FULL DIVERSITY UNDER MULTIPLE CARRIER FREQUENCY OFFSETS OF A FAMILY OF SPACE-FREQUENCY CODES

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ABSTRACT

A cooperative system may have both timing errors and multiple carrier frequency offsets (CFOs). To combat timing errors, space-frequency (SF) coded OFDM systems have been recently proposed for cooperative communications to achieve both full cooperative and full multipath diversities without time synchronization requirement. In this paper, we study the effect of multiple CFOs from relay nodes on a family of rotation based SF codes. We find that they can still achieve full diversities under the condition that the absolute values of normalized CFOs are less than 0.5. We further show that this full diversity property still holds for a complexity-reduced two-stage zero forcing (ZF) aided maximum likelihood (ML) decoding method, for which a ZF method is used to equalize multiple CFOs before ML decoding.

***Index Terms*—** Multiple CFOs, cooperative communications, SF codes, OFDM, ICI.

1. INTRODUCTION

A major challenge for cooperative communications is synchronization because the multiple transmissions in cooperative systems may not be synchronized either in time or frequency. Since OFDM is not sensitive to timing errors, space-frequency (SF) coded OFDM cooperative systems have been proposed to achieve both full cooperative and full multipath diversities without the time synchronization requirement, e.g., in [1–3]. However, these systems still need to face the problem of multiple carrier frequency offsets (CFOs). Since the columns of an SF code matrix are transmitted from different relay nodes with their own oscillators, they may have multiple CFOs that cannot be compensated simultaneously. For SF coded OFDM cooperative systems, this problem becomes greatly stringent because CFO can lead to inter-carrier interference (ICI) in OFDM systems. The problem of multiple CFOs in cooperative communications has been studied recently in [4–7]. All of them adopt the equalization techniques to deal with the ICI problem. However, it is not known if these equalization techniques can guarantee full diversity.

In this paper, we consider the SF codes proposed in [8] for MIMO-OFDM systems. We show that this family of SF codes can still achieve full diversities under the condition that the absolute values of normalized CFOs are less than 0.5. The above full diversity property is based on the maximum likelihood (ML) decoding across all the subcarriers of the OFDM system, which may have a high complexity. To overcome this difficulty, we further study an SF coded OFDM system where the ICI caused by multiple CFOs is first equalized by a zero forcing (ZF) method, followed by ML decoding for the SF codes (we call this method the ZF-ML method). The complexity of this ZF-ML detection method is much reduced compared to the complete ML method described above, and we prove that the ZF-ML method can still achieve the same diversity order as that of the case without CFOs. Therefore, we can conclude that such an SF coded OFDM cooperative system is robust to both timing errors and CFOs from the relay nodes. Throughout this paper, full channel knowledge including CFOs at the destination node is assumed.

Notations: Superscripts $^{\mathcal{T}}$, *, and $^{\mathcal{H}}$ stand for transpose, conjugate, and Hermitian, respectively. $\mathrm{E}[x]$ represents the expectation of variable x. Integer floor are denoted by $\lfloor\cdot\rfloor$. $\mathbf{I}_N$ represents the $N \times N$ identity matrix. The N by 1 all zero and all one vectors are denoted by $\mathbf{1}_N$ and $\mathbf{0}_N$, respectively. $\mathbf{0}_{N\times M}$ stands for an $N \times M$ all zero matrix. $\mathrm{diag}(d_0, \ldots, d_{N-1})$ denotes an $N \times N$ diagonal matrix with diagonal scalar entries $d_0, \ldots, d_{N-1}$, and an $NM \times NM'$ block diagonal matrix with diagonal $M \times M'$ matrix entries $D_0, \ldots, D_{N-1}$ is denoted by $\mathrm{diag}(D_0, \ldots, D_{N-1})$. $\mathbf{F}_N$ is the $N \times N$ normalized FFT matrix. The Kronecker product is denoted by $\otimes$ and the Hadamard product is denoted by $\odot$. The notation $(\cdot)_N$ means modular operation.

2. SYSTEM MODEL

Let us first describe the cooperative protocol and channel models.

2.1. Cooperative Protocol

The cooperative communication system we use in this work includes one source node, one destination node, and a number of relay nodes in the middle. In the first phase, the source node S broadcasts the information while the relay nodes receive the same information. In the second phase, the M_t relay nodes, which have detected the received information symbols correctly, will help the source to transmit. The detected symbols are parsed into blocks of size N and N is also the number of subcarriers in one OFDM symbol. Then, the bth block, $b = 0, 1, \ldots$, is encoded to an SF codeword matrix $\mathbf{C}$ in a distributed fashion [2]. Finally, the mth relay node transmits the $(m-1)$th column of the codeword matrix, denoted as $\mathbf{c}_m$, by the standard OFDM technology. Hence, this is a *decode-and-forward* protocol. We assume that each node has only one transmit/receive antenna.

2.2. Channel Model

The channel impulse response from the mth relay node to the destination node is denoted as $h_m(\tau) = \sum_{l=0}^{L_m-1} \alpha_m(l)\delta(\tau - \tau_m(l))$, where L_m is the number of multipaths of the link from the mth relay node to the destination node. The complex amplitude and delay for the

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lth multipath of the mth relay node are $\alpha_m(l)$ and $\tau_m(l)$, respectively, where $\alpha_m(l)$ is a zero mean complex Gaussian variable with power $\mathrm{E}[|\alpha_m(l)|^2] = \sigma_m^2(l)$ and constraint $\sum_{l=0}^{L_m-1} \sigma_m^2(l) = 1$ for $1 \le m \le M_t$. We further assume that the delays for the relay nodes are rounded to the sampling position, i.e., $\tau_m(l)$ is an integer multiple of $1/F_s$, where F_s is the sampling frequency. The channel taps $\alpha_m(l)$ are assumed independent from each other for different m and l. The $N \times 1$ frequency response vector of the link between the mth relay node and the destination node, can be given by

$$\mathrm{H}_m = \hat{\mathbf{F}}_m \mathbf{h}_m, \tag{1}$$

where $\mathbf{h}_m = [\alpha_m(0), \ldots, \alpha_m(L_m-1)]^{\mathcal{T}}$ and $\hat{\mathbf{F}}_m = [\mathbf{f}^{\tau_m(0)}, \ldots, \mathbf{f}^{\tau_m(L_m-1)}]$. The vector $\mathbf{f}^{\tau_m(l)} = [1, \zeta^{\tau_m(l)}, \ldots, \zeta^{(N-1)\tau_m(l)}]^{\mathcal{T}}$, where $\zeta = \exp(-j\frac{2\pi}{T})$ and T is the duration of an OFDM symbol.

3. STRUCTURE OF SPACE-FREQUENCY CODES

The SF code for the cooperative communication system under investigation is based on the codes in [8], [2], [9]. Here we briefly review the structure and properties of this SF code. For the codes proposed in [8], each $N \times M_t$ SF code matrix $\mathbf{C}$ is a concatenation of some matrices $\mathbf{G}_p$, i.e., $\mathbf{C} = \left[\mathbf{G}_1^{\mathcal{T}}, \mathbf{G}_2^{\mathcal{T}}, \ldots, \mathbf{G}_P^{\mathcal{T}}, \mathbf{0}_{(N-N')\times M_t}^{\mathcal{T}}\right]^{\mathcal{T}}$, where $P = \lfloor N/(\Gamma M_t) \rfloor$, $N' = P\Gamma M_t$, each matrix $\mathbf{G}_p$ ($1 \le p \le P$) is of size ΓM_t by M_t, and Γ is a coding parameter related to the achieveable diversity order for the code. The zero padding matrix $\mathbf{0}_{(N-N')\times M_t}$ is used if the number of subcarriers N is not an integer multiple of ΓM_t. In the remainder of this paper, without loss of generality, we assume that N is an integer multiple of ΓM_t, i.e., $N = P\Gamma M_t$. Each matrix $\mathbf{G}_p$ ($1 \le p \le P$) has the same structure given by $\mathbf{G}_p = \sqrt{M_t}\mathrm{diag}(\mathbf{x}_1^p, \mathbf{x}_2^p, \cdots, \mathbf{x}_{M_t}^p)$, where $\mathbf{x}_m^p = [x_{(m-1)\Gamma+1}^p x_{(m-1)\Gamma+2}^p \ldots x_{m\Gamma}^p]^{\mathcal{T}}$ for $1 \le m \le M_t$, and all x_k^p ($1 \le k \le M_t\Gamma$) are complex symbols and are mapped from an information subvector $\mathbf{s}^p = [s_1^p, s_2^p, \ldots, s_{\Gamma M_t}^p]^{\mathcal{T}}$ by a linear transform $[\mathbf{x}_1^{p\mathcal{T}}, \mathbf{x}_2^{p\mathcal{T}}, \ldots, \mathbf{x}_{M_t}^{p\ \mathcal{T}}]^{\mathcal{T}} = \mathbf{\Theta}\mathbf{s}^p$, where s_l^p is $((p-1)\Gamma M_t + l - 1)$th entry of $\mathbf{s}$ for $1 \le l \le \Gamma M_t$ and $\mathbf{\Theta}$ is the $N \times N$ linear transformation matrix. The energy constraint is $\mathrm{E}\left[\sum_{k=1}^{\Gamma M_t} |x_k|^2\right] = \Gamma M_t$. By properly designing the linear transformation matrix $\mathbf{\Theta}$, the maximum diversity order achieved by these SF codes is $\Gamma M_t M_r$ given $1 \le \Gamma \le L_m$ for $1 \le m \le M_t$, where M_r is the number of receive antennas [8].

One property of the above SF codes is that each subcarrier is only used by one transmit antenna. By considering the structure of $\mathbf{G}_p$, the $(m-1)$th column of $\mathbf{C}$, denoted as $\mathbf{c}_m$, can be written as

$$\mathbf{c}_m = \sqrt{M_t}\mathbf{P}_m (\mathbf{I}_P \otimes \mathbf{\Theta})\, \mathbf{s} = \sqrt{M_t}\mathbf{P}_m \mathbf{x}, \tag{2}$$

where $\mathbf{x} = (\mathbf{I}_P \otimes \mathbf{\Theta})\, \mathbf{s}$ and $\mathbf{P}_m$ is an $N \times N$ diagonal matrix corresponding to the $(m-1)$th column of $\mathbf{C}$ with the following form

$$\mathbf{P}_m(l, l') = \begin{cases} 1, & \text{if } l = l' = (t-1)\Gamma M_t + (m-1)\Gamma + i \\ 0, & \text{else} \end{cases}, \tag{3}$$

where $0 \le i \le \Gamma - 1$ and $1 \le t \le P$. Based on (3) we also have

$$\mathbf{P}_m \mathbf{P}_{m'} = \begin{cases} \mathbf{P}_m, & \text{if } m = m' \\ \mathbf{0}_{N\times N}, & \text{otherwise} \end{cases}. \tag{4}$$

From (2) and (4), we can further get

$$\mathbf{c}_m = \mathbf{P}_m \mathbf{c}_m. \tag{5}$$

4. EFFECT OF MULTIPLE CFOS ON THE SF CODES

We next analyze the effect of the multiple CFOs to the SF codes.

4.1. Receive Signal Model

At the destination node, after standard steps, the bth, $b = 0, 1, 2, \ldots$, received OFDM symbol $\mathbf{z}^b$ in the frequency domain is given by

$$\mathbf{z}^b = \sqrt{\frac{\rho}{M_t}} \sum_{m=1}^{M_t} e^{j\theta_{\varepsilon_m}^b} \mathbf{U}_{\varepsilon_m} \mathrm{diag}(\mathrm{H}_m)\mathbf{c}_m + \mathbf{w}, \tag{6}$$

where $\mathbf{w}$ is an $N \times 1$ vector with each entry being a zero mean unit variance complex Gaussian random variable and ρ stands for the signal-to-noise ratio (SNR) at the destination node. Let Δf_m be the CFO between the mth relay node and the destination node. Then $\varepsilon_m = \Delta f_m T$ is its normalized value by OFDM symbol duration T. In (6), $\theta_{\varepsilon_m}^b = 2\pi\varepsilon_m(bN + bL_{cp} + L_{cp})/N + \theta_{0,m}$ where L_{cp} is the length of cyclic prefix, and $\theta_{0,m}$ is the phase rotation between the phase of the destination node local oscillator and the carrier phase of the mth relay node at the start of the received signal. $\mathbf{U}_{\varepsilon_m}$ is the unitary $N \times N$ ICI matrix induced by ε_m and is given by

$$\mathbf{U}_{\varepsilon_m} = \mathbf{F}_N \mathbf{\Omega}_{\varepsilon_m} \mathbf{F}_N^{\mathcal{H}}, \tag{7}$$

where $\mathbf{\Omega}_{\varepsilon_m} = \mathrm{diag}(1, e^{j2\pi\varepsilon_m/N}, \ldots, e^{j2\pi\varepsilon_m(N-1)/N})$.

Substituting (5) into (6), we have

$$\begin{aligned} \mathbf{z}^b &= \sqrt{\frac{\rho}{M_t}} \sum_{m=1}^{M_t} e^{j\theta_{\varepsilon_m}^b} \mathbf{U}_{\varepsilon_m} \mathrm{diag}(\mathrm{H}_m)\mathbf{P}_m\mathbf{c}_m + \mathbf{w} \\ &= \sqrt{\frac{\rho}{M_t}} \sum_{m=1}^{M_t} e^{j\theta_{\varepsilon_m}^b} \mathbf{U}_{\varepsilon_m} \mathbf{P}_m \mathrm{diag}(\mathrm{H}_m)\mathbf{c}_m + \mathbf{w}, \end{aligned} \tag{8}$$

where the second equality follows from the fact that $\mathrm{diag}(\mathrm{H}_m)$ and $\mathbf{P}_m$ are all diagonal matrices. Based on (4), we have $\mathbf{P}_m \mathrm{diag}(\mathrm{H}_m)\mathbf{c}_m = \mathbf{P}_m \sum_{m'=1}^{M_t} \mathbf{P}_{m'} \mathrm{diag}(\mathrm{H}_{m'})\mathbf{c}_{m'}$. Then, from (8) we further get

$$\begin{aligned} \mathbf{z}^b &= \sqrt{\frac{\rho}{M_t}} \sum_{m=1}^{M_t} e^{j\theta_{\varepsilon_m}^b} \mathbf{U}_{\varepsilon_m} \left(\mathbf{P}_m \sum_{m'=1}^{M_t} \mathbf{P}_{m'} \mathrm{diag}(\mathrm{H}_{m'})\mathbf{c}_{m'} \right) + \mathbf{w} \\ &= \sqrt{\frac{\rho}{M_t}} \mathbf{U}^b \sum_{m=1}^{M_t} \mathrm{diag}(\mathrm{H}_m)\mathbf{c}_m + \mathbf{w}, \end{aligned} \tag{9}$$

where

$$\mathbf{U}^b = \sum_{m=1}^{M_t} e^{j\theta_{\varepsilon_m}^b} \mathbf{U}_{\varepsilon_m} \mathbf{P}_m. \tag{10}$$

From (9) and (10), we can see that due to the property of the SF codes, i.e., each subcarrier is only used by one transmit antenna, the effect of M_t ICI matrix $\mathbf{U}_{\varepsilon_m}$ ($1 \le m \le M_t$) is incorporated into the matrix $\mathbf{U}^b$. Substituting (1) into (9), we further obtain

$$\begin{aligned} \mathbf{z}^b &= \sqrt{\frac{\rho}{M_t}} \mathbf{U}^b \sum_{m=1}^{M_t} \mathrm{diag}(\hat{\mathbf{F}}_m \mathbf{h}_m)\mathbf{c}_m + \mathbf{w} \\ &= \sqrt{\frac{\rho}{M_t}} \mathbf{U}^b \sum_{m=1}^{M_t} [\mathbf{c}_m \odot \mathbf{f}^{\tau_m(0)}, \ldots, \mathbf{c}_m \odot \mathbf{f}^{\tau_m(L_m-1)}]\mathbf{h}_m + \mathbf{w}. \end{aligned} \tag{11}$$

Let $L = \max_m(L_m)$. Define $N \times M_t$ matrix $\mathbf{D}_l \triangleq [\mathbf{f}^{\tau_1(l)}, \mathbf{f}^{\tau_2(l)}, \ldots, \mathbf{f}^{\tau_{M_t}(l)}]$ ($0 \le l \le L-1$), and $M_t L \times 1$ vector $\mathbf{h} \triangleq [\alpha_1(0), \ldots, \alpha_{M_t}(0), \ldots, \alpha_1(L-1), \ldots, \alpha_{M_t}(L-1)]$, where $\alpha_m(l) = 0$ and $\tau_m(l) = 0$, if $l \ge L_m$ for $1 \le m \le M_t$. Further define $N \times M_t L$ matrix $\mathbf{X} = [\mathbf{D}_0 \odot \mathbf{C}, \mathbf{D}_1 \odot \mathbf{C}, \ldots, \mathbf{D}_{L-1} \odot \mathbf{C}]$. Then the received signal, $\mathbf{z}^b$ can be written as:

$$\mathbf{z}^b = \sqrt{\frac{\rho}{M_t}} \mathbf{U}^b \mathbf{X}\mathbf{h} + \mathbf{w}. \tag{12}$$

4.2. Diversity Analysis of SF Codes with Multiple CFOs

It is not hard to see that the signal model (12) is a standard SF coded MIMO-OFDM receive signal model which is examined in [8] and [2]. According to the *Diversity (Rank) criterion* of SF codes design, clearly in the presence of multiple CFOs, the achieved diversity order of SF codes is equal to the minimum rank of the matrix $\mathbf{U}^b(\mathbf{X}-\hat{\mathbf{X}})\mathbf{\Lambda}(\mathbf{X}-\hat{\mathbf{X}})^{\mathcal{H}}\mathbf{U}^{b\mathcal{H}}$ for any distinct $\mathbf{C}$ and $\hat{\mathbf{C}}$, where $\mathbf{\Lambda} = \mathrm{E}[\mathbf{h}\mathbf{h}^{\mathcal{H}}]$. Furthermore we have the inequality

$$\mathrm{rank}\left(\mathbf{U}^b(\mathbf{X}-\hat{\mathbf{X}})\mathbf{\Lambda}(\mathbf{X}-\hat{\mathbf{X}})^{\mathcal{H}}\mathbf{U}^{b\mathcal{H}}\right) \le \mathrm{rank}\left((\mathbf{X}-\hat{\mathbf{X}})\mathbf{\Lambda}(\mathbf{X}-\hat{\mathbf{X}})^{\mathcal{H}}\right), \tag{13}$$

from which we can conclude that in the presence of multiple CFOs, the achieved diversity order by this SF code can only be less than or equal to its achieved diversity order in the case without CFOs. On the other hand, we can see that in the case of $\mathrm{rank}(\mathbf{U}^b) = N$, the equality in (13) holds. That means the achieved diversity order is not decreased by the multiple CFOs.

Since multiple CFOs affect the SF codes through the matrix $\mathbf{U}^b$, let us investigate it in details. Substituting (7) into (10), we have

$$\mathbf{U}^b = \sum_{m=1}^{M_t} e^{j\theta^b_{\varepsilon_m}} \mathbf{F}_N \mathbf{\Omega}_{\varepsilon_m} \mathbf{F}_N^{\mathcal{H}} \mathbf{P}_m = N^{-\frac{1}{2}} \mathbf{F}_N \mathbf{V} \mathbf{P}^b, \tag{14}$$

where $\mathbf{V} = N^{\frac{1}{2}} \sum_{m=1}^{M_t} \mathbf{\Omega}_{\varepsilon_m} \mathbf{F}_N^{\mathcal{H}} \mathbf{P}_m$, $\mathbf{P}^b = \sum_{i=1}^{M_t} e^{j\theta^b_{\varepsilon_i}} \mathbf{P}_i$, and the second equality follows from (4). Here it is easy to verify that $\mathbf{P}^b$ is a diagonal and unitary matrix. Considering the fact that $\mathbf{F}_N$ is a unitary matrix, we have the fact that $\mathrm{rank}(\mathbf{U}^b) = \mathrm{rank}(N^{-\frac{1}{2}}\mathbf{F}_N\mathbf{V}\mathbf{P}^b) = \mathrm{rank}(\mathbf{V})$.

For the SF codeword matrix $\mathbf{C}$, assuming that the kth subcarrier is used by m_kth transmit antenna for $0 \le k \le N-1$ and $1 \le m_k \le M_t$, and considering the expression of $\mathbf{\Omega}_{\varepsilon_m}$, we have the form of the kth column of $\mathbf{V}$, denoted by $\mathbf{v}_k$, as

$$\mathbf{v}_k = [1, e^{j2\pi(\varepsilon_{m_k}+k)/N}, e^{j2\pi(\varepsilon_{m_k}+k)2/N}, \ldots, e^{j2\pi(\varepsilon_{m_k}+k)(N-1)/N}]^{\mathcal{T}}. \tag{15}$$

From (15), it is clear that $\mathbf{V}$ is a Vandermonde matrix. Thus the determinant of $\mathbf{V}$ is calculated by

$$\begin{aligned}\det(\mathbf{V}) &= \prod_{0\le i<l\le N-1} \left(e^{j2\pi(\varepsilon_{m_l}+l)/N} - e^{j2\pi(\varepsilon_{m_i}+i)/N}\right) \\ &= \prod_{0\le i<l\le N-1} e^{j2\pi(\varepsilon_{m_i}+i)/N}\left(e^{j2\pi(l-i+\varepsilon_{m_l}-\varepsilon_{m_i})/N} - 1\right).\end{aligned} \tag{16}$$

From (16), it is easy to see that $\det(\mathbf{V}) = 0$ if and only if we can find a pair of integers i and l so that $e^{j2\pi(l-i+\varepsilon_{m_l}-\varepsilon_{m_i})/N} - 1 = 0$ for $0 \le i < l \le N-1$. Since $2\pi(l-i+\varepsilon_{m_l}-\varepsilon_{m_i}) = 2t\pi \Leftrightarrow \varepsilon_{m_l}-\varepsilon_{m_i} = tN+i-l$ for t is an integer, finally we have

$$\det(\mathbf{V}) = 0 \Leftrightarrow \varepsilon_{m_l}-\varepsilon_{m_i} = tN+i-l, \tag{17}$$

where $0 \le i < l \le N-1$ and all of t, i and l are integers.

We now have the following theorem.

Theorem 1 *If the absolute values of normalized CFOs ε_m, $1 \le m \le M_t$, are all less than 0.5, then the diversity order of the SF codes is not decreased by the multiple CFOs, where the SF code is that described in Section 3.*

The proof can be found in [10].

On the other hand, if there is an m such that $|\varepsilon_m| \ge 0.5$, the maximum achieved diversity order of SF codes may be less than $M_t\Gamma$, although $L_m \ge \Gamma$ for $1 \le m \le M_t$. It can be shown by the analysis in [10] that an counterexample is the case $\varepsilon_{m_l}-\varepsilon_{m_i} = -1$ which is possible if $|\varepsilon_m| \ge 0.5$.

4.3. Diversity Analysis with the ZF-ML Detection Method

The above analysis is based on the ML decoding that needs to jointly consider all the N subcarriers. Its computational complexity will be high when N is not small even for efficient ML detection methods such as the sphere decoding method. We next consider the SF coded system after we equalize the ICI caused by the CFOs using the ZF method, i.e., the two-stage ZF aided ML detection method (ZF-ML). After the ZF equalizer, the ML decoding of the SF coded system is similar to the original SF coded system without CFOs. The ZF-ML detection method is described as follows.

Let us recall the signal model (12), which is given by $\mathbf{z}^b = \sqrt{\frac{\rho}{M_t}}\mathbf{U}^b\mathbf{X}\mathbf{h} + \mathbf{w}$. As we have analyzed that if $|\varepsilon_m| < 0.5$ for $1 \le m \le M_t$, the ICI matrix $\mathbf{U}^b$ is nonsingular. Under this condition we can equalize the ICI matrix $\mathbf{U}^b$ by the ZF method. Thus, after multiplying $\mathbf{z}^b$ by $\mathbf{U}^{b^{-1}}$, we get the ICI free signal model $\tilde{\mathbf{z}}^b = \mathbf{U}^{b^{-1}}\mathbf{z}^b = \sqrt{\frac{\rho}{M_t}}\mathbf{X}\mathbf{h}+\tilde{\mathbf{w}}$, where $\tilde{\mathbf{w}} = \mathbf{U}^{b^{-1}}\mathbf{w}$. Then define the $(p-1)$th $(1 \le p \le P)$ $M_t\Gamma \times 1$ subvector of $\tilde{\mathbf{z}}^b$ as $\tilde{\mathbf{z}}^b_p$ and each $\tilde{\mathbf{z}}^b_p$ is given by

$$\tilde{\mathbf{z}}^b_p = \sqrt{\frac{\rho}{M_t}}\mathbf{X}_p\mathbf{h} + \tilde{\mathbf{w}}_p, \tag{18}$$

where $\mathbf{X}_p$ is the $(p-1)$th $M_t\Gamma \times M_tL$ submatrix of $\mathbf{X}$, i.e, the ith $(0 \le i \le M_t\Gamma-1)$ row of $\mathbf{X}_p$ is the $((p-1)M_t\Gamma+i)$th row of $\mathbf{X}$, and $\tilde{\mathbf{w}}_p = \hat{\mathbf{U}}^b_p\tilde{\mathbf{w}}$ is the $(p-1)$th $M_t\Gamma \times 1$ subvector of $\tilde{\mathbf{w}}$ and $\hat{\mathbf{U}}^b_p$ is the $(p-1)$th $M_t\Gamma \times N$ submatrix of $\mathbf{U}^{b^{-1}}$. Thus the covariance matrix of the new noise $\tilde{\mathbf{w}}_p$ is given by $\mathbf{T}_{b,p} = \mathrm{E}\left[\tilde{\mathbf{w}}_p\tilde{\mathbf{w}}_p^{\mathcal{H}}\right] = \hat{\mathbf{U}}^b_p\left(\hat{\mathbf{U}}^b_p\right)^{\mathcal{H}}$. Since $\hat{\mathbf{U}}^b_p$ has full row rank $M_t\Gamma$, it is clear that $\mathbf{T}_{b,p}$ is nonsingular. So for the signal model (18), if we decode the SF submatrix $\mathbf{G}_p$ by the ML criterion, the achieved diversity order is still equal to the minimum rank of the matrix $(\mathbf{X}_p-\hat{\mathbf{X}}_p)\mathbf{\Lambda}(\mathbf{X}_p-\hat{\mathbf{X}}_p)^{\mathcal{H}}$ for any distinct $\mathbf{G}_p$ and $\hat{\mathbf{G}}_p$, which means that for the ZF-ML detection method, the achieved diversity order is not decreased by multiple CFOs if $|\varepsilon_m| < 0.5$.

Now, we are in a position to state the following theorem.

Theorem 2 *For the SF code described in Section 3, if the absolute values of normalized CFOs ε_m, $1 \le m \le M_t$, are all less than 0.5, then the ZF-ML detection method can still achieve the same diversity order as that of the case without CFOs.*

The proof directly follows from the above discussion.

5. SIMULATION RESULTS

First, an $M_t = 2$ system with 8 OFDM tones, 20 MHz bandwidth and BPSK modulation is simulated. The channels from relay nodes to the destination node are frequency-selective with two equal power rays $[\tau_m(0), \tau_m(1)] = [0, 0.1]\mu s$ for $1 \le m \le 2$. We also assume the destination node has only one receive antenna. For each channel realization, each ε_m is uniformly selected from $[-\varepsilon_{Max}, \varepsilon_{Max}]$. The SF code proposed in [8] is applied. Coding parameter Γ is set as 2. Thus without multiple CFOs, diversity order 4 can be achieved. At the destination node the sphere decoding method is applied to jointly consider all of the subcarriers if CFOs exist.

In Fig.1, we can see that when $\varepsilon_{Max} = 0.4$, since diversity order 4 can still be achieved, the performance loss is very small and the same slop of SER curve as that of the case without CFOs is observed. However when $\varepsilon_{Max} = 0.8$, since when $|\varepsilon_m| > 0.5$ full diversity can not always be achieved for each realization of CFOs, the slop is no longer parallel with the curve without CFOs at the high SNR range.

Finally, we simulated a special case, i.e., $\varepsilon_1 = 0.6$ and $\varepsilon_2 = -0.4$ corresponding to the case $\varepsilon_{m_l} - \varepsilon_{m_i} = -1$, for which full diversity can not be achieved. In Fig.1, we can see that the slope of its SER curve is obvious less than that of the SER curve without CFOs.

To investigate the SER performance of the ZF-ML method, we also consider an $M_t = 2$ system with 64 OFDM tones, 20 MHz bandwidth and QPSK modulation. The channels from relay nodes to the destination node are all frequency-selective fading with two equal power rays and $[\tau_m(0), \tau_m(1)] = [0, 0.5]\mu s$ for $1 \leq m \leq 2$. To achieve full diversity order 4 we set Γ as 2. In Fig.2 we can see that in the presence of multiple CFOs as SNR increases, the OFDM system will quickly suffer from an error floor if we directly decode (referred to as DD) the SF code by only ignoring all the ICI terms due to CFOs. When $\varepsilon_{Max} = 0.2$, for the ZF-ML method, the performance loss is very small and the same slop of SER curve as that of the case without CFOs is observed. As ε_{Max} is increased from 0.2 to 0.4, the performance of the ZF-ML method is degraded. Since ε_{Max} is still less than 0.5, from the previous analysis, we see that when $\varepsilon_{Max} = 0.4$ the ZF-ML method can still achieve full diversity order 4. This is confirmed by simulation results. In Fig.2, its slop is still parallel with the curve without CFOs at the high SNR range.

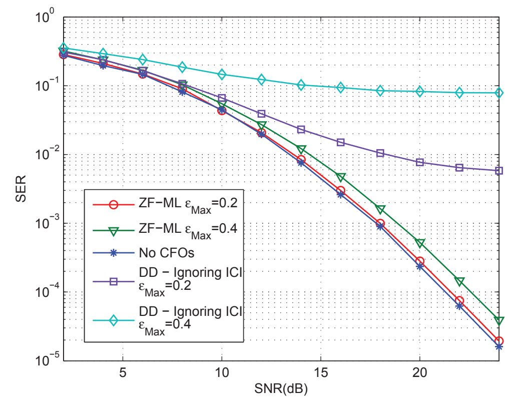

Fig. 2. Performance of the ZF-ML detection method.

6. CONCLUSION

In this paper, we investigated the effect of multiple CFOs in a cooperative OFDM based system on a family of SF codes proposed in [8]. By treating the CFOs as a part of the SF code matrix, we showed that if all the absolute values of normalized CFOs are less than 0.5, the full diversity order for the SF codes are not affected by the multiple CFOs in the SF coded OFDM cooperative system. We further prove that this full diversity property still be preserved if the zero forcing (ZF) method is used to equalize the multiple CFOs. All these imply that the SF codes proposed in [8] for MIMO-OFDM systems are robust to both timing errors and frequency offsets in a cooperative system.

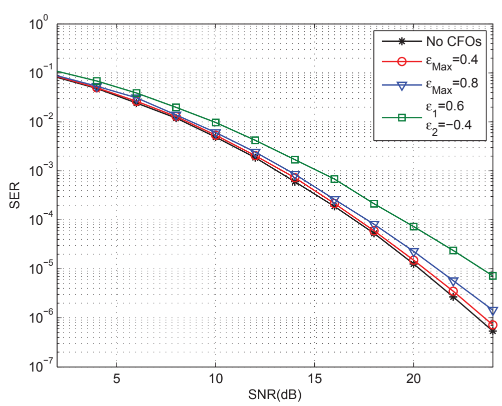

Fig. 1. Performance of the SF codes with multiple CFOs

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