DISTRIBUTED DETECTION OF A NUCLEAR RADIOACTIVE SOURCE BASED ON A HIERARCHICAL SOURCE MODEL

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ABSTRACT

Detection of a nuclear radioactive source is considered using a parallel sensor network architecture and a fusion center. A *Poisson-Gamma hierarchical model* is used to represent the distribution of the count data received by the sensors. Local sensors are assumed to be single threshold binary quantizers that send a vector of sensor decisions over time to the fusion center for global decision-making. Using the developed count model, a generalized likelihood ratio test (GLRT) using a *restricted range MLE (RMLE)* is proposed to declare the global decision. The performance improvement resulting from using the restricted range MLE over the unrestricted MLE while implementing the GLRT is depicted using simulated as well as real data collected from a test-bed using radiation sensors. Using bootstrap, 95% confidence bounds on the ROC curves, evaluated using real data, are obtained.

***Index Terms*—** Distributed Detection, Decision Fusion, Hierarchical Model, Restricted MLE

1. INTRODUCTION

An important application of detecting the presence of a nuclear radioactive source is in countering potential terrorist activities like the explosion of the so called dirty bomb in a densely populated area. The task is to detect the low level radiations from vehicles transporting these sources before they reach their destination. We propose a system comprising a network of radiation counters operating collaboratively to detect the presence of a radioactive source.

Detection of radioactive sources using sensor networks has received some attention over the past couple of years. In [1], the authors examine the increase in signal-to-noise ratio obtained by a simple combination of data from networked sensors compared to a single sensor. The costs and benefits of using a network of radiation detectors for radioactive source detection are analyzed and evaluated in [2]. In [3], the authors propose a Bayesian methodology for source location estimation of a nuclear source from raw sensor measurements. In [4], a distributed detection system for detecting the presence of a nuclear source was presented using the theory of copulas to exploit spatial correlation of sensor observations and in [5], a detection method using sequential probability ratio test (SPRT) was developed. Both methods assumed a Poisson distribution for radiation counts emitted by the radioactive source. In this work, we further investigate the problem of nuclear source detection and make the following contributions:

- **Experiment-driven signal modeling**. We propose the use of a hierarchical signal model to characterize the probability distribution of the received sensor measurements.
- **Distributed detection of the radiation source using the developed signal model**. Here, a GLRT [6] based decision fusion method employing *restricted-range maximum likelihood estimation (RMLE)* is proposed.

2. PROBLEM FORMULATION

We first consider development of an appropriate signal model to represent the measurements collected by the sensors. We make use of data collected from a radiation test-bed set up at Oak Ridge National Laboratory (ORNL). The test-bed setup is described in [5]. Two methods are used to evaluate the developed signal model. The first methods compares the empirical cumulative distribution function (cdf) of sensor observations and the cdf of the proposed distribution obtained by estimating the model parameters from sensor observations. In the second method, the proposed model is evaluated using the chi-square goodness-of-fit (χ^2 GoF) test [7].

We then consider distributed detection of the nuclear radioactive source using a parallel sensor network architecture. The detection problem is formulated as a composite hypothesis testing problem where the H_1 and H_0 hypotheses indicate the presence and absence of a source respectively. L radiation sensors monitor a region for the presence of a nuclear source for N time intervals. Under H_0, the sensor measurements correspond to background radiation only. Under H_1, sensor measurements represent the source as well as background radiation. The parameters of the model representing the background signal under H_0 ($\boldsymbol{\theta_0}$) are assumed to be known *a priori* while those under H_1, that are dependent on the source location and intensity parameters, are unknown ($\boldsymbol{\theta_1}$). The distribution of the observations under both hypotheses differs only in terms of the parameter vector. Hence, the composite hypothesis testing problem can be recast as,

$$\begin{aligned} H_0 &: \boldsymbol{\theta} = \boldsymbol{\theta_0} \\ H_1 &: \boldsymbol{\theta} = \boldsymbol{\theta_1} \neq \boldsymbol{\theta_0} \end{aligned} \tag{1}$$

The local decision makers (sensor nodes) are single threshold binary quantizers that send a vector of one bit decisions over time to a fusion center. In this paper, we assume that the fusion center is equipped with the location information of all sensors. Let τ_i be the quantizer threshold at the i^{th} sensor. The sensor decisions, at any time interval $1 \leq n \leq N$, are quantized versions of sensor observations defined as

$$u_{in} = \mathbb{Q}(z_{in}) = \begin{cases} 0 & \text{if } -\infty < z_{in} \leq \tau_i, \\ 1 & \text{if } \tau_i \leq z_{in} < \infty \end{cases} \tag{2}$$

Sensor thresholds ($\tau_i, \forall i$) are set by constraining the local probability of false alarm at each sensor (determined using the measurement statistics under H_0). Since H_1 is a composite hypothesis and no prior knowledge about the parameter vector $\boldsymbol{\theta_1}$ is assumed to be known, the fusion rule is formulated as a generalized likelihood ratio test (GLRT). In deriving the GLRT, conditional independence of sensor observations, over space as well as time, is assumed.

3. MEASUREMENT MODEL DEVELOPMENT

Radiation counts (Z) are typically assumed to be Poisson(λ) distributed random variables ([8] and [9]). Note that $\mathbb{E}[Z] = \text{Var}[Z] = \lambda$. However, count data that occur in practice are often over-dispersed, i.e., the variance is higher than the mean. This is evident from the statistics of the data collected from the ORNL test-bed. Five datasets containing 167 radiation counts each were collected using a single sensor in both the presence and absence of the source. The sample estimates (SE) of mean and variance, along with their corresponding 95% confidence intervals (CI) are displayed in Table 1. The 95% CIs are obtained by evaluating bootstrap percentiles [10] from 10000 bootstrap samples. From Table 1, we can see that for each dataset, the sample variance is much higher than the sample mean under both hypotheses. Moreover, the bootstrap confidence intervals for the mean and variance are also disjoint and far apart. This further indicates that the mean and variance under either hypothesis are unlikely to have values close to each other. Hence, the Poisson distribution is not be an appropriate model for the measurement counts because it does not take into account the observed over dispersion. We propose to employ a mixture distribution arising from a two-stage hierarchical model to statistically characterize the observed radiation counts. Letting the observed counts at the i^{th} sensor to be represented by z_{in}, where $i = 1, \ldots, L$ and $n = 1, \ldots, N$, the two-stage hierarchy under the j^{th} hypothesis is given as follows.

$$z_{in} \sim \text{Poisson}(\lambda_{ij}) \qquad \lambda_{ij} \sim \text{Gamma}(\alpha_{ij}, \beta_{ij}) \tag{3}$$

where, $j = 0, 1$ corresponding to H_0 and H_1.

The above hierarchical model can be considered as a "more variable" Poisson distribution [11]. The extra variability is accounted for by assuming the rate of the Poisson random variable at each time interval to be random.

DS		*Background*		*Source+Background*	
		μ	σ^2	μ	σ^2
1	SE	7.54	53.18	9.00	64.17
	CI	[6.44, 8.67]	[39.40, 67.94]	[7.84, 10.23]	[48.69, 80.05]
2	SE	8.39	91.59	9.90	88.18
	CI	[6.99, 9.87]	[61.64, 125.31]	[8.52, 11.40]	[56.80, 127.67]
3	SE	6.99	32.38	8.55	47.54
	CI	[6.18, 7.83]	[25.34, 39.57]	[7.54, 9.62]	[35.16,61.95]
4	SE	6.53	45.78	8.28	63.36
	CI	[5.54, 7.59]	[31.51, 61.56]	[7.12, 9.49]	[41.26, 94.03]
5	SE	6.83	41.32	8.28	54.21
	CI	[5.88, 7.84]	[29.30, 54.69]	[7.24, 9.32]	[41.84, 67.02]

Table 1. First and second order statistics of the sensor count data along with their corresponding CIs both in the presence and absence of the source. DS = Dataset number

To validate the proposed model for the statistical distribution of sensor measurements, we evaluate the empirical cdf of the real data and the cdfs obtained by fitting the proposed model and the Poison distribution model to the real data. The cdf plots are shown in Figures 1 and 2 for the background and source-plus-background count respectively for a single dataset. Both figures clearly illustrate that the proposed mixture distribution is a much better fit for the observed measurements when compared to the Poisson distribution.

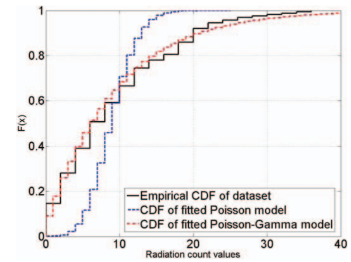

Fig. 1. Empirical cdf of the real background count data and corresponding fitted cdfs of the estimated Poisson-Gamma mixture and simple Poisson models

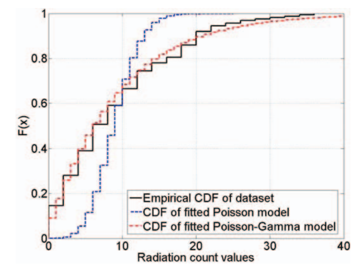

Fig. 2. Empirical cdf of the real background plus source count data and corresponding fitted cdfs of the estimated Poisson-Gamma mixture and simple Poisson models

We next consider the χ^2 GoF test [7] to further investigate the validity of the proposed model. For the sake of brevity, the details about carrying out the test procedure have not been included in this paper. The goodness-fit of test was conducted using real data samples accounting for both background and source plus background radiation counts. The resulting p-values from the test for five datasets are shown in Table 2. It can be seen that the p-values resulting from the proposed mixture distribution are significant whereas those resulting from the Poisson model are always zero. Thus there is a greater probability for the null hypothesis to be accepted during the GoF test (at some significance level) when the null hypothesis distribution is assumed to be the Poisson-Gamma mixture. On the other hand if the null hypothesis distribution is assumed to be Poisson, it will always be rejected. This result provides further justification for using the hierarchical model than the simple Poisson model.

Dataset	*p-value under* H_1		*p-value under* H_0	
	Poisson	*Proposed*	*Poisson*	*Proposed*
1	0.00	0.2262	0.00	0.2759
2	0.00	0.1158	0.00	0.0664
3	0.00	0.0635	0.00	0.0471
4	0.00	0.3095	0.00	0.1289
5	0.00	0.3101	0.00	0.0684

Table 2. p-values for the Poisson and proposed model resulting from χ^2 GoF test for the count data under H_1 and H_0 hypotheses

4. DISTRIBUTED DETECTION SYSTEM DESIGN

Recall that the detection problem is given by Eq. (1). The parameter vector under hypothesis H_j $(j = 0, 1)$ is given by $\boldsymbol{\theta_j} = [\boldsymbol{\alpha_j}, \boldsymbol{\beta_j}]^T$ where $\boldsymbol{\alpha_j} = [\alpha_{1j}, \ldots, \alpha_{Lj}]^T$ and $\boldsymbol{\beta_j} = [\beta_{1j}, \ldots, \beta_{Lj}]^T$. The distribution of the sensor observations is given by the Poisson-Gamma hierarchical model explained in Section 3. Under this model (see Eq.(3)), the probability mass function (pmf) of the radiation counts z_{in} under H_j $(j = 0, 1)$ at the i^{th} sensor, is as follows.

$$P(Z_{in} = z_{in}|H_j) = \frac{1}{\Gamma(\alpha_{ij})} \frac{\Gamma(\alpha_{ij} + z_{in})}{z_{in}!} \frac{\beta_{ij}^{z_{in}}}{(\beta_{ij} + 1)^{\alpha_{ij}+z_{in}}} \quad (4)$$

Let $Pr(u_{in} = 1|H_1) = p_i$ and $Pr(u_{in} = 1|H_0) = q_i$ be the local sensor probability of detection and local sensor probability of false alarm respectively at the i^{th} sensor. Constraining, q_i, the local sensor threshold τ_i can be obtained as follows.

$$\tau_i = \left\lceil F_{Z_i}^{-1}(1 - q_i) \right\rceil \quad (5)$$

where, $F_{Z_i}^{-1}(\cdot)$ is the inverse cdf for $Z_i = z_{in}, \forall n$.

The composite hypothesis test at the fusion center can be reformulated in terms of local sensor statistics (p_i and q_i). Maximizing the pmf of sensor decisions under H_1 over $\boldsymbol{\theta_1}$ is equivalent to maximizing the pmf over p_i for all $i = 1, \ldots, L$. Under the new formulation (see Eq.(6)), let the parameter vector under H_1 be $\boldsymbol{\phi_1} = [p_1, \ldots, p_L]^T$ and that under H_0 be $\boldsymbol{\phi_0} = [q_1, \ldots, q_L]^T$. $\boldsymbol{\phi_1}$ is unknown, since it depends on $\boldsymbol{\theta_1}$.

$$\begin{aligned} H_0 &: \boldsymbol{\phi} = \boldsymbol{\phi_0} \\ H_1 &: \boldsymbol{\phi} = \boldsymbol{\phi_1} \geq \boldsymbol{\phi_0} \end{aligned} \quad (6)$$

Let $\mathbf{u_i}$ be the vector of sensor decisions received over N time periods from sensor i. The collection of decision vectors at the fusion center is given by $\mathbf{u} = [\mathbf{u_1}, \ldots, \mathbf{u_L}]^T$. The generalized likelihood ratio (GLR) [6] based test statistic for fusion is given by Eq. (7), where $\hat{\boldsymbol{\phi_1}}$ denotes the MLE [11] of $\boldsymbol{\phi_1}$.

$$\Lambda_G(\mathbf{u}) = \frac{P(\mathbf{u}|H_1, \hat{\boldsymbol{\phi_1}})}{P(\mathbf{u}|H_0)} \quad (7)$$

Note that p_i is nothing but the success probability of the Bernoulli distributed random variable u_{in}. Consider the likelihood function of p_i given in Eq. (8) where the assumption of temporal independence of sensor decisions has been used. It can be seen that the i^{th} sensors' decisions ($\mathbf{u_i}$) are sufficient statistics [11] for estimating p_i.

$$P(\mathbf{u_i}|p_i) = \prod_{n=1}^{N} p_i^{u_{in}} (1 - p_i)^{(1-u_{in})} = p_i^{\sum u_{in}} (1 - p_i)^{(N - \sum u_{in})} \quad (8)$$

The ML estimate of p_i is given by $\hat{p_i} = \arg\max_{p_i} P(\mathbf{u_i}|p_i)$.

It can be easily shown that $\hat{p_i} = \bar{u_i}$, where $\bar{u_i} = (1/N)\sum_{n=1}^{N} u_{in}$ which is nothing but the mean of the sensor decisions. Thus, the ML estimate of $\boldsymbol{\phi_1}$ can be obtained.

Using the temporal and spatial independence of sensor decisions under both hypotheses, the test statistic becomes

$$\Lambda_G(\mathbf{u}) = \frac{\prod_{i=1}^{L} \prod_{n=1}^{N} P(u_{in}|H_1, \hat{\boldsymbol{\phi_1}})}{P(u_{in}|H_0)} \quad (9)$$

Taking log on both sides and simplifying, the GLR based test statistic is given as follows.

$$\log \Lambda_G(\mathbf{u}) = N \sum_{i=1}^{L} \left[\log \left(\frac{\hat{p_i}(1 - q_i)}{(1 - \hat{p_i}) q_i} \right) \right] \bar{u_i} + \log \left(\frac{1 - \hat{p_i}}{1 - q_i} \right) \quad (10)$$

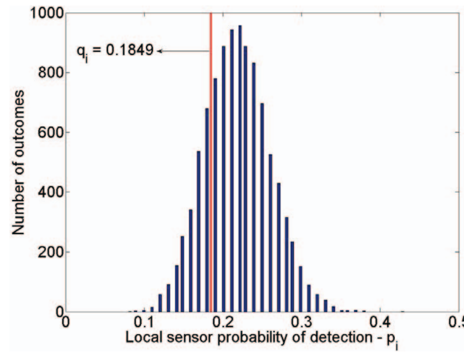

Fig. 3. Histogram of $\hat{p_i}$ with $N = 100$, $q_i \leq 0.2$ which gives $\tau_i = 15$.

It can be seen that Eq.(10) is nothing but the generalized Chair-Varshney test statistic where the local sensor probabilities of detection have been replaced by their ML estimates.

Since the first step of a GLRT is MLE, the detection performance will depend on the quality of the estimate which again is a function of the signal power and the number of observations (N). When the signal power is low and/or N is small, the quality of the estimate will suffer. In Figure 3, we show the histogram of the estimated probability of detection ($\hat{p_i}$) for a particular sensor when $N = 100$ and $q_i \leq 0.2$ after 10000 Monte Carlo runs. The paramter values $(\alpha_{ij}, \beta_{ij})$ used were estimated from the experimental data corresponding to that sensor. It can be noticed that 21.34% of the times, $\hat{p_i}$ is smaller than q_i. Hence it is possible that the estimated value of a particular sensor's probability of detection (p_i) may be smaller than the probability of false alarm (q_i), thereby degrading the decision-making process of that particular sensor. If such a sensor's decisions are used for fusion it will bring down the global performance.

To avoid the degradation in global performance due to poor estimates of the p_i, we propose using the *restricted range* MLE (RMLE) [11] while evaluating the GLR based test statistic. RMLE constrains the range of the parameter of a random variable to be estimated based on some prior knowledge or requirement. For detection problems, it is reasonable and more meaningful to have the requirement that the test is unbiased [12], i.e, the probability of detection is at least greater than or equal to the probability of false alarm. Assuming local sensor tests are unbiased (i.e. $p_i \geq q_i$), the RMLE is given as follows.

$$\hat{p_i} = \max(q_i, \bar{u_i}); \quad (11)$$

The impact of using RMLE instead of the unrestricted MLE can be understood by considering Eq.(10). Suppose $\bar{u_i} < q_i$, then $\hat{p_i} = q_i$ as a result of using the RMLE and the term corresponding to that sensor in the GLR test statistic reduces to zero. Thus, if a sensor is "bad" (i.e. if $\bar{u_i} < q_i$ for some i) then the RMLE has an effect of automatically censoring that particular sensor's decisions during fusion, thereby preventing the degradation in global performance that may occur if the unrestricted MLE is used for GLRT.

5. EXPERIMENT RESULTS

We present the performance comparison of the GLRT based fusion rule using the unrestricted and the restricted MLE by using simulated data first, using $L = 3$ sensors and 100 observations. The parameter values under both hypotheses ($\boldsymbol{\alpha_{ij}}$ and $\boldsymbol{\beta_{ij}}$ as defined in Section 4)

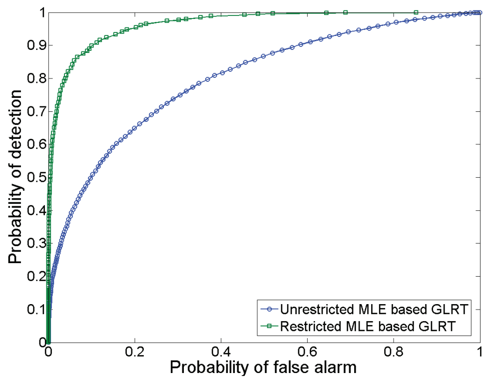

Fig. 4. ROC using simulated dataset

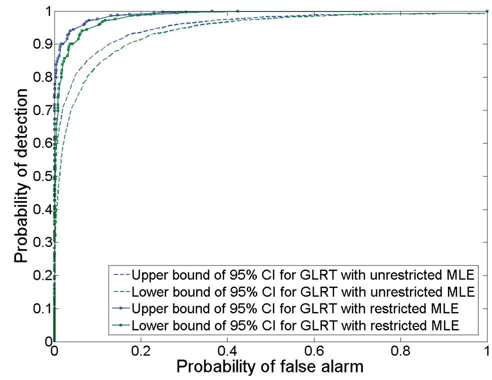

Fig. 5. ROC using real dataset

used to generate the simulated data are given as follows.

$$\boldsymbol{\alpha_{i0}} = [1.0667, 1.3163, 1.0506, 1.0804, 1.2556]$$
$$\boldsymbol{\beta_{i0}} = [9.4374, 9.9764, 9.8210, 9.8180, 9.6980]$$
$$\boldsymbol{\alpha_{i1}} = [0.9868, 0.9880, 0.9951, 0.9518, 0.9807]$$
$$\boldsymbol{\beta_{i1}} = [8.4987, 8.9980, 9.5906, 9.1329, 9.0064]$$

Thresholds for the local binary quantizers are obtained by constraining the local probability of false alarm at each sensor to ≤ 0.2. 10000 Monte Carlo runs are carried out to generate the ROC curves which are shown in Figure 4. From the figure it is clear that, using the GLRT based fusion rule with RMLE results in considerable improvement in the detection performance than the unrestricted MLE.

Next real sensor traces belonging to a particular dataset collected from the ORNL test-bed are used to evaluate the detection performance of both strategies. Decision vectors from 3 sensors are fused. The parameter values under H_0 and H_1 are as below.

$$\boldsymbol{\alpha_{i0}} = [0.6921, 1.8951, 0.8518] \quad \boldsymbol{\beta_{i0}} = [12.1334, 4.2468, 8.1329]$$
$$\boldsymbol{\alpha_{i1}} = [1.2195, 1.0506, 1.0804] \quad \boldsymbol{\beta_{i1}} = [6.6294, 9.4264, 8.8180]$$

The local thresholds are obtained by constraining $q_i \leq 0.1$. Due to limitation in the number of datasets available with the same parameter values, a Monte Carlo approach cannot be adopted to generate the exact ROC curves. Instead, using bootstrap, the 95% confidence interval for the two ROCs corresponding to GLRT using RMLE and unrestricted MLE are shown in Figure 5. Again, the confidence interval obtained using RMLE is much higher than that obtained using the unrestricted MLE demonstrating the superiority of the RMLE based GLRT.

6. CONCLUSIONS

We considered a model based approach for distributed detection of a nuclear radiaoactive source. An experiment-driven model was developed to represent the probability distribution of radioactive counts measured by the sensors. The developed model was validated using a statistical GoF test. Using this model, a distributed detection system was designed. The local sensor thresholds were determined by constraining the local probability of false alarm and a GLRT based fusion strategy was used. In implementing the GLRT, restricted range MLE to estimate the unknown parameters under the H_1 hypothesis was proposed. The results presented using both simulated as well as real datasets demonstrate the superiority of the RMLE based GLRT approach.

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