A LOW COMPLEXITY CHANNEL DECOMPOSITION AND FEEDBACK STRATEGY FOR MIMO PRECODER DESIGN

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ABSTRACT

In classical MIMO systems, singular value decomposition (SVD) is adopted as a common way to decompose the MIMO channel into parallel subchannels. However, in addition to its high complexity, it is also sensitive to the ill-conditioning of the channel matrix. In this paper, we propose a channel decomposition strategy called LDL^H decomposition for low complexity MIMO precoder design. We show that the computation complexity of LDL^H achieves one degree of magnitude lower than that of SVD, as well as some SVD-based decomposition scheme such as geometric mean decomposition (GMD) and uniform channel decomposition (UCD). In addition, we also show that LDL^H -based precoder requires less quantization effort and feedback bandwidth.

Index Terms— MIMO, precoder, SVD, LDL^{*H*}, limited feedback.

1. INTRODUCTION

It is well known that the use of multiple antennas promises substantial capacity gains. In order to exploit these gains, the system must deal with the distortion caused by the channel and the interference. Based on the statistical information of the channel, [2] and [3] have suggested that the use of diagonal precoding can greatly improve the system performance. For channel diagonalization, it has been shown in [1] that the MIMO channel can be decomposed into parallel eigensubchannels, or equivalently, eigen-modes, by means of singular value decomposition (SVD). Moreover, when perfect channel state information (CSI) is available to both the transmitter and the receiver, the performance subjected to minimum mean-square error (MMSE) criterion is improved further. In addition to SVD, there are even more advanced decomposition schemes are devised. In [4], transceiver design based on geometric mean decomposition (GMD) is proposed. Identical parallel subchannels are obtained by GMD, bringing about convenient design in coding/decoding and modulation/demodulation schemes. However, in addition to the inherent SVD operation, it needs extra operations for permutations and Givens rotations. In [5], an improved version originated from GMD is given. The flaw that GMD suffers large capacity loss in low SNR is amended to become strictly capacity lossless for the whole SNR range, called *uniform channel decomposition* (UCD). Starting also from SVD, it is justified that UCD spends tremendous effort in water-filling and the generation of a tunable semi-unitary matrix.

In this paper, we propose to use LDL^{H} as the channel decomposition scheme for low complexity MIMO precoder design in i.i.d. Rayleigh flat fading channel, where **L** is the lower triangular matrix with 1's along the diagonal while **D** is the resultant diagonal matrix. Being endowed with the low complexity properties of triangular decomposition, e.g., LU decomposition, we successfully establish a precoder/decoder architecture based on this apparently succinct matrix decomposition method. An iterative procedure is also developed to realize that in a more efficient way. Quantitative analysis of overhead for both decomposition and feedback quantization are also presented. The implementation simplicity and numerical stability [6] make LDL^H suitable for low complexity MIMO precoder design.

2. SYSTEM MODEL

We consider a MIMO communication system with N transmitting antennas and M receiving antennas. The channel is characterized as a Rayleigh flat fading channel. Fig. 1 shows considered MIMO system. Specifically, a reverse link from receiver to transmitter is available for sending channel state information (CSI) obtained at the receiver. In this *feedback channel*, limited and quantized CSI is carried. In this paper, we assume the feedback is delay-free. In Fig. 1, the sampled baseband signal is represented as

$$y = BHFs + Bn$$
(1)

where $\mathbf{H} \in \mathbb{C}^{M \times N}$ is the channel matrix with rank K. $\mathbf{F} \in \mathbb{C}^{N \times K}$ is the precoder matrix. $\mathbf{B} \in \mathbb{C}^{K \times M}$ is the decoder matrix which matches \mathbf{F} . $\mathbf{s} \in \mathbb{C}^{K \times 1}$ is the source signal. $\mathbf{x} \in \mathbb{C}^{N \times 1}$ is the transmitted signal that $\mathbf{x} = \mathbf{Fs}$. $\mathbf{y} \in \mathbb{C}^{K \times 1}$ is the received signal. $\mathbf{n} \in \mathbb{C}^{M \times 1}$ is the zero-mean additive white Gaussian noise (AWGN) such that $\mathbf{n} \sim N(0, \sigma_{nn}^2 \mathbf{I}_M)$,



Fig. 1. A MIMO system with precoder F and decoder B.

i.e., the covariance $\mathbf{R}_{nn} = \sigma_{nn}^2 \mathbf{I}_M$, where \mathbf{I}_M denotes the identity matrix with dimension M. It is a common situation that the transmitted symbols are uncorrelated with each other. Therefore, the input signal vector \mathbf{s} is assumed to be zeromean and independently distributed with correlation matrix $\mathbf{R}_{ss} = \mathbf{E}[\mathbf{ss}^H] = \sigma_{ss}^2 \mathbf{I}_K$, where $(\cdot)^H$ denotes the conjugate transpose, and $\mathbf{E}[\cdot]$ stands for the expectation value.

3. PRECODER DESIGN USING LDL^H CHANNEL DECOMPOSITION

Traditionally we use SVD to decompose the MIMO channel. However, the computational complexity of SVD amounts to $O(K^4)$. To further reduce the complexity, we introduce the LDL^H decomposition scheme with the obvious advantage that it is a sort of triangular matrix factorization which is more computation-tractable. According to [6], if a matrix $\mathbf{A} \in \mathbb{C}^{N \times N}$ is Hermitian then there exists a *unique* lower triangular matrix \mathbf{L} with *unit diagonal elements* l_{ii} and a *unique diagonal matrix* $\mathbf{D} = \text{diag}\{d_1, \ldots, d_K\}$, that \mathbf{A} can be factorized as

$$\mathbf{A} = \mathbf{L}\mathbf{D}\mathbf{L}^H.$$
 (2)

In which only $O(K^3)$ of computation is required.

In general, the channel matrix \mathbf{H} is not Hermitian. To exploit LDL^H , we pre-multiply \mathbf{H}^H by \mathbf{H} in the receiver. Hence $\mathbf{H}^H \mathbf{H}$ becomes a Hermitian matrix. Assuming that \mathbf{A} equals to $\mathbf{H}^H \mathbf{H}$ and is factorized as

$$\mathbf{A} = \mathbf{H}^H \mathbf{H} = \mathbf{L} \mathbf{D} \mathbf{L}^H. \tag{3}$$

It follows that

$$\mathbf{D} = \left(\mathbf{L}^{-1}\mathbf{H}^{H}\right)\mathbf{H}\left(\mathbf{L}^{-1}\right)^{H}.$$
(4)

The precoder \mathbf{F} and the decoder \mathbf{B} are then realized as

$$\mathbf{F} = (\mathbf{L}^{-1})^H \mathbf{D}^{-\frac{1}{2}} \tag{5}$$

$$\mathbf{B} = \mathbf{L}^{-1} \mathbf{H}^H.$$
(6)

The purpose of mutiplying $D^{-\frac{1}{2}}$ is to normalize the noise level. The equivalent signal model of (1) now becomes

$$\mathbf{y} = \left(\mathbf{L}^{-1}\mathbf{H}^{H}\right)\mathbf{H}\left(\mathbf{L}^{-1}\right)^{H}\mathbf{D}^{-\frac{1}{2}}\mathbf{s} + \left(\mathbf{L}^{-1}\mathbf{H}^{H}\right)\mathbf{D}^{-\frac{1}{2}}\mathbf{n}$$
$$= \mathbf{D}^{\frac{1}{2}}\mathbf{s} + \mathbf{n}_{L}$$
(7)



Fig. 2. Processing steps of LDL^H in receiver: (a) Constructing Hermitian matrix, (b) Decomposition, (c) Matrix inversion and (d) Quantization.

where \mathbf{n}_L is the equivalent noise in LDL^H regime. The noise covariance $\mathbf{R}_{n_L n_L}$ is still $\sigma_{nn}^2 \mathbf{I}_M$. In [8], solid proofs are given to justify that LDL^H is superior to SVD in both the channel condition number and the symbol error rate.

4. COMPLEXITY ANALYSIS

Practical implementation using LDL^{H} is discussed in this section. For comprehensive realization, we divide the flow of LDL^{H} decomposition and quantization feedback into four steps denoted as (a), (b), (c), and (d), as shown in Fig. 2. The $\mathbf{H}^{H}\mathbf{H}$ is constructed in (a). The LDL^{H} decomposition is carried out in (b). The matrix inversion is performed in (c). At last, the quantization for feedback is implemented in (d).

In part (a), normally M complex multiplications (CMs) and (M-1) complex additions (CAs) are required to obtain each a_{ij} , where a_{ij} is the element of **A** in *i*-th row and *j*-th column. Since **A** is Hermitian and with dimension $N \times N$, therefore, only N(N-1)/2 elements of **A** are needed to be calculated. Therefore, totally MN(N+1)/2 of CM and (M-1)N(N+1)/2 of CA are required in part (a).

For operations of $\mathbf{A} = \mathbf{LDL}^{H}$ in part (b), we have developed an iterative algorithm to avoid numerous computations which happens by using traditional way of Gaussian elimination and pivoting. Specifically, during the iteration processes, an iteration variable $r_{ij}(k)$ is involved to efficiently derive each l_{ij} and d_i . It is defined as

$$r_{ij}(k) = \begin{cases} a_{ij} & \text{for } k = 1\\ r_{ij}(k-1) - & \\ \frac{r_{i,k-1}(k-1) \cdot r_{j,k-1}^*(k-1)}{r_{k-1,k-1}(k-1)} & \text{for } k > 1\\ , k \le j \le i \le N \end{cases}$$
(8)

where k is the iteration index, while the $(\cdot)^*$ corresponds to

the conjugate operation. The l_{ij} is then derived as

$$l_{ij} = \begin{cases} \frac{r_{ij}(j)}{\left(\sum_{i=j}^{N} r_{ij}^{2}(j)\right)^{\frac{1}{2}}} & \text{for } j \neq N\\ 1 & \text{for } j = N \end{cases}$$
(9)

and also the d_i :

$$d_i = \begin{cases} \frac{\sum_{i=j}^{N} r_{ij}^2(j)}{r_{jj}(j)} & \text{for } j \neq N \\ r_{NN}(N) & \text{for } j = N. \end{cases}$$
(10)

To show $r_{ij}(k)$, l_{ij} and d_i really construct a_{ij} , the proof is given below.

Lemma 1. Prove that $r_{ij}(k)$, l_{ij} and d_i represented in (8), (9) and (10) construct a_{ij}

Proof. For Hermitian matrix $\mathbf{A} = \mathbf{L}\mathbf{D}\mathbf{L}^{H}$, we have

$$a_{ij} = \begin{cases} j \le i : \sum_{k=1}^{j} l_{ik} d_{kk} l_{jk}^{*} \\ j > i : a_{ij} = a_{ji}^{*} \end{cases}$$
(11)

where k is the number of iteration step. Substitute (9) and (10) into (11), for $j \leq i$, we have

$$a_{ij} = \sum_{k=1}^{j} \left(\frac{r_{ik}(k)}{\left(\sum_{i=k}^{N} r_{ik}^{2}(k)\right)^{\frac{1}{2}}} \right) \cdot \left(\frac{\sum_{i=k}^{N} r_{ik}^{2}(k)}{r_{kk}(k)} \right)$$
$$\cdot \left(\frac{r_{jk}^{*}(k)}{\left(\sum_{j=k}^{N} r_{jk}^{2}(k)\right)^{\frac{1}{2}}} \right)$$
$$= \sum_{k=1}^{j} \frac{r_{ik}(k)r_{jk}^{*}(k)}{r_{kk}(k)}.$$
(12)

In (12), we take out the j-th term from the summation term and obtain

$$a_{ij} = \sum_{k=1}^{j-1} \frac{r_{ik}(k)r_{jk}^{*}(k)}{r_{kk}(k)} + \frac{r_{ij}(j)r_{jj}^{*}(j)}{r_{jj}(j)}$$
$$= \sum_{k=1}^{j-1} \frac{r_{ik}(k)r_{jk}^{*}(k)}{r_{kk}(k)} + r_{ij}(j).$$
(13)

In (13), we separate the term $r_{ij}(j)$ into two parts and take out the (j-1)-th term from the summation, then obtain

$$a_{ij} = \sum_{k=1}^{j-1} \frac{r_{ik}(k)r_{jk}^{*}(k)}{r_{kk}(k)} + r_{ij}(j-1) - \frac{r_{i,j-1}(j-1) \cdot r_{j,j-1}^{*}(j-1)}{r_{j-1,j-1}(j-1)} = \left(\sum_{k=1}^{j-2} \frac{r_{ik}(k)r_{jk}^{*}(k)}{r_{kk}(k)} + \frac{r_{i,j-1}(j-1) \cdot r_{j,j-1}^{*}(j-1)}{r_{j-1,j-1}(j-1)}\right) + r_{ij}(j-1) - \frac{r_{i,j-1}(j-1) \cdot r_{j,j-1}^{*}(j-1)}{r_{j-1,j-1}(j-1)} = \sum_{k=1}^{j-2} \frac{r_{ik}(k)r_{jk}^{*}(k)}{r_{kk}(k)} + r_{ij}(j-1).$$
(14)

Again, in (14), we separate the term $r_{ij}(j-1)$ into two parts and take out the (j-2)-th term from the summation, then obtain

$$a_{ij} = \sum_{k=1}^{j-3} \frac{r_{ik}(k)r_{jk}^*(k)}{r_{kk}(k)} + r_{ij}(j-2).$$
 (15)

Perform the same calculation recursively, we finally reach

$$a_{ij} = \sum_{k=1}^{j-(j-1)} \frac{r_{ik}(k)r_{jk}^*(k)}{r_{kk}(k)} + r_{ij}(2)$$
$$= \frac{a_{i1}a_{j1}^*}{a_{11}} + a_{ij} - \frac{a_{i1}a_{j1}^*}{a_{11}}$$
$$= a_{ij}.$$

We clearly show that the r_{ij} in (8), l_{ij} in (9) and d_{ij} in (10) constitute a_{ij} .

In (8), each $r_{ij}(k)$ requires 2 CMs, 1 CA and N - 1 divisions (DIV) for k > 1. In (9) and (10), the number of $r_{ij}(k)$ needed for both L and D is $\frac{1}{6}N(N-1)(N+1)$. Therefore, total $\frac{1}{3}N(N-1)(N+1)$ CMs, $\frac{1}{6}N(N-1)(N+1)$ CAs, N-1 DIVs and N-1 square roots (SR) are required for both (9) and (10). By using the fact that N DIVs $\equiv N$ CMs and N SRs $\equiv N^2$ CMs. we have total $(\frac{1}{3}N^3 + \frac{1}{2}N^2 + \frac{13}{6}N - 3)$ CMs and $(\frac{1}{6}N^3 + \frac{1}{2}N^2 - \frac{2}{3}N)$ CAs are needed for part (b).

Table 1. Complexity comparison between LDL^{H} and SVD-based decomposition schemes

Operation	Decomposition		Quantization $\{(d)\}$	
Computation	СМ	CA	СМ	CA
SVD	$\frac{3}{2}N^4 - \frac{5}{2}N^3 + 41N^2 + N - 41$	$\frac{3}{2}N^4 - \frac{7}{2}N^3 + 35N^2 - N - 32$	$6N^3 + 2N^2 + N$	$3N^2 + 60N$
GMD	$\frac{3}{2}N^4 - \frac{5}{2}N^3 + 73N^2 + 5N - 77$	$\frac{3}{2}N^4 - \frac{7}{2}N^3 + 65N^2 + N - 64$	$6N^3 + 2N^2 + N$	$3N^2 + 60N$
UCD	$\frac{3}{2}N^4 - \frac{5}{2}N^3 + 110N^2 + 7N - 116$	$\frac{3}{2}N^4 - \frac{7}{2}N^3 + 94N^2 + 2N - 94$	$6N^3 + 2N^2 + N$	$3N^2 + 60N$
$LDL^{H} \{(a)+(b)+(c)\}$	$N^3 + \frac{5}{2}N^2 + \frac{5}{2}N - 6$	$\frac{5}{6}N^3 + 2N^2 - \frac{5}{6}N - 2$	$4N^3 + N^2 + N$	$N^{2} + 15N$

The inversion computation for (c) is straightforward. Total of $\frac{1}{6}N(N-1)(N+4)$ CMs and $\frac{1}{6}N(N-1)(N-2)$ CAs are needed.

The quantization scheme of (d) is the key to precoder design either in complexity or performance. In this paper, we use the process of *constructing Householder-reflection matrix* presented in [6] and the codebook designed in [9] for good resolution over limited feedback bandwidth for all four schemes. Apparently the feedback matrix $(\mathbf{L}^{-1})^H \mathbf{D}^{-\frac{1}{2}}$ of LDL^H scheme can take the advantage that $\frac{N(N-1)}{2}$ of zeros need not be taken into the calculations. Therefore, we evaluate that $2N^3 + N^2$ savings of CM and $2N^2 + 45N$ savings of CA can be obtained.

A quantitative comparison about complexity of decomposition between LDL^H and SVD-based ones such as SVD, GMD and UCD is demonstrated in Table 1 for a $N \times N$ MIMO system and N symbol streams. In addition to the savings of quantization complexity, we clearly see that the decomposition complexity of both CM and CA of proposed LDL^H are lower by one order of magnitude than others.

5. SIMULATIONS AND DISCUSSIONS

Numerical analyses have been conducted to a M = 4, N = 4MIMO system operates in an i.i.d. Rayleigh flat fading channel. Decomposition schemes such as SVD, GMD and UCD are evaluated for comparisons. The modulation is 16-QAM for all subchannels of all cases. MMSE detector is used for SVD and LDL^H while V-BLAST detector is used for UCD and GMD. Performance of open-loop MMSE is also introduced to give a contrast upper bound and is denoted "OL-MMSE" as the legend. The BER performance are shown in Fig. 3. In this figure, UCD is the best one due to its higher diversity gain. Both UCD and GMD outperform SVD and LDL^H by the advantage of subchannel uniformity fitted with constant modulation scheme, though, at the cost of high complexity. It is identified that LDL^H outperforms SVD. This result is consistent with the proof given in [8].

6. CONCLUSION

We have presented a LDL^{H} scheme for MIMO channel decomposition. Quantitative analysis of complexity is given to illustrate that LDL^{H} has lower computation overhead compared with that of SVD, UCD and GMD. As many as one order of magnitude of complexity is achieved by utilizing our iterative algorithm. Furthermore, the low complexity is also verified in the quantization feedback processes. Analytical data shows the error performance of LDL^{H} is better than that of SVD. All those make LDL^{H} an appealing channel decomposition scheme for low-complexity MIMO precoder design.



Fig. 3. Simulation results of BER vs. SNR for SVD, LDL^H , GMD and UCD based on 2000 Monte Carlo trials in N = 4 and M = 4 MIMO system.

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