A SIMPLE DESIGN OF SPACE-TIME BLOCK CODES ACHIEVING FULL DIVERSITY WITH LINEAR RECEIVERS

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ABSTRACT

Orthogonal space-time block codes (OSTBC) are attractive in that they can achieve full diversity and linear complexity of maximum likelihood (ML) decoding. However, the OSTBC have a low symbol rate due to the limitation of the orthogonality of the code structure. Most of the high-rate STBC achieve full diversity based on ML decoding at the receiver that is computationally expensive. In order to achieve full diversity with linear receivers, recently Liu-Zhang-Wong and Shang-Xia introduced new STBC. In this paper, we propose a simple design of STBC which have a high rate and achieve full diversity with linear receivers. The proposed STBC are constructed by embedding Alamouti codes into a Toeplitz matrix. Simulation results show that in comparison with some existing codes for a given codeword length the proposed STBC can give a better bit error rate (BER) performance while having a high rate.

Index Terms— Full diversity, linear receivers, multiple-input multiple-output (MIMO) systems, orthogonal space-time block codes, zero-forcing.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) systems have been under active consideration in the last decade because of their potential for achieving higher data rate and providing more reliable reception performance compared with traditional single-antenna systems for wireless communications. Space-time (ST) coding is a bandwidth-efficient transmission technique that can improve the reliability of data transmission in MIMO wireless systems [1, 2]. It encodes a data stream across different transmit antennas and time slots, so that multiple redundant copies of the data stream can be transmitted through independent fading channels. Orthogonal space-time block coding (OSTBC) is one of the most attractive ST coding approaches because the special structure of orthogonality guarantees a full-diversity and a simple (linear) maximum-likelihood (ML) decoding. The first OSTBC design was proposed by Alamouti in [1] for two transmit antennas and was then extended by Tarokh *et. al.* in [2] for any number of transmit antennas. A class of OSTBC from complex design with the code rate of $1/2$ was also given by Tarokh *et. al.* in [2]. Later, systematic constructions of complex OSTBC of rates $(k+1)/(2k)$ for $M_t = 2k-1$ or $M_t = 2k$ transmit antennas for any positive integer k were proposed in [3]. It has been found that the code rate of the OSTBC is not more than $3/4$ for more than two transmit antennas [4].

In order to improve the code rate while achieving full diversity, various designs of STBC have been proposed including linear dispersion STBC [5] and algebraic STBC [6]. However, those high-rate STBC achieve full diversity based on ML decoding whose complexity increases exponentially with the number of information symbols. Aiming at reducing decoding complexity while obtaining full diversity, Toeplitz STBC [7] and overlapped-Alamouti codes [8] were recently proposed with linear receivers such as zero-forcing (ZF) or minimum mean square error (MMSE) receivers. In this paper, we propose a simple design of STBC which can achieve full diversity with linear receivers and have a higher rate than that of [7]. The proposed codes are constructed by embedding Alamouti codes in a Toeplitz matrix.

This paper is organized as follows. A system model of space-time transmission over MIMO channels with linear receiver is introduced in Section 2. In Section 3, a simple design of full diversity STBC with linear receiver is proposed. Its achievable symbol rate is compared to other existing STBC and full diversity property is shown. Simulation results are presented in Section 4. Finally, in Section 5, we draw our conclusions.

Notations: The superscripts ^{T} and ^{H} stand for transpose and conjugate transpose, respectively. $\det(\mathbf{A})$ stands for the determinant of the matrix $\mathbf{A}$.

2. SYSTEM MODEL

Consider a MIMO system with M transmit antennas and N receive antennas transmitting the symbols $\{s_l\}, l = 1, \cdots, N_s$, which are selected from a given constellation such as QAM or PSK and have unit energy. To be transmitted from the M antennas, the N_s symbols $\mathbf{s} = (s_1, \cdots, s_{N_s})^T$ are encoded into a space-time block codeword matrix $\mathbf{X}(\mathbf{s})$ of size $T \times M$, where T is the block length (coding delay) of the codeword. The (t, m)-th entry of $\mathbf{X}(\mathbf{s})$ will be transmitted to the receiver from the m-th antenna during the t-th symbol period through flat fading channels. The received space-time signal, denoted by the $T \times N$ matrix $\mathbf{Y}$, can be written as

$$\mathbf{Y} = \sqrt{\frac{\rho}{\mu}}\mathbf{X}(\mathbf{s})\mathbf{H} + \mathbf{N}, \tag{1}$$

where $\mathbf{N}$ is the noise matrix whose elements are of i.i.d. with circularly symmetric complex Gaussian distribution $\mathcal{CN}(0, 1)$, $\mathbf{H}$ is the $M \times N$ channel matrix whose entries are also i.i.d. with distribution $\mathcal{CN}(0, 1)$, ρ denotes the average signal-to-noise ratio (SNR) per receive antenna and μ is the normalization factor such that the average energy of the coded symbols transmitting from all antennas during one symbol period is 1.

To decode the transmitted sequence $\mathbf{s}$ with a linear receiver, we need to extract $\mathbf{s}$ from $\mathbf{X}(\mathbf{s})$. Through some operations, we can get an equivalent signal model from (1) as:

$$\mathbf{y} = \sqrt{\frac{\rho}{\mu}}\mathcal{H}\mathbf{s} + \mathbf{n}, \tag{2}$$

where $\mathbf{y}$ denotes a signal vector of length TN, $\mathcal{H}$ is a channel matrix of size $TN \times N_s$ and $\mathbf{n}$ is the noise vector of length TN.

For ZF receiver, the estimate $\hat{\mathbf{s}}_{\mathrm{ZF}}$ of the transmitted symbol sequence $\mathbf{s}$ is, if $(\mathcal{H}^H\mathcal{H})^{-1}$ exits,

$$\hat{\mathbf{s}}_{\mathrm{ZF}} = \sqrt{\frac{\mu}{\rho}}\left(\mathcal{H}^H\mathcal{H}\right)^{-1}\mathcal{H}^H\mathbf{y}. \tag{3}$$

For MMSE receiver,

$$\hat{\mathbf{s}}_{\mathrm{MMSE}} = \sqrt{\frac{\rho}{\mu}}\left(\mathbf{I}_{N_s} + \frac{\rho}{\mu}\mathcal{H}^H\mathcal{H}\right)^{-1}\mathcal{H}^H\mathbf{y}. \tag{4}$$

It was shown in [7, Theorem 1] and [8, Theorem 1] that the STBC $\mathbf{X}(\mathbf{s})$ in (1) can achieve full diversity with ZF/MMSE receivers for QAM, PAM, and PSK signals, if $\mathcal{H}^H\mathcal{H}$ is nonsingular for any nonzero $\mathbf{H}$. To achieve the full diversity with linear receivers, Toeplitz STBC and overlapped-Alamouti codes were proposed in [7] and [8], respectively. In this paper, we propose a design of STBC that can achieve full diversity with linear receiver and has a high rate as well as a simple code structure.

3. A DESIGN OF STBC WITH LINEAR RECEIVERS

3.1. Code Design

Consider a length-L vector $\mathbf{v} = \left[\begin{array}{cccc} v_1 & v_2 & \cdots & v_L \end{array}\right]^T$. Define $\mathcal{T}(\mathbf{v}, L, M)$ as the following $T \times M$ ($T = L + M - 1$) Toeplitz matrix:

$$\mathcal{T}(\mathbf{v}, L, M) = \begin{bmatrix} v_1 & 0 & \cdots & 0 \\ v_2 & v_1 & \ddots & \vdots \\ \vdots & v_2 & \ddots & 0 \\ v_L & \vdots & \ddots & v_1 \\ 0 & v_L & \ddots & v_2 \\ \vdots & 0 & \ddots & \vdots \\ 0 & \vdots & \ddots & v_L \end{bmatrix}. \tag{5}$$

In (5), the m-th column is the circular shift of the first column by $(m-1)$ elements from top to bottom. In [7], Toeplitz STBC were introduced which have the exact form of (5). For the Toeplitz STBC $\mathbf{X}(\mathbf{s}) = \mathcal{T}(\mathbf{v}, L, M)$, the (t, m)th entry of the code matrix is sent from the m-th antenna during the t-th symbol period.

We define

$$\mathbf{B} = \left[\begin{array}{cccc} \mathcal{A}_1^T & \mathcal{A}_2^T & \cdots & \mathcal{A}_L^T \end{array}\right]^T, \tag{6}$$

where $\mathcal{A}_l$ denotes an Alamouti code, i.e.,

$$\mathcal{A}_l = \begin{bmatrix} s_{2l-1} & s_{2l} \\ -s_{2l}^* & s_{2l-1}^* \end{bmatrix}, \quad l = 1, 2, \cdots, L \tag{7}$$

with $s_i (i = 1, 2, \cdots, 2L)$ being the information symbols which are PSK or QAM symbols.

Our proposed design of STBC for M (even number) transmit antennas is given by a block Toeplitz matrix as follows:

$$\mathbb{T}(\mathbf{B}, L, M) = \begin{bmatrix} \mathcal{A}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathcal{A}_2 & \mathcal{A}_1 & \ddots & \vdots \\ \vdots & \mathcal{A}_2 & \ddots & \mathbf{0} \\ \mathcal{A}_L & \vdots & \ddots & \mathcal{A}_1 \\ \mathbf{0} & \mathcal{A}_L & \ddots & \mathcal{A}_2 \\ \vdots & \mathbf{0} & \ddots & \vdots \\ \mathbf{0} & \vdots & \ddots & \mathcal{A}_L \end{bmatrix}, \tag{8}$$

which is a $(2L + M - 2) \times M$ matrix. In fact, the block Toeplitz matrix (8) can be seen as a code design replacing the scalars v_l and 0 in (5) with matrices $\mathcal{A}_l$ and $\mathbf{0}_{2\times 2}$, respectively.

For M (odd number) transmit antennas, the proposed STBC is the first M columns of $\mathbb{T}(\mathbf{B}, L, M + 1)$ where $M + 1$ is an even number.

3.2. Symbol Rate

For a STBC $\mathcal{X}(\mathbf{s})$ of size $T \times M$, symbol rate is defined as $R = \frac{N_s}{T}$, where N_s denotes the number of information symbols that are encoded in $\mathcal{X}(\mathbf{s})$, i.e., $\mathbf{s} = (s_1, s_2, \cdots, s_{N_s})$. The block length (coding delay) of the STBC $\mathcal{X}(\mathbf{s})$ is denoted by T, which is the number of channel uses to transmit a codeword.

For Toeplitz STBC $\mathcal{T}(\mathbf{v}, L, M)$ in (5) proposed by [7], the block length is $L + M - 1$ and L information symbols are encoded. Then, the symbol rate of $\mathcal{T}(\mathbf{v}, L, M)$ is:

$$R_{\mathcal{T}(\mathbf{v},L,M)} \quad = \quad 1 - \frac{M-1}{T}. \tag{9}$$

For our proposed STBC $\mathbb{T}(\mathbf{B}, L, M)$ in (8), the block length is $2L + M - 2$ and $2L$ information symbols are encoded. Then, the symbol rate of $\mathbb{T}(\mathbf{B}, L, M)$ is (even M):

$$R_{\mathbb{T}(\mathbf{B},L,M)} \quad = \quad 1 - \frac{M-2}{T}. \tag{10}$$

For odd M, the symbol rate of our proposed STBC is the same as that for $M + 1$ (even). Then, the symbol rate is $1 - \frac{M-1}{T}$. In summary,

$$R_{\mathbb{T}(\mathbf{B},L,M)} = \begin{cases} \frac{2L}{2L+M-2} = 1 - \frac{M-2}{T}, & M \text{ even} \\ \frac{2L}{2L+(M+1)-2} = 1 - \frac{M-1}{T}, & M \text{ odd} \end{cases} \tag{11}$$

Note that the rate shown above approaches 1 when the space-time codeword length T is sufficiently large.

For a given block length T and even M, the symbol rate comparison between $\mathbb{T}(\mathbf{B}, L, M)$ and $\mathcal{T}(\mathbf{v}, L, M)$ is:

$$\frac{R_{\mathbb{T}(\mathbf{B},L,M)}}{R_{\mathcal{T}(\mathbf{v},L,M)}} = \frac{1 - \frac{M-2}{T}}{1 - \frac{M-1}{T}} = 1 + \frac{1}{T - M + 1}. \tag{12}$$

Considering $T \geq M$ for any STBC, we can see that our proposed STBC $\mathbb{T}(\mathbf{B}, L, M)$ always achieves a higher rate than the Toeplitz STBC in [7] for even M and the rate advantage diminishes when T goes to infinity.

For another recently proposed STBC, namely overlapped-Alamouti code $\mathcal{O}_{M,L}$ in [8], the symbol rate is

$$R_{\mathcal{O}_{M,L}} = \begin{cases} \frac{L}{L+M-2} = 1 - \frac{M-2}{T}, & L \,\&\, M \text{ even} \\ \frac{L}{L+M-1} = 1 - \frac{M-1}{T}, & \text{otherwise} \end{cases} \tag{13}$$

Given the same block length T, our proposed STBC $\mathbb{T}(\mathbf{B}, L, M)$ achieves the same rate as the overlapped-Alamouti code but has a simple code structure.

3.3. Full Diversity Property with Linear Receivers

It was shown in [7, Theorem 1] and [8, Theorem 1] that the STBC $\mathbf{X}(\mathbf{s})$ in (1) can achieve full diversity with ZF/MMSE receivers for QAM, PAM, and PSK signals, if $\mathcal{H}^H\mathcal{H}$ (for $\mathcal{H}$ in (2)) is nonsingular for any nonzero $\mathbf{H}$. In the following, we prove that $\mathcal{H}^H\mathcal{H}$ is nonsingular for the proposed space-time codeword $\mathbb{T}(\mathbf{B}, L, M)$ in (8).

We first consider transmission of the data sequence $\mathbf{s} = (s_1, \cdots, s_{N_s})^T$ via ST coding in (8) over MISO channels. From (1), the received signals can be written as:

$$\mathbf{Y} = \sqrt{\frac{\rho}{\mu}}\mathbb{T}(\mathbf{B}, L, M)\mathbf{h} + \mathbf{N}, \tag{14}$$

where $\mathbf{h} = [\begin{array}{cccc} h_1 & h_2 & \cdots & h_M \end{array}]^T$ with $h_m (m = 1, 2, \cdots, M)$ denoting the channel coefficient of the link from the m-th transmit antenna to the single antenna receiver.

For all elements in the even rows of $\mathbf{Y}$ in (14), we take the negative conjugate. Then, we can obtain

$$\mathbf{y} = \sqrt{\frac{\rho}{\mu}}\mathbb{T}(\mathbf{G}, \frac{M}{2}, N_s)\mathbf{s} + \mathbf{n}, \tag{15}$$

where $\mathbb{T}(\mathbf{G}, \frac{M}{2}, N_s)$ is a channel matrix of size $T \times N_s$ and has the same form of (8) but replacing $\mathbf{B}$ in (8) by $\mathbf{G}$:

$$\mathbf{G} = \left[\begin{array}{cccc} \mathcal{G}_1^T & \mathcal{G}_2^T & \cdots & \mathcal{G}_{\frac{M}{2}}^T \end{array}\right]^T, \tag{16}$$

where

$$\mathcal{G}_k = \left[\begin{array}{cc} h_{2k-1} & h_{2k} \\ -h_{2k}^* & h_{2k-1}^* \end{array}\right] \tag{17}$$

for $k = 1, \cdots, M/2$.

Comparing (2) with (15), we get

$$\begin{aligned} \mathcal{H} &= \mathbb{T}(\mathbf{G}, \frac{M}{2}, N_s) \\ &= \left[\begin{array}{cccc} \mathcal{G}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathcal{G}_2 & \mathcal{G}_1 & \ddots & \vdots \\ \vdots & \mathcal{G}_2 & \ddots & \mathbf{0} \\ \vdots & \vdots & \ddots & \mathcal{G}_1 \\ -- & -- & -- & -- \\ \vdots & \vdots & \ddots & \mathcal{G}_2 \\ \mathcal{G}_{\frac{M}{2}} & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathcal{G}_{\frac{M}{2}} & \ddots & \vdots \\ \vdots & \mathbf{0} & \ddots & \vdots \\ \mathbf{0} & \vdots & \ddots & \mathcal{G}_{\frac{M}{2}} \end{array}\right] \triangleq \left[\begin{array}{c} \mathbf{D}_1 \\ -- \\ \mathbf{D}_2 \end{array}\right]. \end{aligned} \tag{18}$$

Note that $\mathbf{D}_1$ is a lower block triangular matrix having matrix $\mathcal{G}_1$ on the diagonal. Because $\mathcal{H}^H\mathcal{H} = \mathbf{D}_1^H\mathbf{D}_1 + \mathbf{D}_2^H\mathbf{D}_2$. For any nonzero $\mathbf{h}$, we assume that $\mathcal{G}_1 \neq \mathbf{0}$. Then,

$$\begin{aligned} \det\left(\mathcal{H}^H\mathcal{H}\right) &\geq \det(\mathbf{D}_1^H\mathbf{D}_1) + \det(\mathbf{D}_2^H\mathbf{D}_2) \\ &\geq \det(\mathbf{D}_1^H\mathbf{D}_1) \\ &= \left[\det(\mathcal{G}_1^H\mathcal{G}_1)\right]^{N_s/2} \\ &= (|h_1|^2 + |h_2|^2)^{N_s/2} > 0. \end{aligned} \tag{19}$$

For any nonzero $\mathbf{h}$, if $\mathcal{G}_1 = \mathbf{0}$ we then find the smallest index p for $\mathcal{G}_p$ such that $\mathcal{G}_p \neq \mathbf{0}$. Following the similar proof in (19), we can get $\det\left(\mathcal{H}^H\mathcal{H}\right) > 0$. Therefore, for any nonzero $\mathbf{H}$ in MISO, our proposed codes in (8) can achieve full diversity with linear receivers. It can also be proved that the full diversity is obtained for linear receivers in MIMO case.

3.4. Code Examples

3.4.1. $M = 4$, $T = 10$

$$\mathbb{T}(\mathbf{B}, 4, 4) = \left[\begin{array}{cc} \mathcal{A}_1 & \mathbf{0} \\ \mathcal{A}_2 & \mathcal{A}_1 \\ \mathcal{A}_3 & \mathcal{A}_2 \\ \mathcal{A}_4 & \mathcal{A}_3 \\ \mathbf{0} & \mathcal{A}_4 \end{array}\right].$$

The normalization factor μ for the above code is $\mu = \frac{32}{10}$ and the symbol rate is $\frac{8}{10}$. The received signals of MISO systems are $\mathbf{Y} = \sqrt{\frac{\rho}{32/10}}\mathbb{T}(\mathbf{B}, 4, 4)\mathbf{H} + \mathbf{N}$. Equivalently,

$$\mathbf{y} = \sqrt{\frac{\rho}{32/10}}\mathcal{H}\mathbf{s} + \mathbf{n}, \tag{20}$$

where $\mathbf{s} = (s_1, \cdots, s_8)^T$,

$$\mathcal{H} = \left[\begin{array}{cccc} \mathcal{G}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathcal{G}_2 & \mathcal{G}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathcal{G}_2 & \mathcal{G}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathcal{G}_2 & \mathcal{G}_1 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathcal{G}_2 \end{array}\right], \tag{21}$$

and $\mathcal{G}_1$ and $\mathcal{G}_2$ are given by (17).

Under the same scenario, the symbol rate of the Toeplitz STBC is $\frac{7}{10}$.

3.4.2. $M = 5$, $T = 10$

Because our code design for odd M transmit antennas is based on the code design for $M + 1$ (even) transmit antennas, we first design a code for $M + 1 = 6$ and $T = 10$ as follows,

$$\mathbb{T}(\mathbf{B}, 3, 6) = \left[\begin{array}{ccc} \mathcal{A}_1 & \mathbf{0} & \mathbf{0} \\ \mathcal{A}_2 & \mathcal{A}_1 & \mathbf{0} \\ \mathcal{A}_3 & \mathcal{A}_2 & \mathcal{A}_1 \\ \mathbf{0} & \mathcal{A}_3 & \mathcal{A}_2 \\ \mathbf{0} & \mathbf{0} & \mathcal{A}_3 \end{array}\right].$$

Then, the code design for $M = 5$ and $T = 10$, i.e., $\mathbb{T}(\mathbf{B}, 3, 5)$ is given by the first M columns of the code $\mathbb{T}(\mathbf{B}, 3, 6)$. The normalization factor μ for the code $\mathbb{T}(\mathbf{B}, 3, 5)$ is $\mu = \frac{30}{10}$ and the symbol rate is $\frac{6}{10}$. Note that the normalization factor μ for the code

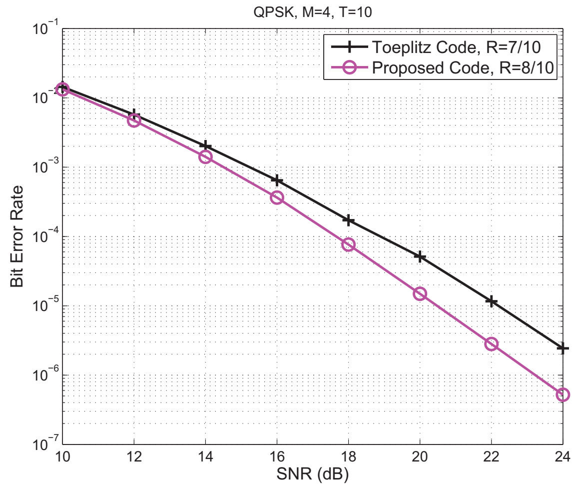

Fig. 1. Comparison of BER performance for Toeplitz codes and the proposed codes in a MISO 4×1 system with with ZF receivers and QPSK. The codes have the same block length $T = 10$.

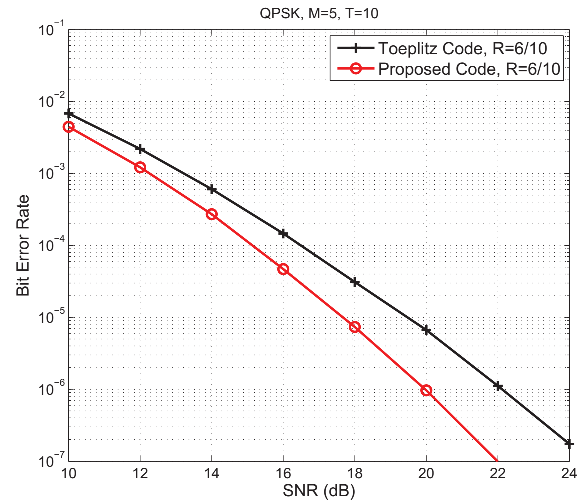

Fig. 2. Comparison of BER performance for Toeplitz codes and the proposed codes in a MISO 5×1 system with with ZF receivers and QPSK. The codes have the same block length $T = 10$.

$\mathbb{T}(\mathbf{B}, 3, 6)$ is $\mu = \frac{36}{10}$. The received signals of MISO systems are $\mathbf{Y} = \sqrt{\frac{\rho}{30/10}}\mathbb{T}(\mathbf{B}, 3, 5)\mathbf{H} + \mathbf{N}$. Equivalently,

$$\mathbf{y} = \sqrt{\frac{\rho}{30/10}}\mathcal{H}\mathbf{s} + \mathbf{n}, \tag{22}$$

where $\mathbf{s} = (s_1, \cdots, s_6)^T$,

$$\mathcal{H} = \begin{bmatrix} \mathcal{G}_1 & \mathbf{0} & \mathbf{0} \\ \mathcal{G}_2 & \mathcal{G}_1 & \mathbf{0} \\ \mathcal{G}_3 & \mathcal{G}_2 & \mathcal{G}_1 \\ \mathbf{0} & \mathcal{G}_3 & \mathcal{G}_2 \\ \mathbf{0} & \mathbf{0} & \mathcal{G}_3 \end{bmatrix}, \tag{23}$$

and $\mathcal{G}_k$ $(k = 1, 2, 3)$ is given by (17) and $h_6 = 0$ in $\mathcal{G}_3$.

Under the same scenario, the symbol rate of the Toeplitz STBC is also $\frac{6}{10}$.

4. SIMULATION RESULTS

To show the performance comparison between our proposed STBC and the Toeplitz STBC in [7], we present simulation results of the two codes in a MISO system with M transmit antennas over flat Rayleigh fading channels. QPSK and ZF receiver are adopted. When the block length (i.e., coding delay) is 10 and $M = 4$, the symbol rates for the proposed STBC and the Toeplitz STBC are $\frac{8}{10}$ and $\frac{7}{10}$, respectively. When $T = 10$ and $M = 5$, the symbol rates for the two codes are both $\frac{6}{10}$.

Fig. 1 shows the bit error rate (BER) performances of the proposed STBC and the Toeplitz STBC for 4 transmit antennas. It is clear to see that our proposed STBC outperform the Toeplitz STBC. It should be emphasized that our proposed STBC for $M = 4$ has a slightly higher rate (0.1 in symbol rate advantage) than the Toeplitz STBC. It is expected that if we keep the same transmission efficiency, the BER performance advantage of our STBC is increased. Fig. 2 shows the BER performances of the two codes for 5 transmit antennas. It clearly demonstrates that the proposed STBC is superior to the Toeplitz STBC while achieving the same rate of $\frac{6}{10}$.

5. CONCLUSIONS

A simple design of high-rate STBC that can achieve full diversity with linear receivers was proposed. The proposed STBC were constructed by embedding Alamouti codes into a Toeplitz matrix. Simulation results demonstrated a better BER performance of the proposed codes while having a slightly higher rate than the Toeplitz codes given the identical block length (coding delay).

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