DIVERSITY ANALYSIS OF ANTENNA SELECTION OVER FREQUENCY-SELECTIVE MIMO CHANNELS

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ABSTRACT

Antenna selection is a simple yet efficient technique to obtain diversity advantage from Multi-Input and Multi-Output (MIMO) communication systems. Without specifying system factors such as antenna selection criterion, coding and decoding, equalization, and channel statistics, we evaluate diversity orders of antenna selections over frequency-selective MIMO channels. We show that the diversity order of any configuration approximately scales in the numbers of transmit and receive antennas. Numerical simulations are provided to validate our analysis.

***Index Terms*— MIMO, diversity, antenna selection**

1. INTRODUCTION

Multi-Input Multi-Output (MIMO) technologies that employ multiple antennas at the transmitter and/or at the receiver improve the overall capacity and/or the error-rate performances in wireless communication systems. One major drawback of MIMO is the presence of multiple radio-frequency (RF) chains which consist of amplifiers, downconverters, and A/D converters. These multiple RF chains increase the complexity and hardware cost of the system.

To capture the benefits of multiple antennas system and to reduce the implementation costs, antenna selection, which selects some antennas from an array of multiple antennas based on a certain criterion, has been developed [1, 2]. For flat (non frequency-selective) channels, transmit antenna is selected to maximize channel capacity [3], or to minimize the error-rate using linear receivers [4]. The antennas for space-time block coding (STBC) are selected based on the maximization of the Frobenius channel norm (or equivalently received signal-to-noise ratio (SNR)) [5]. Antenna selection is also applicable to MIMO over frequency-selective channels. For orthogonal frequency-division multiplexing (OFDM) systems, channel norms are used to select receive antennas in coherent [6] and in non-coherent [7] STC-OFDM systems, as well as to select transmit antennas in STBC-MISO-OFDM systems [8].

The symbol error rate (SER) performance of an antenna selection depends on several factors, e.g., selection criterion, coding/decoding, equalization, and channel statistics. It is not easy to obtain an (even approximate) analytical expression of SER of the antenna selection. Thus, the diversity order, which is the slope of SER curve at high SNR, is utilized to roughly evaluate the performance. As the diversity order increases, SER decreases rapidly as a function of SNR.

For flat fading channels, it is shown in [9] that the diversity order of a single antenna selection is identical with the diversity order of maximum ratio combining. For MIMO-OFDM with receive antenna selection by the norm criterion and ML decoding, the pairwise error probability (PEP) analysis reveals that the diversity order scales in the number of receive antennas [6,7]. However, except for some cases, it is not straightforward to analyze SER in general, since the relation between SER and receive SNR is complicated. Moreover, different criteria can be developed for frequency-selective channels. Thus, it needs to evaluate diversity orders of MIMO systems with different equalization, detection, and channel statistics.

In this paper, we develop a technique to roughly evaluate the diversity advantage without specifying equalization, detection, and channel statistics. Our idea is simple: We define the diversity order of a system with a fixed configuration and compare it with a system with antenna selection of the same numbers of transmit and receive antennas with the original system. This avoids complicated SER derivations. Then, under some mild conditions, we show that the diversity order approximately scales in the numbers of transmit and receive antennas. Since we do not specify equalization, detection, and channel statistics, our result can be applicable to many MIMO systems with antenna selection. As a practically important example, we consider two transmit antenna selection in the downlink from the base station to the mobile terminal. Performances of three different criteria are assessed by simulations to validate our theoretical finding.

2. ANTENNA SELECTION OVER FREQUENCY-SELECTIVE CHANNELS

Let us consider a Multi-Input and Multiple-Output (MIMO) communication system with M_t transmit and N_r receive antennas over frequency-selective channels. The channel is assumed to be quasi-static fading such that the channel tap coefficients are invariant over a sufficient number of symbols and independent, identically distributed (i.i.d.) of each other.

Let $\{h^{(m,n)}(l)\}_{l=0}^{L}$ be the discrete-time baseband equivalent channel impulse responses of the taps from transmit antenna $m(\in [1, M_t])$ to the receive antenna $n(\in [1, N_t])$, where L is the maximum order of the FIR channels. At the receiver terminal, we assume perfect timing and carrier synchronization.

We utilize the cyclic prefix (CP) to mitigate frequency-selective multipath effects. In OFDM [10] and so-called single carrier (SC) CP or CP only block transmission [11], a CP of length $N_{cp} \geq L$ is appended to the tail of a data sequence of length N to avoid interference between consecutive blocks. We assume that CP is sufficiently long so that there is no inter block interference.

Antenna selection is a simple technique to enhance system performance, which is especially beneficial for the system with limited number RF chains. In antenna selection, a predetermined number of antennas with *good* channel condition are selected for transmission so as to enjoy antenna diversity. In this paper, we select $\bar{M}_t$ transmit and $\bar{N}_r$ receive antennas from M_t transmit and N_r receive antennas.

The analytical error-rate analysis for antenna selection is complicated even for flat MIMO channels. Moreover, unlike flat MIMO channels, many criteria for frequency-selective MIMO channels can be developed with/without space time coding. To capture the error-rates of many systems with antenna selection, we develop a technique to analyze their diversity orders.

3. DIVERSITY ANALYSIS OF ANTENNA SELECTION

At high (receive) SNR $\bar{\gamma}$, the average symbol error probability (SEP) of a system is often approximated as

$$P_E \approx (G_c \bar{\gamma})^{-G_d}, \tag{1}$$

where G_c is the coding gain and G_d is the diversity gain.

For our diversity analysis, we borrow useful results in [9] on the coding and the diversity gain. For a given transmission scheme with an antenna selection criterion, suppose that the SEP of an $\bar{M}_t$ transmit and $\bar{N}_r$ receive antenna system depends on a positive value x ranging from 0 to ∞ , which is a function of the setting, e.g., channel, equalization and STC coding/encoding. Then, we express the SEP as $P_E(x)$. The variable x is random resulted from channel fading. For example, we may take the instantaneous receive SNR for x. We correspond poor condition to small x and vice versa.

We assume that the probability density function (PDF) $p(x)$ of x can be approximated when x tend to 0 from above as

$$p(x) = ax^t + o(x^{t+\epsilon}), \tag{2}$$

where $\epsilon > 0$ and a and t are positive constants, which are dependent on the setting. Let $Q(y)$ denote the Gaussian-Q function $Q(x) := (1/\sqrt{2\pi}) \int_y^\infty e^{-t^2/2} dt$.

It is proved in [9] that if the SEP can be expressed for a positive k as

$$P_E(x) = Q(\sqrt{kx}), \tag{3}$$

then the diversity order is

$$G_d = t + 1. \tag{4}$$

As pointed out in [9], the result can be applicable to more general $P_E(x)$.

The average SER can be expressed as

$$\int_0^\infty P_E(x) p(x) dx. \tag{5}$$

In place of (3), we assume that for any finite $t \geq 0$, $P_E(x)$ meets

$$\int_0^\infty P_E(x) a x^t dx = (G_c \bar{\gamma})^{-(t+1)} + o(\bar{\gamma}^{-(t+1)}), \tag{6}$$

and

$$\int_0^\infty P_E(x) o(x^{t+\epsilon}) dx = o(\bar{\gamma}^{-(t+1)}). \tag{7}$$

Now let us return to antenna selection. We select $\bar{M}_t$ transmit antennas among M_t transmit antennas and $\bar{N}_r$ receive antennas among N_r receive antennas that has the largest value of x.

There are ${}_{M_t}C_{\bar{M}_t}$ candidates for for transmit antennas selection and ${}_{N_r}C_{\bar{N}_r}$ candidates for receive antennas selection. Thus, we have ${}_{M_t}C_{\bar{M}_t} \cdot {}_{N_r}C_{\bar{N}_r}$ candidates for selection in total.

For simplicity, we assume that $\bar{M}_t$ is a divisor of M_t and $\bar{N}_r$ is a divisor of N_r. One can partition transmit antennas into $\tilde{M}_t := M_t/\bar{M}_t$ sets such that each transmit antenna belongs to only one set. Similarly, one can divide receive antennas into $\tilde{N}_r = N_r/\bar{N}_r$ sets. In this case, we have $\tilde{M}_t \tilde{N}_r$ combinations of transmit and receive antenna in total. Suppose that we select the best set from these combinations, which has the largest value of x. Then, its SEP $\bar{P}_e$ is an upper bound of the SEP of the original antenna selection.

Since we assume i.i.d. channels, each combination has statistically independent x. Let y be the largest of $\tilde{M}_t \tilde{N}_r$ x's. Since y is the largest among $\tilde{M}_t \tilde{N}_r$ independent random values, its PDF can be expressed as

$$\tilde{M}_t \tilde{N}_r \left(\int_0^y f(x) dx \right)^{\tilde{M}_t \tilde{N}_r - 1} f(y). \tag{8}$$

Substituting (2) into (8) leads to

$$\tilde{M}_t \tilde{N}_r \left[\frac{a^{\tilde{M}_t \tilde{N}_r}}{(t+1)^{M_t - 1}} \right] y^{\tilde{M}_t \tilde{N}_r (t+1) - 1} + o(y^{\tilde{M}_t \tilde{N}_r (t+1) - 1 + \epsilon}). \tag{9}$$

It follows from our assumption (6) and (7) and

$$\bar{P}_e = \int_0^\infty P_E(y)p(y)dy, \tag{10}$$

that the diversity order of $\bar{P}_e$ is given by

$$\tilde{M}_t\tilde{N}_r(t+1) = \tilde{M}_t\tilde{N}_r G_d \tag{11}$$

Therefore we can conclude that the diversity order of our antenna selection is at least $\tilde{M}_t\tilde{N}_r G_d$. This means that for fixed $\bar{M}_t$ and $\bar{N}_r$, the diversity order is scaled approximately lineally in the numbers M_r and N_r of equipped antennas.

For a single antenna selection, our result coincides with the selection diversity order in [9]. It also subsumes the result that the diversity order of OFDM with STC for receive antenna selection with ML equalization and the norm criterion scales in the number of receive antennas [6, 7] and is applicable to wider class of MIMO systems.

4. TWO TRANSMIT ANTENNA SELECTION

To validate our analysis, we take an example. For the practical importance and for the simplicity of presentation, we consider a downlink from the base station to the mobile terminal where multiple antennas are possible at the base station but the mobile terminal has only one antenna.

In receive antenna selection, since the transmission scheme is fixed, the increase of the number of receive antennas results in the increase of the received SNR and hence improves the SER performance. On the other hand, in transmit antenna selection, two antenna selection does not necessarily outperform one antenna selection. However, except for some special cases, two antenna selection with Alamouti's orthogonal STBC exhibits better performance than one antenna selection without space-time block coding (STBC) [8].

Let us select two transmit antennas with indices i and j ($i, j \in [1, N_t]$) be used for transmission. After Alamouti's orthogonal STBC, coded symbols are transmitted through the selected antennas. More precisely, at time $2k$ for k integer, the information symbols $s_n(2k)$ and $s_n(2k+1)$ are sent from antenna i at the nth subcarrier and from antenna j at the nth subcarrier, respectively, while at time $2k+1$, $s_n^*(2k+1)$ and $-s_n^*(2k)$ are sent over from antenna i at the nth subcarrier and from antenna j at the nth subcarrier, where $(\cdot)^*$ denotes conjugation.

We define E_s/N_0 as the transmit SNR per symbol, where E_s is the transmit energy per symbol N_0 is the variance of the additive white Gaussian noise (AWGN). Then, the receive SNR at subcarrier n, denoted $\gamma_n^{(i,j)}$, is found to be

$$\gamma_n^{(i,j)} = \frac{\gamma}{2}(|H_n^{(i)}|^2 + |H_n^{(j)}|^2), \tag{12}$$

where $H_n^{(k)}$ denotes the frequency response of the channel from transmit antenna k at frequency $2\pi n/N$.

The SER of the system with a pair (i, j) of transmit antenna depends on the signal constellation and is often (approximately) expressed as a function in SNR or signal-to-interference-noise-ratio (SINR), which is also dependent on the equalization. Let $\text{SER}^{(i,j)}$ be the SER of the system with active transmit antennas i and j. Then, the SER can be minimized by selecting a pair of transmit antennas such that

$$\arg \min_{i,j\in[1,N_t],i\neq j} \text{SER}^{(i,j)}. \tag{13}$$

If we can express $\text{SER}^{(i,j)}$ as a function of SNR and channel state information (CSI), then the optimal two transmit antennas can at least be obtained by exhaustive numerical search. Take QPSK constellation and hard-detection of linearly zero-forcing (ZF) equalized signals for example.

For QPSK constellation, the bit-error rate (BER) can be utilized in place of SER. The BER averaged over one OFDM symbols related to the SNR of each subcarrier $\gamma_n^{(i,j)}$ as follows:

$$\text{BER}^{(i,j)} = \frac{1}{N}\sum_{n=0}^{N-1} Q(\sqrt{\gamma_n^{(i,j)}}). \tag{14}$$

To apply our result, we may take $x = 1/\text{BER}^{(i,j)} - 1$ but details are omitted here.

To reduce the numerical complexity, sub-optimal antenna selection criteria have been also proposed. The receive power is utilized for OFDM with receive antenna selection [6, 7] and with transmit antenna selection [8], where a pair of transmit antennas is selected that has the maximum channel norm

$$\arg \max_{i,j\in[1,N_t],i\neq j} (\|\boldsymbol{h}^{(i)}\|^2 + \|\boldsymbol{h}^{(j)}\|^2), \tag{15}$$

with $\|\boldsymbol{h}^{(i)}\|^2$ being the squared norm of channel impulse response from transmit antenna i. Another possible criterion is to select a pair of transmit antennas that maximizes the minimum of $\gamma_n^{(i,j)}$ such that

$$\arg \max_{i,j\in[1,N_t],i\neq j} \min_{n\in[0,N-1]} \gamma_n^{(i,j)}. \tag{16}$$

4.1. Numerical simulations

We assess the BER performances of the antenna selection methods by numerical simulations. The symbols are drawn from a QPSK constellation and sent through an uncoded MISO OFDM system of block size $N = 64$. The channels are i.i.d. with $L+1 = 8$ component channel taps of exponential power profile. The results are averaged over 100 random channel realizations.

For OFDM with STBC, two antennas out of N_t transmit antennas are chosen based on BER criterion (13), norm criterion (15), and max-min criterion (16), which are labeled with BER(2), Norm(2), and maxmin(2), respectively.

Similarly, one antenna out of N_t transmit antennas is selected, whose labels are BER(1), Norm(1), and maxmin(1),

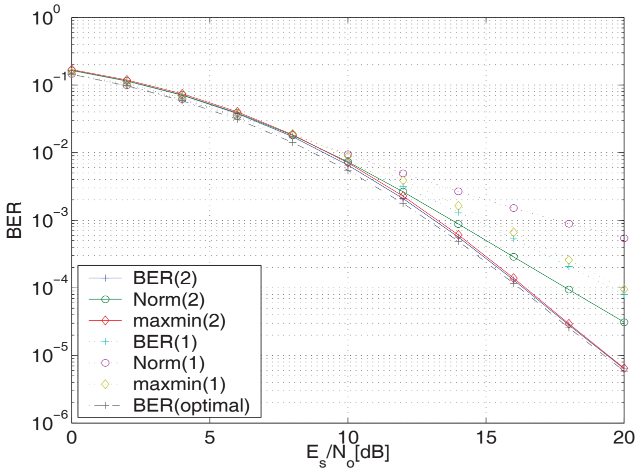

Fig. 1. BER of OFDM with transmit antenna selection.

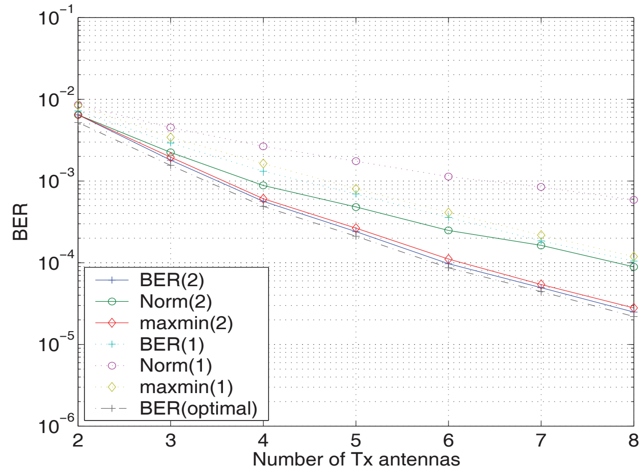

Fig. 2. BER of OFDM vs number of equipped transmit antennas at 14dB.

respectively. The optimal BER is obtained by comparing the BER of one transmit antenna system without STBC by the BER criterion and the BER of two transmit antenna system with STBC by the BER criterion and selecting the better one, which is denoted by BER(optimal).

We assume the perfect knowledge, i.e., channel frequency responses and SNR for BER criterion (13), channel norms for norm criterion (15), and channel frequency responses for max-min criterion (16), for selection.

Fig. 1 illustrates the BERs of antenna selection methods for $N_t = 4$. It can be observed that the diversity order, i.e., the slope of the BER curve at high SNR, of each selection method is different. BER and maxmin selection has almost the same diversity order, while norm selection has the least.

At a fix $E_s/N_0 = 14$dB with E_s the energy per symbol, we evaluate the BER performance by varying the number N_t of transmit antennas from 2 to 8. Fig. 2 depicts the results. For every method, the performance approximately scales in the number of transmit antennas, which clearly validates our analysis and shows the benefits of transmit antenna selection diversity.

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