NEW HYBRID ADAPTIVE BLIND EQUALIZATION ALGORITHMS FOR QAM SIGNALS

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ABSTRACT

This paper introduces new hybrid blind equalization algorithms for QAM signals, the first term of which is the constant modulus criterion (CMA) or its extended version (ECMA) and the second are a penalty term that vanishes at constellation points coordinates. Several penalties, based on cosine, gaussian and polynomial ℓ_1-norm fonctions respectively are investigated. Simulations show the effectiveness of these algorithms.

***Index Terms*—** Adaptive equalizers.

1. INTRODUCTION

Blind equalization is an important issue in modern communications. It is suitable for communication applications for which sending periodically learning sequences may substantially affect the quality of service. Blind equalizers endeavor to filter transmitted signals without need to initializing sequences but only suppose a priori knowledge about some statistical properties of these signals.

To deal with this problem, the constant modulus algorithm (CMA), pioneered by Godard [1] and extended later on [2–8], has been extensively used over the last two decades, since it is easy to implement and has good convergence properties [9, 10]. CMA minimizes a cost function related to signal amplitude, generally via the stochastic gradient descent algorithm. However, with the evolution of requirements in communications, CMA has shown its limits. Many extensions of the CMA, together with new algorithms, have been proposed for transmission of signals with non constant modulus, such as quadrature amplitude modulated (QAM) signals. The minimization of the CMA criterion does not solve satisfactorily the equalization problem for QAM signals, since it leads to large residual errors and constellation is recovered up to a phase rotation. To overcome these drawbacks, other algorithms like the multimodulus algorithm (MMA) [11], the square contour algorithm (SCA) [12, 13] and the extended CMA (ECMA) [14] have been proposed. The most interesting extensions of the CMA are those based on hybrid approaches [6–8], where it is proposed to augment the standard CMA cost function by a penalty term that takes into account the location of constellation symbols. This penalty term is often referred to as the constellation matching error (CME). The algorithms based on criteria with CMA+CME structure, are also called modified CMA (MCMA). CME functions are generally, positive function, constrained to meet certain properties : symmetry around constellation coordinates, uniformity (that is, even level of local maxima) and zero value at constellation points. As CME examples, we can cite powers of sine and cosine functions and polynomials [6].

In this work, we propose two CME terms leading to new criteria. The first CME, is the ℓ_1-norm of a monic polynomial, the zeros of which are constellation coordinates. So, it can be considered as a particular criterion of the MCMA family proposed in [7] and we will call it ℓ_1-MCMA. The second, CME consists in a modified version of the Gaussian criterion proposed in [5], where it was used in a dual mode and used later as a CME in a hybrid form in [7], to the extended CMA (ECMA) proposed by Li and Zhang [14] and obtain what we will call Gauss-ECMA. The paper is organized as follows: in section 2, we briefly recall the conventional CMA, the ECMA and the MCMA. In section 3, we introduce our ℓ_1-MCMA and Gauss-ECMA criteria, among others. Section 4 supplies simulations that show the good performance of the proposed algorithms.

2. PROBLEM STATEMENT AND BACKGROUND

The mathematical model for the receiver input at time k is

$$x_k = \sum_{l=0}^{L_h} h_l s_{k-l} + n_k \tag{1}$$

and the equalizer output is

$$z_k = \sum_{l=0}^{L_e} w_l^* x_{k-l}, \tag{2}$$

where s_k, h_k, n_k and w_k denote respectively the symbols transmitted over the channel, the impulse response of the discrete channel, the noise and the impulse response of equalizer at time index k. We introduce vectors $\boldsymbol{w} = [w_1, w_2, ..., w_{L_e}]^T$ and $\boldsymbol{x}_k = [x_k, x_{k-l}, ..., x_{k-L_e+1}]^T$, where $(\cdot)^T$ denotes transpose. In the following, transpose conjugate will be denoted by $(\cdot)^H$.

2.1. Constant Modulus Algorithm (CMA)

The standard CMA criterion for received symbols z_k to be minimized according to $\boldsymbol{w}$ is

$$J(\boldsymbol{w}) = \mathbb{E}\left\{\left(\left|\boldsymbol{w}^H\boldsymbol{x}_k\right|^2 - R_m\right)^2\right\}$$

with $R_m = \frac{\mathbb{E}\left\{|s_k|^4\right\}}{\mathbb{E}\left\{|s_k|^2\right\}}$. We can be updated bu means of a gradient descent algorithm as follows:

$$\boldsymbol{w}_{k+1} = \boldsymbol{w}_k - \mu e_k \boldsymbol{x}_k \boldsymbol{x}_k^H \boldsymbol{w}_k, \tag{3}$$

with $e_k = |z_k|^2 - R_m$ and μ the step size. It is known that the CMA is blind to carrier phase.

2.2. Extended Constant Modulus Algorithm (ECMA)

The ECMA algorithm is a particular case of the generalized constant modulus algorithms proposed in [14]. Introducing the generalized complex modulus as

$$|z_k|_p = \left(|z_{kr}|^p + |z_{ki}|^p\right)^{\frac{1}{p}} \ , \tag{4}$$

for $p \geq 1$, z_{kr} and z_{ki} denoting the real and imaginary part of z_k respectively, the ECMA criterion is given by

$$J_{\text{ECMA}}(\boldsymbol{w}) = \mathbb{E}\left\{\left|\left|\boldsymbol{w}^H\boldsymbol{x}_k\right|_4^2 - R_{4,2}\right|^2\right\} \ , \tag{5}$$

with $R_{4,2} = \frac{\mathbb{E}\left\{|s_k|_4^4\right\}}{\mathbb{E}\left\{|s_k|_4^2\right\}}$. The learning rule for this algorithm is

$$\boldsymbol{w}_{k+1} = \boldsymbol{w}_k - 4\mu\left(|z_k|_4^2 - R_{4,2}\right)\frac{z_{kr}^3 - jz_{ki}^3}{|z_k|_4^2}\boldsymbol{x}_k \ . \tag{6}$$

Unlike CMA, the ECMA, is phase sensitive and does not require additional phase recovery in steady-state operation.

2.3. Modified Constant Modulus Algorithm (MCMA)

MCMA criteria [6] are designed for QAM modulations. They consist in adding a positive penalty term to the CMA. They are of the form

$$J(\boldsymbol{w}) = \mathbb{E}\left\{\left(\left|\boldsymbol{w}^H\boldsymbol{x}_k\right|^2 - R_m\right)^2\right\} + \beta\left(g\left(z_{kr}\right) + g\left(z_{ki}\right)\right) \tag{7}$$

where $g(\cdot)$ is a positive function that cancels at constellation coordinates and β is a tuning parameter. The corresponding updating procedure is given by

$$\boldsymbol{w}_{k+1} = \boldsymbol{w}_k - \mu\left(e_k\boldsymbol{x}_k\boldsymbol{x}_k^H\boldsymbol{w}_k + \beta\psi_k\boldsymbol{x}_k\right) \ , \tag{8}$$

with

$$\psi_k = g'(z_{kr}) - jg'(z_{ki}) \ . \tag{9}$$

There are several possible choices for g among which

$$g_c(x) = \cos^2\left(\frac{x\pi}{2d}\right) \ ,$$

with $2d$ the minimum distance between symbols.

$$g_G(x) = 1 - \sum_{l=1}^{M} e^{\frac{-|x-s_l|^2}{2\sigma^2}} \ .$$

Corresponding ψ_k for g_c can be found in [6]. We shall refer to MCMA with $g_c(x)$ and $g_G(x)$ penalty terms, as the Cos-MCMA and the Gauss-MCMA respectively.

3. NEW CRITERIA

3.1. The ℓ_1-MCMA criterion

Here, the penalty term is the ℓ_1-norm of the monic polynomial with zeros at constellation points coordinates. It is inspired both by hybrid approaches [6] and by Li's cost function [5] given by

$$J(\boldsymbol{w}) = \mathbb{E}\left\{\prod_{l=1}^{M}\left|\boldsymbol{w}^H\boldsymbol{x}_k - s_l\right|^2\right\}. \tag{10}$$

where M is the constellation order (16, 32, 64,...). But, it has been shown in [5] that, without an extremely good initialization, the adaptation rule using this criterion may diverge, for high order constellations.
Our ℓ_1-MCMA criterion is given by equation (7) with

$$g_{\ell_1}(x) = \prod_{c_i \in A}|x - c_i| \,, \tag{11}$$

where c_i are inphase or quadrature coordinate values. For the ℓ_1-norm CME, we get

$$g'_{\ell_1}(z) = \sum_{c_j \in A} sign(z - c_j) \prod_{c_i \in A, c_i \neq c_j} |z - c_i| \ .$$

We shall see on simulations, that despite it slightly violates uniformity property, the ℓ_1-MCMA has good convergence properties and high equalization performance compared to other MCMA criteria. We can also notice that its CME is somewhat similar to the MMA criterion, so we expect that it will be able to correct phase rotations.

3.2. The g-ECMA criterion

We define the g-ECMA criterion as the sum of the ECMA criterion J_{ECMA} (subsection 2.2) and the CME function g (subsection 2.3) :

$$J_{g-\text{ECMA}}(\boldsymbol{w}) = J_{\text{ECMA}}(\boldsymbol{w}) + \beta\left(g(z_{kr}) + g(z_{ki})\right) \ .$$

Depending on the choice of g, we get the Cos-ECMA, Gauss-ECMA or ℓ_1-ECMA criteria. With these criteria we cumulate advantages of the ECMA criterion that better accounts for the QAM constellation shape than the standard CMA which is specifically dedicated to PSK modulations, with constellation zeros location constraint. We are going to show in the simulation part that high performance gain is achieved in particular with Gauss-ECMA.

4. SIMULATIONS

We consider 16-QAM symbols with uniform distributions and chose a 20 taps long ($L_e = 20$) equalizer. First, we compare performance of the ℓ_1-MCMA criterion to those of the conventional CMA, the Cos-MCMA and the Gauss-MCMA. The channel we use, is frequency selective with impulse response $h_1 = [1 \quad 0.1294 + 0.483j]$ [7]. In figures 1 to 4 steps sizes are ajusted to ensure the same asymptotic variance. We take $\beta = \frac{100}{\pi}$ for Cos-MCMA and Gauss-MCMA as in [7] and $\beta = 2$ for the ℓ_1-MCMA and for the Gauss-MCMA, $\sigma = 0, 2$. These choice of parameters are designed to get good performance. All convergence curves, that give the mean square error (MSE) in dB vs the iteration number, are averaged over 200 Monte Carlo independent runs. In figure 1, the convergence properties of CMA, Cos-MCMA, Gauss-MCMA and ℓ_1-MCMA are compared. We notice that ℓ_1-MCMA performs slightly better than criteria with Cosine and Gaussian CME penalties. It has faster convergence and lower mis-adjustment. CMA alone cannot be adjusted to get as low variance as other criteria. Simulations in figures 2 and 3 are

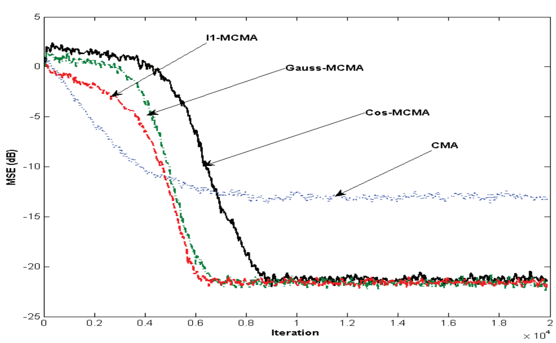

Fig. 1. Performance comparison of CMA, Cos-MCMA, Gauss-MCMA and ℓ_1-MCMA for 16-QAM and channel h_1 (SNR=30dB).

obtained with channel $h_2 = [0.9063 + 0.4226j \quad 0.3214 + 0.3830j]$. Parameters β and σ are the same as for h_1 channel. As it can be seen from the scatters of figure 2, in this case, a phase correction loop is needed for CMA and Cos-MCMA, while ℓ_1-MCMA, as expected, and Gauss-MCMA algorithms converge without need of such a phase correction. We can

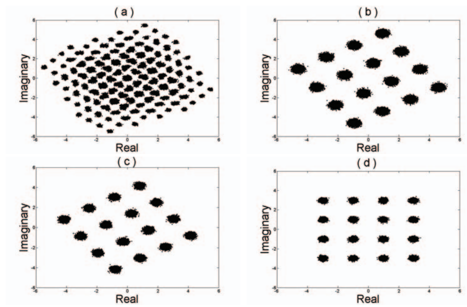

Fig. 2. (a) Constellation at the receiver side (16-QAM), (b) the equalized constellation for MCMA without phase correction, (c) the equalized constellation for Cos-MCMA without phase correction and (d) the equalized constellation for ℓ_1-MCMA which automatically corrects phase rotation.

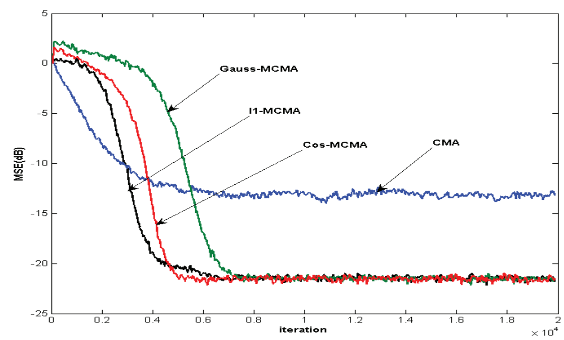

Fig. 3. Performance comparison of CMA, Cos-MCMA, Gauss-MCMA and ℓ_1-MCMA for 16-QAM and channel h_2 (SNR=30dB)

observe that the ℓ_1-MCMA convergence is faster than other g-MCMA criteria.

Comparison of Gauss-ECMA, Cos-ECMA and ℓ_1-ECMA for 16-QAM symbols at SNR=30 dB is proposed in figure 4. It illustrates the MSE (dB) vs the iteration number, for channel h_1. Here, $\mu = 4.5 \times 10^{-6}$ and $\beta = 3$ for ℓ_1-ECMA, $\mu = 7.5 \times 10^{-6}$ for Cos-ECMA and $\mu = 1.5 \times 10^{-5}$, $\beta = 3$ and $\sigma = 0.2$ for Gauss-ECMA. Figure 4 shows that Gauss-ECMA converges faster than ℓ_1-ECMA and Cos-ECMA.

The last simulations are concerned with the comparison of mis-adjustments of ECMA, Gauss-ECMA, Cos-ECMA and ℓ_1-ECMA using the channel h_2. In figure 5, the parameters μ, β and σ are chosen to ensure similar speed of convergence and we compare the level of misadjustments of these algorithms at steady-state. On figure 5, it is obvious that for the same speed of convergence, the Gauss-ECMA shows best performance since it converges to a much lower MSE. On the other hand, contrarily to the MCMA case, both of the four algorithms converge without need of a phase correction. Finally,

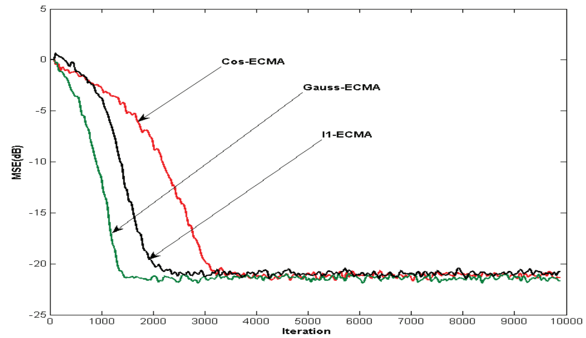

Fig. 4. Performance comparison of Cos-ECMA, Gauss-ECMA and ℓ_1-ECMA for 16-QAM and channel h_1.

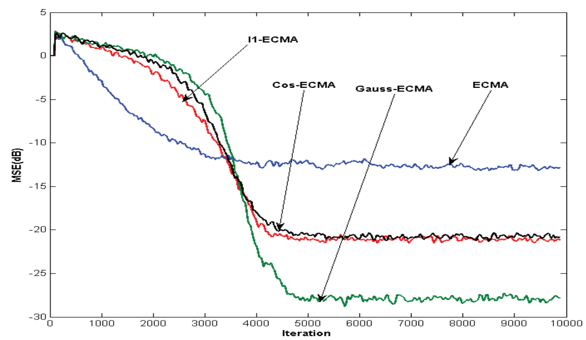

Fig. 5. Performance comparison of ECMA, Cos-ECMA, Gauss-ECMA and ℓ_1-ECMA for 16-QAM and channel h_2.

considering all the simulation (with CMA and ECMA), we can assert that the Gauss-ECMA algorithm has the best performance.

5. CONCLUSIONS

We have proposed two new hybrid adaptive blind equalization criteria for QAM Signals. The first one (ℓ_1-MCMA), is new in the sense that it adds a new CME to the CMA criterion. The ℓ_1-MCMA has been compared to Cos-MCMA and Gauss-MCMA and it shows good performance. Besides its fast convergence, it is able to correct phase rotations. Our second criterion (Gauss-ECMA) is new, since it is the first time that a CME term is associated to the extended constant modulus criterion, resulting in higher performance. A simulation compared to ECMA and other g-ECMA criteria, show the overall superiority of the Gauss-ECMA criterion. In the other hand, using the ECMA as the amplitude term together with a constellation matching error, has the advantage to correct the phase rotations independently of the CME choice.

6. REFERENCES

[1] D. N. Godard, “Self-recovering equalization and carrier tracking in two-dimensional data communication systems,” *IEEE Transactions on Communications*, vol. 28, no. 11, pp. 1867–1875, November 1980.

[2] J. A. Garcia and A. R. F. Vidal, “Improved blind equalization via adaptive combination of constant modulus algorithms,” in *Proc. ICASSP*, Toulouse, France, May 2006, vol. 3, pp. 756–759.

[3] Y. Kim, S. Kim, and M. Kim, “The derivation of a new blind equalization algorithm,” *Electronics and Telecommunications Research Institute Journal*, vol. 18, no. 2, pp. 53–60, July 1996.

[4] Z. Xiong, Z. Dongfeng, J. Zhigang, W. Anhong, and Z. Liyi, “A modified blind equalization algorithm based on kurtosis of output signal,” in *Proc. Asia-Pacific Radio Science Conference*, August 2004, pp. 228–231.

[5] S. Barbarossa and A. Scaglione, “Blind equalization using cost function matched to the signal constellation,” in *Proc. Asilomar*, Pacific Grove, CA, November 1997, vol. 1, pp. 550–554.

[6] L. He, M. G. Amin, and C. Reed, “Adaptive equalization techniques for indoor dynamic wireless communication channels,” *SPIE proceedings series*, vol. 4395, no. 3, pp. 28–38, April 2001.

[7] L. He, M. G. Amin, C. Reed, and R. C. Malkemes, “A hybrid adaptive blind equalization algorithm for qam signals in wireless communications,” *IEEE Transactions on Signal Processing*, vol. 52, no. 7, pp. 2058–2069, July 2004.

[8] K. Banovic, E. Abdel-Raheem, and M.A.S. Khalid, “Hybrid methods for blind adaptive equalization: New results and comparisons,” in *Proc. 10th ISCC*, June 2005, pp. 275–280.

[9] C. R. Johnson, P. Schniter, T. J. Endres, J. D. Behm, D. R. Brown, and R. A. Casas, “Blind equalization using the constant modulus criterion: A review,” *Proceedings of The IEEE*, vol. 86, no. 10, pp. 1927–1950, October 1998.

[10] R. Cusani and A. Laurenti, “Convergence analysis of the cma blind equalizer,” *IEEE Transactions on Communications*, vol. 43, no. 2-3-4, pp. 1304–1307, February-March-April 1995.

[11] J. Yang, J.J. Werner, and G.A. Dumont, “The multimodulus blind equalization algorithm,” in *Proc. Int. Conf. DSP*, Santorini, Greece, July 1997, pp. 127–130.

[12] T. Thaiupathump and S.A. Kassam, “Square contour algorithm: A new algorithm for blind equalization and carrier phase recovery,” in *Proc. IEEE Asilomar Conference on Signals, Systems and Computers*, 2003, pp. 647–651.

[13] S.A. Sheikh and P. Fan, “Improved square contour algorithm for blind equalization of qam signals,” in *Proc. INMIC*, December 2006, pp. 116–119.

[14] X.L. Li and X.D. Zhang, “A family of generalized constant modulus algorithms for blind equalization,” *IEEE Transactions on Communications*, vol. 54, no. 11, pp. 1913–1917, November 2006.