the DOA estimate at the target azimuth angle to the sum of the square amplitude of DOA estimates at other angles as the peak-to-ripple ratio (PRR). As shown in Fig. 2, although the increase in L can improve the PPR of these four methods, the increase is much faster for the CS method.

6. CONCLUSION

A compressive sensing method has been proposed to estimate the DOA of targets for MIMO radar systems. The DOA of targets can construct a sparse vector in the angle space. Therefore, we can solve for this sparse vector by ℓ_l -norm minimization with many fewer samples than conventional methods, i.e. the Capon, APES and GLRT techniques. The proposed method is superior to these conventional methods when one receive antenna is active. If multiple receive antennas are used, the proposed approach can yield similar performance to the other three methods, but by using far fewer samples.

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7. REFERENCES

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Fig. 1. DOA estimation using (a) 1 receive antenna and (b) 10 receive antennas. In both (a) and (b), the top three curves were obtained using 512 snapshots. The bottom curve was obtained using 15 snapshots only.



Fig. 2. PRR using one receive antenna.