A NEW SCHEME FOR SYNCHRONIZATION OF INACTIVE NODES IN A SENDER-RECEIVER PROTOCOL

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ABSTRACT

This paper targets the problem of clock synchronization for a set of receivers lying within the broadcast range of two nodes implementing a general sender-receiver protocol using a wireless channel. The maximum likelihood estimate of the clock offset of the inactive node (and mean link delay) hearing the broadcasts from both the master and slave nodes was derived in [1] assuming symmetric exponential link delays. In this paper, the minimum variance unbiased estimate for the clock offset of such nodes is derived by applying the Rao-Blackwell-Lehmann-Scheffé theorem. The result is important in the realm of wireless sensor networks, where a tight network synchronization along with a conservative energy utilization plays a major role in network performance.

Index Terms— Clock Synchronization, Wireless Sensor Networks.

1. INTRODUCTION

A wireless sensor node is equipped with a tiny microprocessor, sensor, battery and a radio for communication capabilities. Due to the deployment of the network usually for the lifetime, the power consumption in the sensor nodes assumes a paramount importance and the central player in designing of algorithms.

Various protocols addressing the clock synchronization problem in Wireless Sensor Network (WSN) are mainly based on packet synchronization techniques among which the sender-receiver approach [2] is widely utilized. Owing to the wireless nature of the broadcast medium, the nodes located in the common broadcast region of a master and slave node can overhear the time synchronization packets between them and exploit the received observations for synchronizing their clocks with the master node. In [1], the Maximum Likelihood Estimates (MLE) for the clock offset and mean link delays of the inactive nodes were derived under the symmetric exponential delay model. This paper not only derives the more attractive minimum variance unbiased estimator for the clock offset, but also addresses the problem under the more realistic asymmetric delay model.

A WSN consists of several nodes, with a master node r chosen as the reference. Fig. 1 depcits a series of messages exchanged between node r and another node s (chosen as the slave node) whose clock offset is ψ_o^s with respect to node r. As illustrated in Fig. 1, the timestamps m_k^r and m_k^{sr} are recorded by node r at pre-transmission and post-reception of timing messages, while node s records m_k^{rs} and m_k^s according to its own time reference at post-reception and pre-transmission of timing messages.

It is also clear from Fig. 1 that many nodes like node t, whose clock offset with respect to node r is ψ_o^t, lie within the intersection of the broadcast regions of nodes r and s and hence receive the packet exchange through the channel between nodes r and s. They can synchronize their clocks with the reference conserving considerable power.

For simplicity, it is assumed that the deterministic part of link delays τ is unknown but same for all the nodes receiving the messages from nodes r and s due to similar hardware specifications and characteristics of the nodes. The solution for different τ follows along the similar lines. In addition, the random link delays, ε_k^{rs}, ε_k^{rt} and ε_k^{st}, are modeled as coming from the exponential distribution and both cases with similar and dissimilar means will be studied in detail. The justifications behind these assumptions can be found in detail in the literature. The following equations summarize the model depicted above for $k = 1, \cdots, N$.

$$\begin{aligned} m_k^{rs} &= m_k^r + \psi_o^s + \tau + \varepsilon_k^{rs}, \\ m_k^{rt} &= m_k^r + \psi_o^t + \tau + \varepsilon_k^{rt}, \\ m_k^{st} &= m_k^s - \psi_o^s + \psi_o^t + \tau + \varepsilon_k^{st}, \end{aligned}$$

where ε_k^{rs}, ε_k^{rt} and ε_k^{st} are independent and identically distributed exponential random variables with means α, β and γ, respectively. Rearranging the equations and introducing the notations $U_k \triangleq m_k^{rs} - m_k^r$, $V_k \triangleq m_k^{rt} - m_k^r$ and $W_k \triangleq$

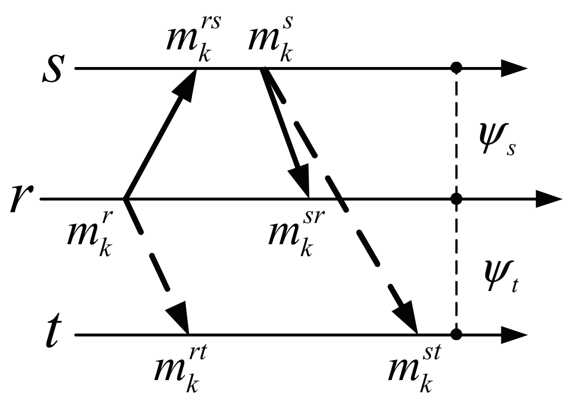

Fig. 1. Timing message exchange between nodes r and s, which node t is also receiving

$m_k^{st} - m_k^s$ yields

$$\begin{aligned} U_k &= \psi_o^s + \tau + \varepsilon_k^{rs}, & (1)\\ V_k &= \psi_o^t + \tau + \varepsilon_k^{rt}, & (2)\\ W_k &= \psi_o^t - \psi_o^s + \tau + \varepsilon_k^{st}. & (3) \end{aligned}$$

Next, we discuss the derivation of the minimum variance unbiased estimator for the clock offset of the inactive nodes.

2. CLOCK OFFSET ESTIMATION

Since the Mean Square Error (MSE) usually depends on the unknown parameter, a technique chosen to attain realizable yet best estimators is to constrain the bias to be zero, because since the dependance of minimum MSE estimator on the unknown parameter typically comes from the bias. Therefore, restricting the possible estimators to be unbiased and then finding the estimator with the smallest variance for all values of the unknown parameter yields the optimal solution within the class of unbiased estimators. Hence, we proceed towards deriving the MVUE for the clock offset and mean link delays for the problem at hand.

Herein, the MVUE is obtained based on the Rao-Blackwell-Lehmann-Scheffé theorem. First, the likelihood function is factored according to Neymann-Fisher factorization theorem which gives the sufficient statistics $\mathbf{T}$. Second, the completeness of the sufficient statistics is checked. In case it is complete, any of the following two approaches yields the desired result $\hat{\theta}$ as the MVUE: either for any unbiased estimator $\check{\theta}$, $\hat{\theta} = E[\check{\theta}|\mathbf{T}]$ is evaluated, or a function $g(\mathbf{T})$ of the sufficient statistics is found such that $\hat{\theta} = g(\mathbf{T})$ is an unbiased estimator. The MVUE in the current scenario is obtained following these lines.

Let $\mathbf{\Psi_A} \triangleq [\psi_o^t \ \psi_o^s \ \tau \ \alpha \ \beta \ \gamma]^T$. In the asymmetric delays case, the likelihood function for the clock offset as a function of observations $\{U_k\}_{k=1}^N$, $\{V_k\}_{k=1}^N$ and $\{W_k\}_{k=1}^N$ from (1), (2) and (3) is given by

$$\begin{aligned} L\left(\psi_o^t, \psi_o^s, \tau, \alpha, \beta, \gamma\right) &= (\alpha\beta\gamma)^{-N} e^{-\sum_{k=1}^N \left[\frac{1}{\alpha}\left(U_k - \psi_o^s - \tau\right)\right]} \cdot \\ & e^{-\sum_{k=1}^N \left[\frac{1}{\beta}\left(V_k - \psi_o^t - \tau\right) + \frac{1}{\gamma}\left(W_k - \psi_o^t + \psi_o^s - \tau\right)\right]} \cdot \\ & \cdot I_{\left[U_{(1)} - \psi_o^s - \tau\right]} \, I_{\left[V_{(1)} - \psi_o^t - \tau\right]} \, I_{\left[W_{(1)} - \psi_o^t + \psi_o^s - \tau\right]}, \end{aligned}$$

where $I_{[\cdot]}$ denotes the unit step function. The likelihood function can now be factored as

$$\begin{aligned} L\left(\psi_o^t, \psi_o^s, \tau, \alpha, \beta, \gamma\right) &= h_1(\tau, \alpha, \beta, \gamma) g_1\left(\sum_{k=1}^N U_k, \psi_o^s, \alpha\right) \cdot \\ & g_2\left(\sum_{k=1}^N V_k, \psi_o^t, \beta\right) g_3\left(\sum_{k=1}^N W_k, \psi_o^t, \psi_o^s, \gamma\right) \cdot \\ & g_4\left(U_{(1)}, \psi_o^s, \tau\right) g_5\left(V_{(1)}, \psi_o^t, \tau\right) g_6\left(W_{(1)}, \psi_o^t, \psi_o^s, \tau\right) \end{aligned}$$

where

$$\begin{aligned} g_1\left(\sum_{k=1}^N U_k, \psi_o^s, \alpha\right) &= e^{-\frac{1}{\alpha}\sum_{k=1}^N \left(U_k - \psi_o^s\right)}, \\ g_2\left(\sum_{k=1}^N V_k, \psi_o^t, \beta\right) &= e^{-\frac{1}{\beta}\sum_{k=1}^N \left(V_k - \psi_o^t\right)}, \\ g_3\left(\sum_{k=1}^N W_k, \psi_o^t, \psi_o^s, \gamma\right) &= e^{-\frac{1}{\gamma}\sum_{k=1}^N \left(W_k - \psi_o^t + \psi_o^s\right)}, \\ g_4\left(U_{(1)}, \psi_o^s, \tau\right) &= I\left[U_{(1)} - \psi_o^s - \tau\right], \\ g_5\left(V_{(1)}, \psi_o^t, \tau\right) &= I\left[V_{(1)} - \psi_o^t - \tau\right], \\ g_6\left(W_{(1)}, \psi_o^t, \psi_o^s, \tau\right) &= I\left[W_{(1)} - \psi_o^t + \psi_o^s - \tau\right], \\ h_1\left(\tau, \alpha, \beta, \gamma\right) &= (\alpha\beta\gamma)^{-N} e^{N\tau\left[\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right]}. \end{aligned}$$

Note that the above functions depend on the data only through $\mathbf{T} = \{\sum_{k=1}^N U_k, U_{(1)}, \sum_{k=1}^N V_k, V_{(1)}, \sum_{k=1}^N W_k, W_{(1)}\}$. Therefore, according to the Neyman-Fisher factorization theorem, $\mathbf{T}$ is a sufficient statistic for $\mathbf{\Psi_A}$. Since $\dim(\mathbf{T}) = \dim(\mathbf{\Psi_A})$, we have to find a 6×1 vector function $\hat{\mathbf{\Psi}}_\mathbf{A}$ such that $E[\hat{\mathbf{\Psi}}_\mathbf{A}] = \mathbf{\Psi_A}$, provided that $\mathbf{T}$ is a complete sufficient statistic. Since the probability density function (pdf) of $\mathbf{T}$ is required to check whether $\mathbf{T}$ is complete, and $\sum_{k=1}^N U_k$ and $U_{(1)}$, $\sum_{k=1}^N V_k$ and $V_{(1)}$, and $\sum_{k=1}^N W_k$ and $W_{(1)}$ are not independent, we proceed as follows. Considering into account only the data set $\{V_k\}_{k=1}^N$ first, it is evident that the pdf of the minimum order statistic $V_{(1)}$ is exponential with mean β/N, whereas the joint pdf of $V_{(1)}, V_{(2)}, \ldots, V_{(N)}$ is given by

$$\begin{aligned} f\left(V_{(1)}, V_{(2)}, \cdots, V_{(N)}\right) &= N!\beta^{-N} e^{-\frac{1}{\beta}\sum_{k=1}^N \left\{V_k - \psi_o^t - \tau\right\}} \\ & \prod_{k=1}^N I\left[V_k - \psi_o^t - \tau\right]. \qquad (4) \end{aligned}$$

Now consider the transformation

$$\eta_k = (N-k+1)\left(V_k - V_{(k-1)}\right), \quad k = 1, 2, \cdots, N,$$

where $V_{(0)} = \psi_o^t + \tau$. Since $\sum_{k=1}^{N}(V_k - \psi_o^t - \tau) = \sum_{k=1}^{N}\eta_k$ and the Jacobian of the transformation is $N!$, a substitution in (4) reveals that

$$p\left(\eta_1, \eta_2, \cdots, \eta_N\right) = \beta^{-N} e^{-\frac{1}{\beta}\sum_{k=1}^{N}\eta_k} \cdot \prod_{k=1}^{N} I\left[\eta_k\right],$$

i.e., η_k are *independent* exponential RVs with similar mean β. In addition, since each $\eta_k \sim \exp(\beta)$, each η_k also assumes a Gamma distribution $\eta_k \sim \Gamma(1,\beta)$. Using the relationship $\sum_{k=1}^{N}(V_k - V_{(1)}) = \sum_{k=2}^{N}\eta_k$, and the fact that each of $\eta_2, \eta_3, \cdots, \eta_N$ is independent of η_1 (and hence of $V_{(1)}$, since $\eta_1 = N(V_{(1)} - \psi_o^t - \tau)$), $\sum_{k=1}^{N}(V_k - V_{(1)}) \sim \Gamma(N-1,\beta)$ and is independent of $V_{(1)}$.

By a similar reasoning, it can be deduced that $\sum_{k=1}^{N}(U_k - U_{(1)}) \sim \Gamma(N-1,\alpha)$ and $\sum_{k=1}^{N}(W_k - W_{(1)}) \sim \Gamma(N-1,\gamma)$, and are independent of $U_{(1)}$ and $W_{(1)}$, respectively. Therefore, the one-to-one function $\mathbf{T}' = \{\sum_{k=1}^{N}(U_k - U_{(1)}), U_{(1)}, \sum_{k=1}^{N}(V_k - V_{(1)}), V_{(1)}, \sum_{k=1}^{N}(W_k - W_{(1)}), W_{(1)}\}$ of $\mathbf{T}$ is also sufficient for estimating $\mathbf{\Psi_A}$ because the sufficient statistics are unique within one-to-one transformations. Consequently, $\mathbf{T}'$ comprises of six independent random variables, which in terms of the three-parameter Gamma distribution are given by

$$\begin{aligned}
u = \sum_{k=1}^{N}(U_k - U_{(1)}) &\sim \Gamma\left(N-1, \alpha, 0\right), \\
U_{(1)} &\sim \Gamma\left(1, \alpha/N, \psi_o^s + \tau\right) \\
v = \sum_{k=1}^{N}(V_k - V_{(1)}) &\sim \Gamma\left(N-1, \beta, 0\right), \\
V_{(1)} &\sim \Gamma\left(1, \beta/N, \psi_o^t + \tau\right) \\
w = \sum_{k=1}^{N}(W_k - W_{(1)}) &\sim \Gamma\left(N-1, \gamma, 0\right), \\
W_{(1)} &\sim \Gamma\left(1, \gamma/N, \psi_o^t - \psi_o^s + \tau\right)
\end{aligned}$$

Note that the domains of u, v and w are controlled by $U_{(1)}$, $V_{(1)}$ and $W_{(1)}$, respectively. Next, it has to be checked whether $\mathbf{T}'$, or equivalently $\mathbf{T}$, is complete. Completeness implies that there is but one function of $\mathbf{T}$ that is unbiased. Let $g(\mathbf{T}')$ be a function of $\mathbf{T}'$ such that $E[g(\mathbf{T}')] = \mathbf{\Psi_A}$. Suppose that there exists another function h for which $E[h(\mathbf{T}')] = \mathbf{\Psi_A}$ is also true. Then,

$$E\left[g\left(\mathbf{T}'\right) - h\left(\mathbf{T}'\right)\right] = E\left[\pi\left(\mathbf{T}'\right)\right] = 0 \qquad \forall\ \mathbf{\Psi_A}$$

where $\pi(\mathbf{T}') \triangleq g(\mathbf{T}') - h(\mathbf{T}')$ and the expectation is taken with respect to $p(\mathbf{T}'; \mathbf{\Psi_A})$. As a result,

$$\begin{aligned}
&\iiiint\!\!\iint_{R_{\{U_{(1)},V_{(1)},W_{(1)}\}}} \pi\left(u, U_{(1)}, v, V_{(1)}, w, W_{(1)}\right) \cdot \frac{\alpha^{-(N-1)}}{\Gamma\left(N-1\right)} \\
&u^{N-2} e^{-\frac{u}{\alpha}} \cdot \frac{N}{\alpha} e^{-\frac{N}{\alpha}\left\{U_{(1)} - \psi_o^s - \tau\right\}} \cdot \frac{\beta^{-(N-1)}}{\Gamma\left(N-1\right)} v^{N-2} e^{-\frac{v}{\beta}} \cdot \frac{N}{\beta} \\
&e^{-\frac{N}{\beta}\left\{V_{(1)} - \psi_o^t - \tau\right\}} \cdot \frac{\gamma^{-(N-1)}}{\Gamma\left(N-1\right)} w^{N-2} e^{-\frac{w}{\gamma}} \cdot \frac{N}{\gamma} \\
&e^{-\frac{N}{\gamma}\left\{W_{(1)} - \psi_o^t + \psi_o^s - \tau\right\}} du dU_{(1)} dv dV_{(1)}\, dw\, dW_{(1)} = 0,
\end{aligned}$$

which is true for all $\mathbf{\Psi_A}$, and where $R_{U_{(1)},V_{(1)},W_{(1)}}$ is the region defined by $I[U_{(1)} - \psi_o^s - \tau]$, $I[V_{(1)} - \psi_o^t - \tau]$ and $I[W_{(1)} - \psi_o^t + \psi_o^s - \tau]$. The above relation can be expressed as

$$\begin{aligned}
&\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \pi\left(u, U_{(1)}, v, V_{(1)}, w, W_{(1)}\right) \cdot \\
&\left(uvw\right)^{N-2} e^{-\left\{\frac{u+NU_{(1)}}{\alpha} + \frac{v+NV_{(1)}}{\beta} + \frac{w+NW_{(1)}}{\gamma}\right\}} \cdot \\
&du\, dU_{(1)}\, dv\, dV_{(1)}\, dw\, dW_{(1)} \;=\; 0 \qquad \forall\ \mathbf{\Psi_A}
\end{aligned}$$

The left side of the above equation is the six-dimensional Laplace transform of $\pi(\mathbf{T}')$. It follows from the uniqueness theorem for two-sided Laplace transform that $\pi(\mathbf{T}') = 0$ almost everywhere, leading to $g(\mathbf{T}') = h(\mathbf{T}')$ and hence there is only one unbiased function of $\mathbf{T}'$. This proves that the statistic $\mathbf{T}'$, or equivalently $\mathbf{T}$, is complete for estimating $\mathbf{\Psi_A}$ when the links are asymmetric and all of α, β and γ are unknown. Finally, the complete sufficient statistic $\mathbf{T}$ is also minimal owing to Bahadur's theorem which states that *if* $\mathbf{T}$*, taking values in* $\Re^k$*, is sufficient for* $\mathbf{\Psi_A}$ *and boundedly complete, then* $\mathbf{T}$ *is minimal sufficient*.

Consequently, finding an unbiased estimator for $\mathbf{\Psi_A}$ as a function of $\mathbf{T}$ yields the MVUE, according to the Rao-Blackwell-Lehmann-Scheffé theorem. Therefore, the six unbiased functions of $\mathbf{T}$ for each of ψ_o^t, ψ_o^s, τ, α, β and γ just by inspection are

$$\hat{\mathbf{\Psi}}_{\mathbf{A}} = \frac{1}{N-1}\begin{bmatrix} N\left(2V_{(1)} - U_{(1)} - W_{(1)}\right) - \left(2\overline{V} - \overline{U} - \overline{W}\right) \\ N\left(V_{(1)} - W_{(1)}\right) - \left(\overline{V} - \overline{W}\right) \\ N\left(U_{(1)} - V_{(1)} + W_{(1)}\right) - \left(\overline{U} - \overline{V} + \overline{W}\right) \\ N\left(\overline{U} - U_{(1)}\right) \\ N\left(\overline{V} - V_{(1)}\right) \\ N\left(\overline{W} - W_{(1)}\right) \end{bmatrix}.$$

As a result, the MVUE for the desired parameter, the clock offset of the inactive nodes, for asymmetric unknown network delays is expressed as

$$\hat{\psi}_o^t = \frac{1}{(N-1)}\left[N\left(2V_{(1)} - U_{(1)} - W_{(1)}\right) - \left(2\overline{V} - \overline{U} - \overline{W}\right)\right],$$

and its variance, equal to its MSE, is

$$\mathrm{var}(\hat{\psi}_o^t) = \frac{1}{N\left(N-1\right)}\left(\alpha^2 + 4\beta^2 + \gamma^2\right).$$

Now turning to the symmetric case when $\alpha = \beta = \gamma \triangleq \lambda$, the likelihood function for the clock offset as a function of observations $\{U_k\}_{k=1}^N$, $\{V_k\}_{k=1}^N$ and $\{W_k\}_{k=1}^N$ from (1), (2) and (3) can be expressed as

$$L\left(\psi_o^t, \psi_o^s, \tau, \lambda\right) = \lambda^{-3N} e^{-\frac{1}{\lambda}\sum\limits_{k=1}^{N}\left[U_k+V_k+W_k-2\psi_o^t-3\tau\right]}. \\ I_{\left[U_{(1)}-\psi_o^s-\tau\right]} I_{\left[V_{(1)}-\psi_o^t-\tau\right]} I_{\left[W_{(1)}-\psi_o^t+\psi_o^s-\tau\right]}.$$

Note that the above likelihood function can be factored as

$$L\left(\psi_o^t, \psi_o^s, \tau, \lambda\right) = g_1\left(\sum_{k=1}^N U_k, \sum_{k=1}^N V_k, \sum_{k=1}^N W_k, \lambda\right) \\ g_2\left(U_{(1)}, \psi_o^s, \tau\right) g_3\left(V_{(1)}, \psi_o^t, \tau\right) g_4\left(W_{(1)}, \psi_o^t, \psi_o^s, \tau\right) \\ h_1\left(\psi_o^t, \tau, \lambda\right),$$

where

$$g_1\left(\sum_{k=1}^N U_k, \sum_{k=1}^N V_k, \sum_{k=1}^N W_k, \lambda\right) = e^{-\frac{1}{\lambda}\sum\limits_{k=1}^{N}\left[U_k+V_k+W_k\right]},$$
$$g_2(U_{(1)}, \psi_o^s, \tau) = I[U_{(1)} - \psi_o^s - \tau],$$
$$g_3(V_{(1)}, \psi_o^t, \tau) = I[V_{(1)} - \psi_o^t - \tau],$$
$$g_4(W_{(1)}, \psi_o^t, \psi_o^s, \tau) = I[W_{(1)} - \psi_o^t + \psi_o^s - \tau],$$
$$h_1(\psi_o^t, \tau, \lambda) = \lambda^{-3N} e^{\frac{N}{\lambda}[2\psi_o^t+3\tau]}.$$

It is evident that $\mathbf{T} = \{\sum_{k=1}^N (U_k + V_k + W_k), U_{(1)}, V_{(1)}, W_{(1)}\}$ are the minimal sufficient statistics according to Neymann-Fisher Factorization theorem. Now proceeding similarly as before, $\sum_{k=1}^N (U_k + V_k + W_k)$ is dependent on $U_{(1)}$, $V_{(1)}$ and $W_{(1)}$. As a result, $\mathbf{T}$ can be transformed into $\mathbf{T}' = \{\sum_{k=1}^N (U_k - U_{(1)} + V_k - V_{(1)} + W_k - W_{(1)}), U_{(1)}, V_{(1)}, W_{(1)}\}$. It can be concluded from the discussion in the last section that $\sum_{k=1}^N (U_k - U_{(1)} + V_k - V_{(1)} + W_k - W_{(1)})$ is Gamma distributed with parameters $(3(N-1), \lambda)$. Hence, $\mathbf{T}'$ is a combination of four independent random variables, which in terms of the three parameter Gamma distribution are

$$q = \sum_{k=1}^N (U_k - U_{(1)} + V_k - V_{(1)} + W_k - W_{(1)}) \\ \sim \Gamma\left(3\left(N-1\right), \lambda, 0\right), \\ U_{(1)} \sim \Gamma\left(1, \lambda/N, \psi_o^s + \tau\right), V_{(1)} \sim \Gamma\left(1, \lambda/N, \psi_o^t + \tau\right), \\ W_{(1)} \sim \Gamma\left(1, \lambda/N, \psi_o^t - \psi_o^s + \tau\right).$$

Next, defining $g(\mathbf{T}')$ and $h(\mathbf{T}')$ as functions of $\mathbf{T}'$ such that $E[g(\mathbf{T}')] = E[h(\mathbf{T}')] = \mathbf{\Psi_S}$,

$$E\left[g\left(\mathbf{T}'\right) - h\left(\mathbf{T}'\right)\right] = E\left[\pi\left(\mathbf{T}'\right)\right] = 0 \qquad \forall\ \mathbf{\Psi_S}$$

where the expectation is taken with respect to $p(\mathbf{T}'; \mathbf{\Psi_S})$. As a result, since the domain of q is also dictated by $U_{(1)}$, $V_{(1)}$ and $W_{(1)}$, it follows that

$$\iiiint\limits_{R_{U_{(1)},V_{(1)},W_{(1)}}} \pi\left(q, U_{(1)}, V_{(1)}, W_{(1)}\right) \frac{\lambda^{-3(N-1)}}{\Gamma\left[3\left(N-1\right)\right]} q^{3N-4} e^{-\frac{q}{\lambda}} \\ \left(\frac{N}{\lambda}\right)^3 e^{-\frac{N}{\lambda}\left\{U_{(1)}+V_{(1)}+W_{(1)}-2\psi_o^t-3\tau\right\}} dq dU_{(1)} dV_{(1)} dW_{(1)} = 0,$$

which is true for all $\mathbf{\Psi_S}$ and where $R_{U_{(1)},V_{(1)},W_{(1)}}$ is the region defined by $I[U_{(1)} - \psi_o^s - \tau]$, $I[V_{(1)} - \psi_o^t - \tau]$ and $I[W_{(1)} - \psi_o^t + \psi_o^s - \tau]$. It follows that for any $\mathbf{\Psi_S}$ the following equation must hold:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \pi\left(q, U_{(1)}, V_{(1)}, W_{(1)}\right) q^{3N-4} \\ e^{-\frac{N}{\lambda}\left\{\frac{q}{N}+U_{(1)}+V_{(1)}+W_{(1)}\right\}} dq dU_{(1)} dV_{(1)} dW_{(1)} = 0.$$

From the uniqueness of the two-sided Laplace transform, it follows that $\pi(\mathbf{T}') = 0$ almost everywhere, resulting in the completeness of $\mathbf{T}'$, or equivalently $\mathbf{T}$. Hence, $\mathbf{T}$ is also the minimal sufficient statistics from Bahadur's theorem and the MVUE is an unbiased estimator of $\mathbf{T}$ expressed as

$$\hat{\mathbf{\Psi}}_{\mathbf{S}} = \begin{bmatrix} 2V_{(1)} - U_{(1)} - W_{(1)} \\ V_{(1)} - W_{(1)} \\ \frac{1}{3(N-1)}\left\{3N\left(U_{(1)} + W_{(1)} - V_{(1)}\right) + \right. \\ \left. 2\left(2V_{(1)} - U_{(1)} - W_{(1)}\right) - \left(\overline{U} + \overline{V} + \overline{W}\right)\right\} \\ \frac{N}{3(N-1)}\left\{\left(\overline{U} + \overline{V} + \overline{W}\right) - \left(U_{(1)} + V_{(1)} + W_{(1)}\right)\right\} \end{bmatrix}.$$

Hence, the MVUE for the clock offset of the inactive node, in the case of symmetric unknown delays, is given by

$$\hat{\psi}_o^t = 2V_{(1)} - U_{(1)} - W_{(1)},$$

and its variance can be expressed as $\mathrm{var}\left(\psi_\mathrm{o}^\mathrm{t}\right) = 6\lambda^2/\mathrm{N}^2$.

3. CONCLUSION

For a general sender-receiver protocol, the expression for the minimum variance unbiased estimator has been derived, which is important for synchronization in the energy conserving wireless sensor networks.

4. REFERENCES

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