BLIND SOFT-OUTPUT EQUALIZATION OF BLOCK-ORIENTED WIRELESS COMMUNICATIONS

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ABSTRACT

We consider blind equalization for block transmissions over the frequency selective Rayleigh fading channel. In the absence of pilot symbols, the receiver must be able to perform joint equalization and blind channel identification. Relying on a mixed discrete-continuous state-space representation of the communication system, we introduce a blind Bayesian equalization algorithm based on a Gaussian mixture parameterization of the a posteriori probability density function (pdf) of the transmitted data and the channel. The performances of the proposed algorithm are compared with existing blind equalization techniques using numerical simulations.

Index Terms— Frequency selective channels, wireless fading channels, block transmissions, blind equalization.

1. INTRODUCTION

The main advantage of blind transmissions is that they maximize the throughput, since no pilot symbols are needed to estimate the channel.

Blind equalizers were first proposed by Sato [1] and Godard [2] using finite impulse response filters. However, these methods suffer from local and slow convergence and may fail on ill-conditioned or time-varying channels.

Since the advent of turbo-equalization [3], there has been a renewed interest in symbol-by-symbol soft-input soft-output (SISO) equalization. Two main approaches have been proposed so far to achieve SISO equalization in a blind or semi-blind context. The first approach relies on fixed-lag smoothing [4]. The second approach uses fixed-interval smoothing [5]-[6]. All the aforementioned methods employ a trellis description of the intersymbol interference (ISI) [7], where each discrete ISI state has its associated channel estimate.

In this paper, we will consider fixed interval smoothing, which is adapted to block-oriented communications. After modeling the ISI and the unknown channel at the receiver side, we obtain a combined state-space formulation of our communication system. Then, we introduce a smoother based on Gaussian mixtures to perform blind SISO equalization. Finally, our method is compared with several existing blind equalization techniques through numerical simulations. $\mathcal{N}_{\mathcal{C}}(\mathbf{x} : \mathbf{m}, \mathbf{P})$ will denote a complex Gaussian distribution of the variable $\mathbf{x}$, with mean $\mathbf{m}$ and covariance matrix $\mathbf{P}$.

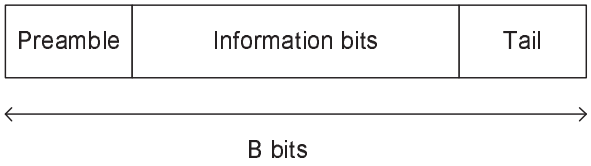

Fig. 1. Data format.

2. SYSTEM MODEL

The transmitted data are organized in bursts containing B bits, as illustrated in Fig. 1. For the ease of exposition, we consider binary phase shift keying (BPSK) modulation, so that the bit transmitted at instant k, $b_k \in \{-1, +1\}$. The preamble (resp. tail) is a short all-one vector of length equal to the channel memory, used to initialize the first (resp. final) ISI state to a known value. Since blind equalization is of interest, no additional pilot symbols are inserted in the data stream.

We assume a discrete Rayleigh fading channel of memory L. The elements of the impulse response $\{c_k^i\}_{i=0}^L$ are modeled as independent zero-mean complex Gaussian random variables. The received complex noisy observation at instant k has the form

$$y_k = \sum_{i=0}^{L} c_k^i b_{k-i} + n_k, \tag{1}$$

where n_k is a complex white Gaussian noise sample with single-sided power spectral density N_0.

We define the ISI state as the subsequence taking 2^L discrete values, $s_k = [b_k, b_{k-1}, \ldots, b_{k-L+1}]^T$. Let f_k denote the ISI state transition function defined by the relation

$$s_k = f_k(s_{k-1}, b_k).$$

It is well known that f_k can be represented graphically by a trellis diagram containing 2^L states [7]. We define the channel state as $\mathbf{x}_k = [c_k^0, c_{k-1}^0, c_k^1, c_{k-1}^1, \ldots, c_k^L, c_{k-1}^L]^T$. A state space representation is obtained for (1) as

$$\begin{cases} s_k = f_k\left(s_{k-1}, b_k\right) \\ \mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \boldsymbol{\pi}_k \\ y_k = \mathbf{H}_k(s_{k-1}, s_k)\mathbf{x}_k + n_k, \end{cases} \tag{2}$$

where the second equation corresponds to an order two autoregressive channel model [8] with a Gaussian process noise vector $\boldsymbol{\pi}_k$ and the observation matrix has the form

$$\mathbf{H}_k(s_{k-1}, s_k) = [b_k, 0, b_{k-1}, 0, \ldots, b_{k-L}, 0]^T.$$

3. BLIND SISO EQUALIZATION USING A GAUSSIAN MIXTURE APPROACH

In this section, we derive a fixed-interval smoother by propagating a mixture of N Gaussians per ISI state forward and backward in the ISI trellis, following an idea originally introduced in [9]. The ISI state s_k and the channel state $\mathbf{x}_k$ will be jointly estimated. A posteriori probabilities for the bits b_k are obtained by a simple marginalization step.

3.1. Forward filtering

A recursive expression of $p(s_k, \mathbf{x}_k|\mathbf{y}_{1:k})$, where $\mathbf{y}_{1:k} = (\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_k)$ is obtained by noting that

$$p(s_k, \mathbf{x}_k, \mathbf{y}_{1:k}) = \sum_{s_{k-1}} p(s_k|s_{k-1}) p(\mathbf{y}_k|\mathbf{d}_k(s_{k-1}, s_k), \mathbf{x}_k) \times \int p(\mathbf{x}_k|\mathbf{x}_{k-1}) p(s_{k-1}, \mathbf{x}_{k-1}, \mathbf{y}_{1:k-1}) d\mathbf{x}_{k-1}, \tag{3}$$

where the discrete summation extends over the states s_{k-1}, for which a valid transition (s_{k-1}, s_k) exists. In general, the multiplications and integration in (3) cannot be expressed in closed form, therefore we introduce the following Gaussian mixture parameterization at instant $k-1$

$$p(s_{k-1}, \mathbf{x}_{k-1}, \mathbf{y}_{1:k-1}) = \sum_{i=1}^{N} \alpha^i(s_{k-1}) \mathcal{N}_{\mathcal{C}}\left(\mathbf{x}_{k-1} : \mathbf{x}_{k-1|k-1}^i(s_{k-1}), \mathbf{P}_{k-1|k-1}^i(s_{k-1})\right). \tag{4}$$

In (4), each discrete state s_{k-1} is associated with a mixture of N Gaussians, where N is a design parameter of choice.

Theorem 3.1 *A closed form expression of $p(s_k, \mathbf{x}_k, \mathbf{y}_{1:k})$ is obtained as*

$$p(s_k, \mathbf{x}_k, \mathbf{y}_{1:k}) = \sum_{s_{k-1}} \sum_{i=1}^{N} \alpha^i(s_{k-1}, s_k) \mathcal{N}_{\mathcal{C}}\left(\mathbf{x}_k : \mathbf{x}_{k|k}^i(s_{k-1}, s_k), \mathbf{P}_{k|k}^i(s_{k-1}, s_k)\right) \tag{5}$$

where the means $\mathbf{x}_{k|k}^i(s_{k-1}, s_k)$ and covariance matrices $\mathbf{P}_{k|k}^i(s_{k-1}, s_k)$ associated with the state transition (s_{k-1}, s_k) are obtained from the following recursions

$$\begin{cases} \mathbf{x}_{k|k-1}^i(s_{k-1}) = \mathbf{F}\mathbf{x}_{k-1|k-1}^i(s_{k-1}) \\ \mathbf{P}_{k|k-1}^i(s_{k-1}) = \mathbf{F}\mathbf{P}_{k-1|k-1}^i(s_{k-1})\mathbf{F}^H + \mathbf{Q} \\ \mathbf{K}_k^i(s_{k-1}, s_k) = \mathbf{P}_{k|k-1}^i(s_{k-1})\mathbf{H}_k(s_{k-1}, s_k)^H \\ \qquad \left(\mathbf{H}_k(s_{k-1}, s_k)\mathbf{P}_{k|k-1}^i(s_{k-1})\mathbf{H}_k(s_{k-1}, s_k)^H + \mathbf{R}\right)^{-1} \\ \mathbf{x}_{k|k}^i(s_{k-1}, s_k) = \mathbf{x}_{k|k-1}^i(s_{k-1}) + \\ \qquad \mathbf{K}_k^i(s_{k-1}, s_k)\left(\mathbf{y}_k - \mathbf{H}_k(s_{k-1}, s_k)\mathbf{x}_{k|k-1}^i(s_{k-1})\right) \\ \mathbf{P}_{k|k}^i(s_{k-1}, s_k) = \mathbf{P}_{k|k-1}^i(s_{k-1}) \\ \qquad - \mathbf{K}_k^i(s_{k-1}, s_k)\mathbf{H}_k(s_{k-1}, s_k)\mathbf{P}_{k|k-1}^i(s_{k-1}) \end{cases}$$

and the weights $\alpha^i(s_{k-1}, s_k)$ are given by

$$\alpha^i(s_{k-1}, s_k) = \alpha^i(s_{k-1}) p(s_k|s_{k-1}) \times \mathcal{N}_{\mathcal{C}}(\mathbf{y}_k : \mathbf{H}_k(s_{k-1}, s_k)\mathbf{x}_{k|k-1}^i(s_{k-1}), \mathbf{H}_k(s_{k-1}, s_k)\mathbf{P}_{k|k-1}^i(s_{k-1})\mathbf{H}_k(s_{k-1}, s_k)^H + \mathbf{R}). \tag{6}$$

The proof is obtained from standard Kalman filtering techniques after injecting (4) into (3).

3.2. Complexity reduction algorithm (CRA)

A problem with (5) is that each discrete state s_k is now associated with a mixture of more than N Gaussians. This means that the number of terms in the Gaussian mixture will grow with time. In order to keep the computational complexity constant for each time instant, we need to approximate the exact expression given by (5) as

$$p(s_k, \mathbf{x}_k, \mathbf{y}_{1:k}) \approx \sum_{i=1}^{N} \alpha^i(s_k) \mathcal{N}_{\mathcal{C}}\left(\mathbf{x}_k : \mathbf{x}_{k|k}^i(s_k), \mathbf{P}_{k|k}^i(s_k)\right),$$

so that again N Gaussians with weight $\alpha^i(s_k)$, mean $\mathbf{x}_{k|k}^i(s_k)$ and covariance $\mathbf{P}_{k|k}^i(s_k)$, $i = 1, \ldots, N$ are associated with each state s_k, as in (4). We do this by applying the CRA proposed in [10]. Assume that $\mathcal{N}_1$ (resp. $\mathcal{N}_2$) is a multivariate Gaussian, whose weight, mean and covariance are given by $w_1, \mathbf{x}_1, \mathbf{P}_1$ (resp. $w_2, \mathbf{x}_2, \mathbf{P}_2$). In [10], a practical measure of similarity between the two densities is given by

$$D = w_1 w_2 \left[\mathcal{I}(\mathcal{N}_1||\mathcal{N}_2) + \mathcal{I}(\mathcal{N}_2||\mathcal{N}_1)\right],$$

where $\mathcal{I}(.||.)$ denotes the Kullback-Leibler distance. Then, pairs of similar Gaussians with minimal D are repeatedly merged until N Gaussians subsist using the approximation

$$w_1\mathcal{N}_{\mathcal{C}}(\mathbf{x}_k : \mathbf{x}_1, \mathbf{P}_1) + w_2\mathcal{N}_{\mathcal{C}}(\mathbf{x}_k : \mathbf{x}_2, \mathbf{P}_2) \approx w\mathcal{N}_{\mathcal{C}}(\mathbf{x}_k : \mathbf{x}, \mathbf{P}),$$

where

$$w = w_1 + w_2$$
$$\mathbf{x} = \frac{w_1\mathbf{x}_1 + w_2\mathbf{x}_2}{w_1 + w_2}$$

$$\mathbf{P} = \frac{w_1}{w_1+w_2}\left[\mathbf{P}_1 + (\mathbf{x}_1-\mathbf{x})(\mathbf{x}_1-\mathbf{x})^H\right] + \frac{w_2}{w_1+w_2}\left[\mathbf{P}_2 + (\mathbf{x}_2-\mathbf{x})(\mathbf{x}_2-\mathbf{x})^H\right].$$

3.3. Backward filtering

Let T denote the total number of available observations and $\mathbf{y}_{k+1:T} = (\mathbf{y}_{k+1}, \mathbf{y}_{k+2}, \ldots, \mathbf{y}_T)$. A time-reversed version of the forward filter in Sec. 3.1 can also be derived.

Theorem 3.2 *Assume that the following Gaussian mixture parameterization*

$$p(\mathbf{y}_{k+2:T}|s_{k+1}, \mathbf{x}_{k+1}) = \sum_{i=1}^{N} \beta^i(s_{k+1}) \times \mathcal{N}_{\mathcal{C}}\left(\mathbf{x}_{k+1} : \mathbf{x}^i_{k+1|k+2:T}(s_{k+1}), \mathbf{P}^i_{k+1|k+2:T}(s_{k+1})\right). \quad (7)$$

is available at instant $k+1$. A closed form expression of $p(\mathbf{y}_{k+1:T}|s_k, \mathbf{x}_k)$ is obtained as

$$p(\mathbf{y}_{k+1:T}|s_k, \mathbf{x}_k) = \sum_{s_{k+1}} \sum_{i=1}^{N} \beta^i(s_k, s_{k+1}) \times \mathcal{N}_{\mathcal{C}}\left(\mathbf{x}_k : \mathbf{x}^i_{k|k+1:T}(s_k, s_{k+1}), \mathbf{P}^i_{k|k+1:T}(s_k, s_{k+1})\right), \quad (8)$$

where the means $\mathbf{x}^i_{k|k+1:T}(s_k, s_{k+1})$ and covariance matrices $\mathbf{P}^i_{k|k+1:T}(s_k, s_{k+1})$ associated with the state transition (s_k, s_{k+1}) are obtained from the following recursions

$$\begin{cases} \mathbf{K}^i_{k+1}(s_k, s_{k+1}) = \mathbf{P}^i_{k+1|k+2:T}(s_{k+1})\mathbf{H}_{k+1}(s_k, s_{k+1})^H \times \\ \quad \left(\mathbf{H}_{k+1}(s_k, s_{k+1})\mathbf{P}^i_{k+1|k+2:T}(s_{k+1})\mathbf{H}_{k+1}(s_k, s_{k+1})^H + \mathbf{R}\right)^{-1} \\ \mathbf{x}^i_{k+1|k+1:T}(s_k, s_{k+1}) = \mathbf{x}^i_{k+1|k+2:T}(s_{k+1}) + \\ \quad \mathbf{K}^i_{k+1}(s_k, s_{k+1})\left(\mathbf{y}_{k+1} - \mathbf{H}_{k+1}(s_k, s_{k+1})\mathbf{x}^i_{k+1|k+2:T}(s_{k+1})\right) \\ \mathbf{P}^i_{k+1|k+1:T}(s_k, s_{k+1}) = \mathbf{P}^i_{k+1|k+2:T}(s_{k+1}) - \\ \quad \mathbf{K}^i_{k+1}(s_k, s_{k+1})\mathbf{H}_{k+1}(s_k, s_{k+1})\mathbf{P}^i_{k+1|k+2:T}(s_{k+1}) \\ \mathbf{x}^i_{k|k+1:T}(s_k, s_{k+1}) = \mathbf{F}\mathbf{x}^i_{k+1|k+1:T}(s_k, s_{k+1}) \\ \mathbf{P}^i_{k|k+1:T}(s_k, s_{k+1}) = \mathbf{F}\mathbf{P}^i_{k+1|k+1:T}(s_k, s_{k+1})\mathbf{F}^H + \mathbf{Q} \end{cases}$$

and the weights $\beta^i(s_k, s_{k+1})$ are given by

$$\beta^i(s_k, s_{k+1}) = \beta^i(s_{k+1}) p(s_{k+1}|s_k) \times \mathcal{N}_{\mathcal{C}}(\mathbf{y}_{k+1} : \mathbf{H}_{k+1}(s_k, s_{k+1})\mathbf{x}^i_{k+1|k+2:T}(s_{k+1}), \mathbf{H}_{k+1}(s_k, s_{k+1})\mathbf{P}^i_{k+1|k+2:T}(s_{k+1})\mathbf{H}_{k+1}(s_k, s_{k+1})^H + \mathbf{R}).$$

Again, we need to apply the CRA of Sec. 3.2 to (8), so that $p(\mathbf{y}_{k+1:T}|s_k, \mathbf{x}_k)$ admits the desired form

$$p(\mathbf{y}_{k+1:T}|s_k, \mathbf{x}_k) \approx \sum_{i=1}^{N} \beta^i(s_k) \mathcal{N}_{\mathcal{C}}\left(\mathbf{x}_k : \mathbf{x}^i_{k|k+1:T}(s_k), \mathbf{P}^i_{k|k+1:T}(s_k)\right).$$

3.4. Smoothing

Following [9], a two-fi lter smoothing formula is obtained as

$$p(s_k, s_{k+1}, \mathbf{x}_k, \mathbf{y}_{1:T}) = p(s_{k+1}|s_k) p(s_k, \mathbf{x}_k, \mathbf{y}_{1:k}) \times \int p(\mathbf{x}_{k+1}|\mathbf{x}_k) p(\mathbf{y}_{k+1}|d_{k+1}(s_k, s_{k+1}), \mathbf{x}_{k+1}) \times p(\mathbf{y}_{k+2:T}|s_{k+1}, \mathbf{x}_{k+1}) d\mathbf{x}_{k+1}$$

Theorem 3.3 *Using the Gaussian mixture approximations for the forward and the backward filter introduced in Sec. 3.1 and Sec. 3.3, respectively, a closed form expression of $p(s_k, s_{k+1}, \mathbf{x}_k, \mathbf{y}_{1:T})$ is obtained as*

$$p(s_k, s_{k+1}, \mathbf{x}_k, \mathbf{y}_{1:T}) = \sum_{i=1}^{N}\sum_{j=1}^{N} \sigma^{i,j}(s_k, s_{k+1}) \mathcal{N}_{\mathcal{C}}\left(\mathbf{x}_k : \mathbf{x}^{i,j}_{k|1:T}(s_k, s_{k+1}), \mathbf{P}^{i,j}_{k|1:T}(s_k, s_{k+1})\right), \quad (9)$$

where the covariances associated to transition (s_k, s_{k+1}) are

$$\mathbf{P}^{i,j}_{k|1:T}(s_k, s_{k+1}) = \mathbf{P}^j_{k|k+1:T}(s_k, s_{k+1}) \times [\mathbf{P}^j_{k|k+1:T}(s_k, s_{k+1}) + \mathbf{P}^i_{k|k}(s_k)]^{-1} \mathbf{P}^i_{k|k}(s_k)$$

and the means associated to transition (s_k, s_{k+1}) are

$$\mathbf{x}^{i,j}_{k|1:T}(s_k, s_{k+1}) = \mathbf{P}^{i,j}_{k|1:T}(s_k, s_{k+1}) \times \left[\mathbf{P}^i_{k|k}(s_k)^{-1}\mathbf{x}^i_{k|k}(s_k) + \mathbf{P}^j_{k|k+1:T}(s_k, s_{k+1})^{-1}\mathbf{x}^j_{k|k+1:T}(s_k, s_{k+1})\right],$$

for $1 \le i, j \le N$. The expression of the weights is given by

$$\sigma^{i,j}(s_k, s_{k+1}) = \alpha^i(s_k)\beta^j(s_{k+1}) p(s_{k+1}|s_k) b^{i,j}(s_k, s_{k+1}) \times \mathcal{N}_{\mathcal{C}}(\mathbf{y}_{k+1} : \mathbf{H}_{k+1}(s_k, s_{k+1})\mathbf{x}^j_{k+1|k+2:T}(s_{k+1}), \mathbf{H}_{k+1}(s_k, s_{k+1})\mathbf{P}^j_{k+1|k+2:T}(s_{k+1})\mathbf{H}_{k+1}(s_k, s_{k+1})^H + \mathbf{R}).$$

The coefficient $b^{i,j}(s_k, s_{k+1})$ has the form (10) at the bottom of the page.

$$b^{i,j}(s_k, s_{k+1}) = \frac{1}{\pi^{2(L+1)} \det\left[\mathbf{P}^j_{k|k+1:T}(s_k, s_{k+1}) + \mathbf{P}^i_{k|k}(s_k)\right]} \times \exp\left\{-\left[\mathbf{x}^i_{k|k}(s_k) - \mathbf{x}^j_{k|k+1:T}(s_k, s_{k+1})\right]^H \left[\mathbf{P}^j_{k|k+1:T}(s_k, s_{k+1}) + \mathbf{P}^i_{k|k}(s_k)\right]^{-1} \left[\mathbf{x}^i_{k|k}(s_k) - \mathbf{x}^j_{k|k+1:T}(s_k, s_{k+1})\right]\right\}. \quad (10)$$

Since we are interested in soft-output equalization, we must compute smoothed bit-by-bit marginal probabilities. Let $B_k^{(m)}$ be the set of state transitions (s_{k-1}, s_k) such that the information bit $b_k = m$, with $m = 0, 1$. Taking Eq. (9) at instant $k-1$ and marginalizing out the vector $\mathbf{x}_{k-1}$, we obtain

$$p(b_k = m|\mathbf{y}_{1:T}) \propto \sum_{(s_{k-1},s_k)\in B_k^{(m)}} \sum_{i=1}^{N}\sum_{j=1}^{N} \sigma^{i,j}(s_{k-1}, s_k). \tag{11}$$

4. NUMERICAL RESULTS

We consider a memory-2 Rayleigh fading channel simulated with the method introduced in [11]. The standard deviations of the resulting three complex processes (c_k^0, c_k^1, c_k^2) are set at $(0.407, 0.815, 0.407)$. The block size is $B = 100$ bits, with a preamble and a tail of length 2 bits. We assume that each data block is affected by an independent channel realization. E_b denotes the average energy per bit.

We compare the bit error rate (BER) of three blind equalizers. The first equalizer is the constant modulus algorithm (CMA) [2], iterated 50 times back and forth on each data block, in order to avoid the slow convergence problem.Differential encoding of the transmitted data was used only for the CMA equalizer, to solve the phase ambiguity inherent in this method. The second blind equalizer is the proposed Gaussian mixture smoother in the degenerate case where the channel estimation is performed with only $N = 1$ Gaussian per discrete ISI state. Our method then reduces to a fixed-interval (instead of fixed-lag) version of the algorithm proposed in [4]. Finally, the third blind equalizer is the proposed Gaussian mixture smoother, with a mixture of $N = 2$ Gaussians associated with each discrete ISI state. The second and the third equalizer operate on the conventional 4-state ISI trellis [7]. Fig. 2 illustrates the BER on a fast fading channel with normalized fading rate $B_dT = 10^{-2}$. Our method with $N = 2$ attains performances close to equalization with perfect channel state information (CSI).

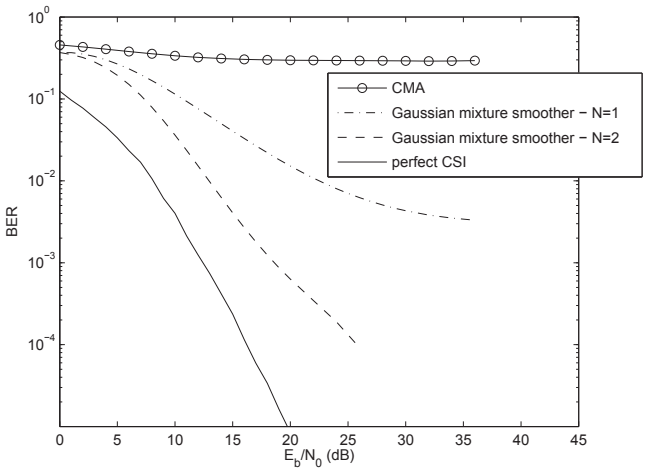

Fig. 2. BER performances of blind SISO equalization at $B_dT = 10^{-2}$.

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