

ADAPTIVE PREDISTORTION OF NONLINEAR VOLTERRA SYSTEMS USING SPECTRAL MAGNITUDE MATCHING

Emad Abd-Elrady, Li Gan and Gernot Kubin

Christian Doppler Laboratory for Nonlinear Signal Processing
Institute of Signal Processing and Speech Communication
Graz University of Technology, Inffeldgasse 16b, A-8010 Graz, Austria.
E-mail: {emad.abd-elrady, li.gan, gernot.kubin}@TUGraz.at

ABSTRACT

Digital compensation of nonlinear systems is an important topic in many practical applications. This paper considers the problem of predistortion of nonlinear systems described using Volterra series by connecting in tandem an adaptive Volterra predistorter. The suggested Direct Learning Architecture (DLA) approach utilizes the Spectral Magnitude Matching (SMM) method that minimizes the sum squared error between the spectral magnitudes of the output signal of the nonlinear system and the desired signal. The coefficients of the predistorter are estimated recursively using the generalized Newton iterative algorithm. A comparative simulation study with the Nonlinear Filtered-x Least Mean Squares (NFxLMS) algorithm shows that the suggested SMM approach achieves much better performance but with higher computation complexity.

Index Terms— Adaptive systems, nonlinear systems, parameter estimation, spectral analysis, Volterra series.

1. INTRODUCTION

In many areas, cancelling or reducing the nonlinear distortion due to the nonlinearity characteristic of the electronic or electromechanical devices are becoming more and more important. Examples can be found in communication systems, speech processing and control engineering, see [1-3]. Two kinds of adaptive compensation techniques can be used to reduce the nonlinear distortion, which are adaptive postdistortion, also named as adaptive equalization, and adaptive predistortion [1]. Although the postdistortion is an effective way to compensate the nonlinear distortion, the predistortion is more efficient and necessary in many other situations, such as compensation of the nonlinear distortion for the power amplifiers in satellite communication [4] and the active noise cancellation for loudspeakers [5].

Predistortion of nonlinear Volterra systems based on the Direct Learning Architecture (DLA) approach and using the Nonlinear Filtered-x Least Mean Squares (NFxLMS) algorithm has been considered in [3, 6]. The idea in [3, 6] is to connect a nonlinear Volterra predistorter tandemly with the nonlinear Volterra system and adaptively adjusting the coefficients of the predistorter in order to minimize the mean square distortion. These coefficients were estimated recursively using the NFxLMS algorithm. The problems usually encountered while using the NFxLMS algorithm are slow convergence and need of accurate identification of the nonlinear Volterra system.

In [7], a method was introduced for estimating telephone handset nonlinearity by matching the spectral magnitude of the distorted signal to the output of a nonlinear channel model. The nonlinear

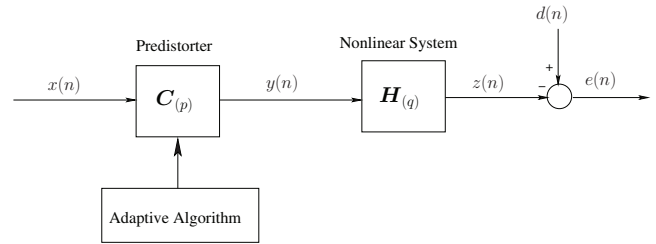


Fig. 1. Adaptive predistortion of nonlinear systems.

model was chosen as a Wiener-Hammerstein cascade system with a static nonlinearity described by a finite-order polynomial. The nonlinear model coefficients were estimated using the generalized Newton iterative algorithm [8, 9] that minimizes a cost function of the sum squared error between the spectral magnitudes - evaluated for a number of short-time frames - of the measured distorted signal and the output signal of the nonlinear model.

In this paper, the same approach of [7] is used for the purpose of predistortion of nonlinear Volterra systems. Also here, the coefficients of the Volterra predistorter are estimated recursively using the generalized Newton iteration algorithm to minimize the sum squared error between the spectral magnitudes of the output signal of the nonlinear Volterra system and the desired signal. The suggested Spectral Magnitude matching (SMM) approach presented in this paper does not require the identification of the nonlinear Volterra system as in [3, 6].

The paper is organized as follows. In Sec. 2, the DLA approach is discussed. A brief review of the NFxLMS algorithm is given in Sec. 3. The SMM method is introduced in Sec. 4. In Sec. 5, simulation results are given. Conclusions are presented in Sec. 6.

2. THE DIRECT LEARNING ARCHITECTURE

The DLA approach of this paper, see Fig. 1, assumes that the nonlinear system $H_{(q)}$ to be compensated is a discrete-time invariant causal system. Also, the system $H_{(q)}$ with input and output signals $y(n)$ and $z(n)$ can be modeled by q th-order Volterra series with M -tap memories. Hence, the output $z(n)$ is given by

$$z(n) = \sum_{k=1}^q \left(\sum_{i_1=0}^{M-1} \dots \sum_{i_k=0}^{M-1} h_k(i_1, \dots, i_k) y(n-i_1) \dots y(n-i_k) \right) \quad (1)$$

where $h_k(i_1, \dots, i_k)$ are the k th-order kernels of the nonlinear system.

Similarly, the relation between the input and output of the adaptive Volterra predistorter $C_{(p)}$ is given by

$$y(n) = \sum_{k=1}^p \left(\sum_{i_1=0}^{N-1} \dots \sum_{i_k=0}^{N-1} c_k(i_1, \dots, i_k; n) x(n-i_1) \dots x(n-i_k) \right) \quad (2)$$

where N is the number of memories and $c_k(i_1, \dots, i_k; n)$ are the k th-order kernels of this predistorter. According to the p th-order Volterra theorem [10], the Volterra filter $C_{(p)}$ can remove nonlinearities up to p th-order provided that the inverse of the first-order Volterra system is causal and stable.

Let us define the parameter vector C of the adaptive Volterra predistorter as

$$C = \begin{pmatrix} C_1 \\ \vdots \\ C_p \end{pmatrix}, \quad (3)$$

where C_k is given by

$$C_k = \begin{pmatrix} c_k(0, \dots, 0) \\ \vdots \\ c_k(N-1, \dots, N-1) \end{pmatrix}. \quad (4)$$

Also, assume that the desired signal $d(n)$ is given as

$$d(n) = x(n - \tau) + v(n), \quad (5)$$

where τ is the time delay necessary to have a causal Volterra predistorter and $v(n)$ is AWGN.

Remark 1: The time delay τ equals zero in case the system to be compensated is minimum phase [3].

The main goal of digital predistortion is to estimate the parameter vector C such that the output signal $z(n)$ becomes very close to the desired signal $d(n)$. This estimation process can be done directly or indirectly. Indirect learning Architecture (ILA) approach for predistortion of nonlinear Volterra systems was introduced in [11, 12] using Recursive Least Squares (RLS), Kalman Filter (KF) and Recursive Prediction Error Method (RPEM) algorithms. The work done in [3, 6] considers direct estimation of the parameter vector C using the NFxLMS algorithm. Also in this paper, the SMM method is used for direct estimation of C . The NFxLMS algorithm and the SMM method are discussed in the next sections.

3. THE NFxLMS ALGORITHM

The kernels of the adaptive Volterra filter were estimated in [3], see Fig. 2, by minimizing the mean square distortion defined as

$$E\{e^2(n)\} = E\{[d(n) - z(n)]^2\} \quad (6)$$

where E denotes the expectation and $d(n)$ is the desired signal defined in Eq. (5).

The NFxLMS algorithm of [3] was obtained by applying the stochastic gradient algorithm [13]:

$$\hat{C}_k(n+1) = \hat{C}_k(n) - \frac{\mu_k}{2} \Delta_k(n) \quad (7)$$

where μ_k is a small positive constant that controls stability and rate of convergence of the adaptive algorithm and usually is defined as

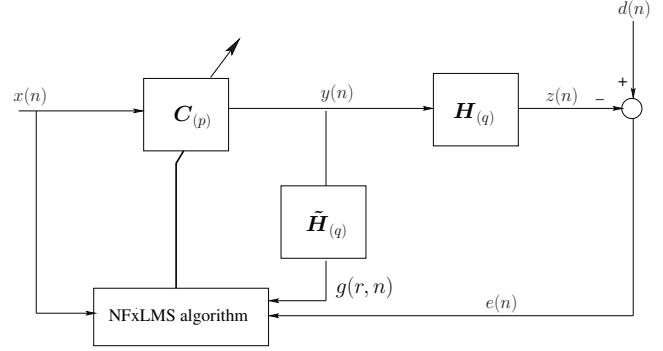


Fig. 2. Adaptive predistortion using the NFxLMS algorithm.

the *step-size parameter*. Also, the gradient vector $\Delta_k(n)$ is defined as

$$\Delta_k(n) = \begin{pmatrix} \frac{\partial e^2(n)}{\partial c_k(0, \dots, 0; n)} \\ \vdots \\ \frac{\partial e^2(n)}{\partial c_k(N-1, \dots, N-1; n)} \end{pmatrix}. \quad (8)$$

Taking into consideration that (cf. Eq. (6))

$$\frac{\partial e^2(n)}{\partial c_k(i_1, \dots, i_k; n)} = -2e(n) \frac{\partial z(n)}{\partial c_k(i_1, \dots, i_k; n)} \quad (9)$$

where $\frac{\partial z(n)}{\partial c_k(i_1, \dots, i_k; n)}$ can be written as

$$\frac{\partial z(n)}{\partial c_k(i_1, \dots, i_k; n)} = \sum_{r=0}^{M-1} g(r; n) \frac{\partial y(n-r)}{\partial c_k(i_1, \dots, i_k; n)}. \quad (10)$$

Here $g(r; n)$ is given as

$$g(r; n) = \frac{\partial z(n)}{\partial y(n-r)} = h_1(r) + 2 \sum_{i=0}^{M-1} h_2(r, i) y(n-i) + 3 \sum_{i_1=0}^{M-1} \sum_{i_2=0}^{M-1} h_3(r, i_1, i_2) y(n-i_1) y(n-i_2) + \dots \quad (11)$$

Assuming that μ_k is chosen sufficiently small, $\frac{\partial y(n-r)}{\partial c_k(i_1, \dots, i_k; n)}$ can be approximated as (cf. Eq. (2))

$$\frac{\partial y(n-r)}{\partial c_k(i_1, \dots, i_k; n)} \approx \frac{\partial y(n-r)}{\partial c_k(i_1, \dots, i_k; n-r)} = x(n-r-i_1) \dots x(n-r-i_k). \quad (12)$$

Substituting by Eqs. (10)-(12) in Eq. (9), we have

$$\frac{\partial e^2(n)}{\partial c_k(i_1, \dots, i_k; n)} = -2e(n) \sum_{r=0}^{M-1} g(r; n) x(n-r-i_1) \dots x(n-r-i_k). \quad (13)$$

Remark 2: In Eq. (11), it is assumed that the correct kernels of the nonlinear system $H_{(q)}$ are known or have been estimated. The problem of estimating Volterra kernels for nonlinear systems is discussed, e.g., in [14].

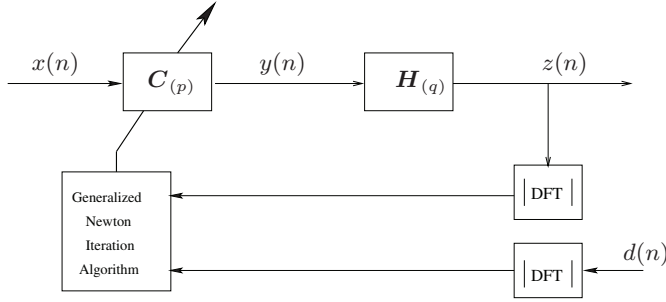


Fig. 3. Adaptive predistortion using the SMM approach.

4. THE SPECTRAL MAGNITUDE MATCHING METHOD

The Spectral Magnitude Matching (SMM) method [7], see Fig. 3, minimizes the sum squared error between the spectral magnitude of the desired signal $d(n)$ and the spectral magnitude of the received signal $z(n)$ through the following cost function:

$$V_C = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} [|D(\omega_l; k)| - |Z(\omega_l; k; \mathbf{C})|]^2 \quad (14)$$

where $D(\omega_l; k)$ and $Z(\omega_l; k; \mathbf{C})$ are the short-time DFT of the desired and output signals, respectively. K is the number of uniformly-spaced short-time frames and L is the DFT length.

The cost function V_C can be written as

$$V_C = \mathbf{\Gamma}_C^T \mathbf{\Gamma}_C \quad (15)$$

where

$$\mathbf{\Gamma}_C = \begin{pmatrix} \gamma^0(\mathbf{C}) \\ \gamma^1(\mathbf{C}) \\ \vdots \\ \gamma^{K-1}(\mathbf{C}) \end{pmatrix} \quad (16)$$

and

$$\gamma^k(\mathbf{C}) = \begin{pmatrix} |D(\omega_0; k)| - |Z(\omega_0; k; \mathbf{C})| \\ \vdots \\ |D(\omega_{L-1}; k)| - |Z(\omega_{L-1}; k; \mathbf{C})| \end{pmatrix}, \quad (17)$$

$k = 0, \dots, K-1$.

The parameter vector \mathbf{C} that minimizes the cost function V_C can be estimated using the generalized Newton iteration [8, 9]. In this case, we have

$$\hat{\mathbf{C}}(m+1) = \hat{\mathbf{C}}(m) + \mu \mathbf{\Delta}(m) \quad (18)$$

where m is the iteration index, μ is the adaptation gain, and the gradient $\mathbf{\Delta}(m)$ is given by [7, 13, 15]

$$\begin{aligned} \mathbf{\Delta}(m) &= - \left[\frac{d^2 V_C}{d\mathbf{C}^2} \right]^{-1} \left[\frac{dV_C}{d\mathbf{C}} \right] \\ &= - \left(\mathbf{J}^T(m) \mathbf{J}(m) \right)^{-1} \mathbf{J}^T(m) \mathbf{\Gamma}_C \big|_{\mathbf{C}=\hat{\mathbf{C}}(m)}. \end{aligned} \quad (19)$$

Here $\mathbf{J}(m)$ is the Jacobian matrix of first derivative of $\mathbf{\Gamma}_C$ with respect to \mathbf{C} evaluated at $\mathbf{C} = \hat{\mathbf{C}}(m)$, i.e.

$$\mathbf{J}(m) = \frac{d\mathbf{\Gamma}_C}{d\mathbf{C}} \big|_{\mathbf{C}=\hat{\mathbf{C}}(m)} = \begin{pmatrix} \mathbf{J}^0(m) \\ \mathbf{J}^1(m) \\ \vdots \\ \mathbf{J}^{K-1}(m) \end{pmatrix} \quad (20)$$

where

$$\begin{aligned} \mathbf{J}^k(m) &= \frac{d\gamma^k(\mathbf{C})}{d\mathbf{C}} \big|_{\mathbf{C}=\hat{\mathbf{C}}(m)} \\ &= - \begin{pmatrix} \frac{d|Z(\omega_0; k; \mathbf{C})|}{d\mathbf{C}} \\ \vdots \\ \frac{d|Z(\omega_{L-1}; k; \mathbf{C})|}{d\mathbf{C}} \end{pmatrix} \big|_{\mathbf{C}=\hat{\mathbf{C}}(m)}, \end{aligned} \quad (21)$$

$k = 0, \dots, K-1$.

Due to the fact that there is no close form expression for the gradient $\mathbf{\Delta}(m)$, an approximate gradient was evaluated in [7] by finite element approximation. The same approach is considered in this paper. The approximation follows the following lines:

1. Initiate with a parameter vector $\hat{\mathbf{C}}(0)$ and compute the DFT magnitude $|D(\omega_l; k)|$.
2. Compute the DFT magnitude $|Z(\omega_l; k; \hat{\mathbf{C}})|$ based on the current value of the parameter vector $\hat{\mathbf{C}}(m)$ and form $\mathbf{\Gamma}_C$.
3. Recalculate $z(n; \hat{\mathbf{C}})$ for each perturbed component of $\hat{\mathbf{C}}(m)$ and then compute its DFT magnitude. The (i, j) element of the matrix element $\mathbf{J}^k(m)$ denoted as $J_{i,j}^k(m)$ is evaluated using a first backward difference for each element of $\hat{\mathbf{C}}(m)$ as

$$\begin{aligned} J_{i,j}^k(m) &= \frac{\partial \gamma_i^k(\hat{\mathbf{C}}; m)}{\partial \hat{c}_j(m)} \\ &\approx - \frac{1}{\varepsilon_m} (|Z(\omega_i; k; \hat{c}_1(m), \dots, \hat{c}_j(m) + \varepsilon_m, \dots)| \\ &\quad - |Z(\omega_i; k; \hat{c}_1(m), \dots, \hat{c}_j(m), \dots)|) \end{aligned} \quad (22)$$

where $\gamma_i^k(\hat{\mathbf{C}}; m)$ is the i th element of $\gamma^k(\hat{\mathbf{C}})$, $\hat{c}_j(m)$ is the j th element of the parameter vector $\hat{\mathbf{C}}(m)$ and ε_m is a small adaptive perturbation evaluated as

$$\varepsilon_m = \frac{V_C(m)}{V_C(0)} \varepsilon_0 \quad (23)$$

where ε_0 is the initial perturbation, $V_C(0)$ is the initial value of V_C , and $V_C(m)$ is the m th step value of V_C . This means that the perturbation decreases proportionally with the error.

4. Finally, evaluate the correction term $\mathbf{\Delta}(m)$ from Eq. (19) and update the parameter vector $\hat{\mathbf{C}}$ using Eq. (18).

Remark 3: Regardless the fact that the SMM approach does not require the nonlinear Volterra system to be first identified like the NFxLMS algorithm, its computation complexity is higher than the NFxLMS algorithm. Future research will consider the possibility of reducing the computation complexity of the SMM method.

5. SIMULATION STUDY

In order to investigate the performance of the suggested SMM approach as compared to the NFxLMS algorithm for predistortion of nonlinear Volterra systems using adaptive Volterra predistorter, the following simulations were performed.

The nonlinear system $\mathbf{H}_{(q)}$ is assumed to be a known second-order Volterra system. The adaptive Volterra predistorter $\mathbf{C}_{(p)}$ is also assumed to be a second-order Volterra filter, i.e. $q = p = 2$. Also, the number of memories in the adaptive predistorter was chosen as

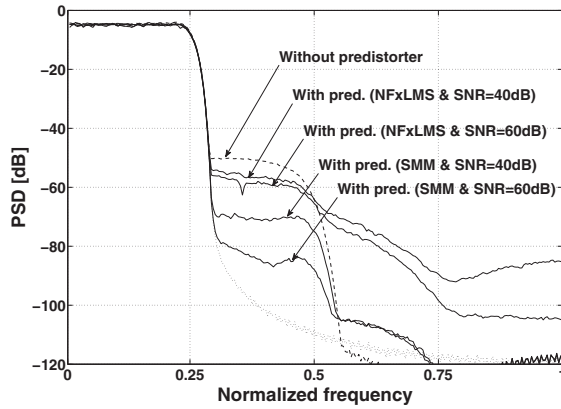


Fig. 4. PSDs for input and output signals.

$N = 4$. The input-output relation of the nonlinear system $\mathbf{H}_{(2)}$ is given by

$$z(n) = \mathbf{H}_{(2)}[y(n)] = \mathbf{H}_1[y(n)] + \mathbf{H}_2[y(n)] \quad (24)$$

where the first-order kernels vector \mathbf{H}_1 and the second-order kernels matrix \mathbf{H}_2 are given as

$$\mathbf{H}_1 = \begin{pmatrix} 0.5625 & 0.4810 & 0.1124 & -0.1669 \end{pmatrix} \quad (25)$$

$$\mathbf{H}_2 = 0.01 \times \begin{pmatrix} 1.7499 & 0 & 0 \\ 0 & 0 & -0.8750 \\ 0 & -0.8750 & 0 \end{pmatrix}. \quad (26)$$

The input signal was chosen as a random signal with uniform distribution over $(-1, 1)$. The frequency band of the input is limited to prevent aliasing [10]. For the SMM approach, a data length of 10^4 samples divided into 20 short-time frames have been used each with length 500 samples. The DFT length L was 250 and a Hanning window [16] was used. The SMM method was initialized with $\hat{\mathbf{C}}(0) = \mathbf{1}$. An initial perturbation of $\varepsilon_0 = 0.1$ and adaptation gain of $\mu = 0.9$ were used. For the NFxLMS algorithm, a data length of 5×10^4 and step sizes of $\mu_1 = \mu_2 = 0.1$ have been used.

Figure 4 shows power spectral densities (PSDs) of the output signals of the nonlinear Volterra system with and without the predistorter. The performance of the SMM approach and the NFxLMS algorithm are given for signal to noise ratios (SNRs) of 40 dB and 60 dB. From this figure, we can see that the Volterra predistorter using the SMM approach significantly reduce spectral regrowth and achieves much better performance than the NFxLMS algorithm.

6. CONCLUSIONS

Adaptive predistortion of nonlinear Volterra systems based on the Spectral Magnitude Matching (SMM) method has been considered in this paper. The SMM method is based on minimizing the difference between the spectral magnitudes of the output and the desired signals. The coefficients of the adaptive Volterra predistorter are estimated using the generalized iterative Newton algorithm. Simulation results show that the suggested approach can significantly suppress spectral regrowth and achieves much better results than the NFxLMS algorithm. The drawback of the suggested SMM approach is the high computation complexity. Future research will focus on reducing the computation complexity of the SMM method.

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