# RECURSIVE LEAST-SQUARES DECISION-DIRECTED TRACKING OF DOUBLY-SELECTIVE CHANNELS USING EXPONENTIAL BASIS MODELS

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## ABSTRACT

We present a decision-directed tracking approach to doubly-selective channel estimation exploiting the complex exponential basis expansion model (CE-BEM). The time-varying nature of the channel is well captured by the CE-BEM while the time-variations of the (unknown) BEM coefficients are likely much slower than those of the channel. We track the BEM coefficients via the exponentially-weighted recursive least-squares (RLS) algorithm, aided by symbol decisions from a decision-feedback equalizer (DFE). Simulation examples demonstrate its superior performance over an existing subblock-wise channel tracking scheme.

***Index Terms*—** Doubly-selective channels, adaptive channel estimation, recursive least-squares, basis expansion models

## 1. INTRODUCTION

Recently basis expansion models (BEM) have been investigated to represent doubly-selective channels in wireless applications [1, 9, 13]. In contrast to the symbol-wise AR models that describe the channel variations on a symbol-by-symbol basis, a BEM depicts evolution of the channel over a period (block) of time. Using time-multiplexed (TM) training, in [2], a subblock-wise tracking approach based on a CE-BEM for the channel and an AR model for the BEM coefficients was proposed. This approach outperforms the symbol-wise AR model-based approach in fast-fading environments. In [3], decision-directed tracking of CE-BEM-based doubly-selective channels was proposed based on the subblock tracking approach of [2]. The approaches of [2, 3] assume that each BEM coefficient follows a first-order AR process, which is not necessarily true for a "real-world" channel, and this assumption possibly incurs modeling error in estimation. To circumvent this problem, an adaptive channel estimation scheme with no a priori model for the BEM coefficients was proposed in [4], where two finite-memory adaptive filtering algorithms, the exponentially-weighted and the sliding-window recursive least-squares (RLS) algorithm, are considered for subblock-wise channel tracking.

In this paper, we propose an RLS decision-directed approach to track the channel and to detect information symbols. It is based on the exponentially-weighted RLS (EW-RLS) algorithm of [4] and the decision-directed tracking proposed in [3]. Decision-directed channel tracking using a polynomial BEM has been investigated in [5], where the BEM coefficients are updated via the RLS algorithm within a sliding window. Decision-directed channel estimation using Kalman filtering and polynomial or CE-BEM for OFDM systems has been explored in [6, 8]. The contributions [5, 6, 8] consider block-by-block updating and/or a priori stochastic models for BEM coefficients, whereas our decision-directed scheme updates the BEM coefficients much more frequently and without using any "arbitrary" model for variations of the BEM coefficients.

*Notation:* Superscripts $*$, $T$, and $H$ denote the complex conjugation, transpose, and complex conjugate transpose respectively. $\mathbf{I}_N$ is the $N \times N$ identity matrix, $\mathbf{0}_{M\times N}$ is the $M \times N$ null matrix and $\otimes$ denotes the Kronecker product. $\delta(\tau)$ is the Kronecker delta.

## 2. SYSTEM MODEL

Consider a doubly-selective, single-input multi-output (SIMO), FIR linear channel with $N$ outputs and discrete-time response $\{\mathbf{h}(n;l)\}$ ($N$-column vector channel response at time instance $n$ to a unit input at time instance $n-l$). With $\{s(n)\}$ as the scalar information sequence, the symbol-rate noisy $N$-column channel output vector is given by ($n = 0, 1, \ldots$)

$$\mathbf{y}(n) = \sum_{l=0}^{L} \mathbf{h}(n;l)\, s(n-l) + \mathbf{v}(n) \tag{1}$$

where $\mathbf{v}(n)$ is zero-mean, white noise, uncorrelated with $s(n)$, with $E\{\mathbf{v}(n+\tau)\mathbf{v}^H(n)\} = \sigma_v^2 \mathbf{I}_N \delta(\tau)$. In TM training schemes, $s(n)$ can be either a training or an information symbol.

In CE-BEM [1, 9, 10], over the $k'$-th block consisting of an observation window of $T_B$ symbols, the channel is represented as ($n = (k'-1)T_B, (k'-1)T_B + 1, \cdots, k'T_B - 1$ and $l = 0, 1, \cdots, L$ )

$$\mathbf{h}(n;l) = \sum_{q=1}^{Q} \mathbf{h}_q^{(l)} e^{j\omega_q n}, \tag{2}$$

where one chooses ($q = 1, 2, \ldots, Q$ and $K \geq 1$ is an integer)

$$T := KT_B, \quad Q \geq 2\left\lceil f_d T T_s \right\rceil + 1, \tag{3}$$

$$\omega_q := \frac{2\pi}{T}\left[q - (Q+1)/2\right], \quad L := \left\lfloor \tau_d / T_s \right\rfloor, \tag{4}$$

$\tau_d$ and $f_d$ are respectively the delay spread and the Doppler spread, and $T_s$ is the symbol duration. The BEM coefficients $\mathbf{h}_q^{(l)}$'s remain invariant during this block, but are allowed to change at the next block; the functions $\left\{e^{j\omega_q n}\right\}$

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$(q = 1, 2, \cdots, Q)$ are common for every block. If $K \geq 2$, the Doppler spectrum is over-sampled (therefore the representation (2) is called over-sampled CE-BEM) [10], compared with the critically sampled case $K = 1$ [1,9].

## 3. DECISION-DIRECTED TRACKING

Consider two overlapping blocks (each consisting of $T_B$ symbols) that differ by only $m_s$ ($1 \leq m_s \ll T_B$) symbols: the "past" block beginning at time $n_0$, and the "present" block beginning at time $n_0 + m_s$. Thanks to the significant overlapping of the two blocks, one can expect the BEM coefficients to vary only a little from the past block to the current overlapping one. Therefore, rather than estimate $\mathbf{h}_q^{(l)}$ anew at every non-overlapping block as in [9], we propose to update the BEM coefficients every $m_s$ ("step-size") symbols using TM training and detected symbols. [In the following $\hat{s}(n)$ denotes the symbol estimate and $\breve{s}(n)$ denotes the detected symbol.]

### 3.1. State-Space Modeling using CE-BEM

Stack the BEM coefficients in (2) into vectors

$$\mathbf{h}^{(l)} := \begin{bmatrix} \mathbf{h}_1^{(l)T} & \mathbf{h}_2^{(l)T} & \cdots & \mathbf{h}_Q^{(l)T} \end{bmatrix}^T \tag{5}$$

$$\mathbf{h} := \begin{bmatrix} \mathbf{h}^{(0)T} & \mathbf{h}^{(1)T} & \cdots & \mathbf{h}^{(L)T} \end{bmatrix}^T \tag{6}$$

of size $NQ$ and $M := NQ(L+1)$ respectively. The coefficient vectors in (5) and (6) of the $p$-th overlapping block will be denoted by $\mathbf{h}^{(l)}(p)$ and $\mathbf{h}(p)$. Again, we emphasize that the $p$-th block and the $(p+1)$-st block differ by just $m_s$ symbols. For $pm_s \leq n < (p+1)m_s$, by (1), (2), (5) and (6), the received signal at time $n$ can be written as

$$\mathbf{y}(n) = \mathbf{S}^T(n)\left[\mathbf{I}_{L+1} \otimes \boldsymbol{\mathcal{E}}(n)\right]^H \mathbf{h}(p) + \mathbf{v}(n)$$

where

$$\boldsymbol{\mathcal{E}}(n) := \begin{bmatrix} e^{-j\omega_1 n} & e^{-j\omega_2 n} & \cdots & e^{-j\omega_Q n} \end{bmatrix}^T \otimes \mathbf{I}_N,$$
$$\mathbf{S}(n) := \begin{bmatrix} s(n) & s(n-1) & \cdots & s(n-L) \end{bmatrix}^T \otimes \mathbf{I}_N.$$

Further defining

$$\mathbf{C}_i(p) := \mathbf{S}^T(pm_s + i)\left[\mathbf{I}_{L+1} \otimes \boldsymbol{\mathcal{E}}(pm_s + i)\right]^H, \tag{7}$$

$$\mathbf{C}(p) := \begin{bmatrix} \mathbf{C}_0^T(p) & \mathbf{C}_1^T(p) & \cdots & \mathbf{C}_{m_s-1}^T(p) \end{bmatrix}^T, \tag{8}$$

we have

$$\mathbf{y}_{ms}(p) = \mathbf{C}(p)\mathbf{h}(p) + \mathbf{v}_{ms}(p) \tag{9}$$

where $\mathbf{y}_{ms}(p) :=$

$$\begin{bmatrix} \mathbf{y}^T(pm_s) & \mathbf{y}^T(pm_s+1) & \cdots & \mathbf{y}^T((p+1)m_s - 1) \end{bmatrix}^T$$

and $\mathbf{v}_{ms}(p)$ is defined likewise. Using TM training or detected symbols in $\mathbf{C}(p)$, our objective is to devise an RLS scheme for estimating $\mathbf{h}(p)$.

### 3.2. Exponentially-Weighted RLS Tracking

Based on (9), we can apply exponentially-weighted regularized RLS (EW-RLS) algorithm [11, Chapter 12] to track an unknown $\mathbf{h}(p)$. Choose $\mathbf{h}$ to minimize the cost function

$$\lambda^{p+1}\beta\|\mathbf{h}\|^2 + \sum_{i=0}^{p}\lambda^{p-i}\|\mathbf{y}_{m_s}(i) - \mathbf{C}(i)\mathbf{h}\|^2 \tag{10}$$

where $\beta > 0$ is a regularization parameter and $0 < \lambda < 1$ is the forgetting factor. Mimicking [11, Algorithm 12.3.1] (see also [4]), EW-RLS tracking comprises the following steps:

1) Initialization: $\hat{\mathbf{h}}(-1) = \mathbf{0}_{M\times 1}$ and $\mathbf{P}(-1) = \beta^{-1}\mathbf{I}_M$
2) RLS recursion: For $p = 0, 1, \cdots$

$$\boldsymbol{\Gamma}(p) = \lambda\mathbf{I}_{Nm_s} + \mathbf{C}(p)\mathbf{P}(p-1)\mathbf{C}^H(p),$$
$$\mathbf{G}(p) = \mathbf{P}(p-1)\mathbf{C}^H(p)\boldsymbol{\Gamma}^{-1}(p),$$
$$\hat{\mathbf{h}}(p) = \hat{\mathbf{h}}(p-1) + \mathbf{G}(p)\left[\mathbf{y}_{m_s}(p) - \mathbf{C}(p)\hat{\mathbf{h}}(p-1)\right],$$
$$\mathbf{P}(p) = \lambda^{-1}\left[\mathbf{I}_M - \mathbf{G}(p)\mathbf{C}(p)\right]\mathbf{P}(p-1),$$

where $\hat{\mathbf{h}}(p)$ denotes the estimate of $\mathbf{h}(p)$ given the observations $\{\mathbf{y}_{m_s}(0), \mathbf{y}_{m_s}(1), \cdots, \mathbf{y}_{m_s}(p)\}$.

After RLS recursion for every $p$, we can generate the channel by the estimated $\hat{\mathbf{h}}(p)$ via the CE-BEM (2).

### 3.3. Channel Prediction

We employ a DFE [12] with equalization delay $d > 0$ to equalize the estimated channel at the receiver. Its output symbol decisions are used as a pseudo-training. We need to "predict" the channel up to time $n$ to obtain the detected symbol $\breve{s}(n-d)$ at the DFE. We use the "current" BEM coefficient vector estimate in the CE-BEM (2) to predict the channel $\hat{\mathbf{h}}(n;l)$ for values of $n$ as needed, i.e., the channel estimates in our receiver are given by

$$\hat{\mathbf{h}}(n;l) = \boldsymbol{\mathcal{E}}^H(n)\hat{\mathbf{h}}^{(l)}(p), \tag{11}$$

for $n = pm_s, pm_s + 1, \cdots, (p+2)m_s + d - 1$ where the definition of $\hat{\mathbf{h}}^{(l)}(p)$ is similar to (5) and $\hat{\mathbf{h}}^{(l)}(p)$ is based on observations up to time $n = (p+1)m_s - 1$.

### 3.4. Minimum Mean-Square Error (MMSE)-DFE

Using the estimated channel, the symbol decisions are made by an FIR MMSE-DFE [12]. Given the lengths of the feedforward (FF) and feedback (FB) filters as $l_f$ and $l_b$, respectively, the estimate of the information symbol $\hat{s}(n-d)$ is obtained by combining the outputs of FF and FB filters and can be written as

$$\hat{s}(n-d) = \sum_{m=0}^{l_f-1}\mathbf{f}_m^T(n)\,\mathbf{y}(n-m) - \sum_{k=1}^{l_b} b_k(n)\,\breve{s}(n-d-k) \tag{12}$$

where $N \times 1$ $\mathbf{f}_m(n)$'s and scalar $b_k(n)$'s are the taps of FF and FB time-varying filters at time $n$, and $\breve{s}(n-d-k)$ is the hard decision of $\hat{s}(n-d-k)$. The estimate $\hat{s}(n-d)$ is also fed into the quantizer to obtain the symbol decision $\breve{s}(n-d)$.

Stack the inputs of the FF filter at time $n$ as

$$\mathbf{y}_f(n) := \begin{bmatrix}\mathbf{y}^T(n) & \mathbf{y}^T(n-1) & \cdots & \mathbf{y}^T(n-l_f+1)\end{bmatrix}^T$$

and also define $\mathbf{v}_f(n)$ likewise. By (1), we have

$$\mathbf{y}_f(n) = \mathbf{H}(n)\,\mathbf{s}_f(n) + \mathbf{v}_f(n) \qquad (13)$$

where $\mathbf{H}(n) :=$

$$\begin{bmatrix}\mathbf{h}(n;0) & \cdots & \mathbf{h}(n;L) & & \\ & \ddots & & \ddots & \ddots \\ & & \mathbf{h}(n-l_f+1;0) & \cdots & \mathbf{h}(n-l_f+1;L)\end{bmatrix}$$

$$\mathbf{s}_f(n) := [s(n) \quad s(n-1) \quad \cdots \quad s(n-l_f-L+1)]^T.$$

Further define

$$\mathbf{s}_b(n) := [\check{s}(n-d) \quad \check{s}(n-d-1) \quad \cdots \quad \check{s}(n-d-l_b)]^T.$$

Since $\{s(n)\}$ is i.i.d. with variance $\sigma_s^2$, from (13) we have

$$\mathbf{R}_{ss}(n) := E\left\{\mathbf{s}_b(n)\,\mathbf{s}_b^H(n)\right\} = \sigma_s^2\mathbf{I}_{(l_b+1)},$$
$$\mathbf{R}_{sy}(n) := E\left\{\mathbf{s}_b(n)\,\mathbf{y}_f^H(n)\right\} = \sigma_s^2\mathbf{\Phi}\mathbf{H}^H(n),$$
$$\mathbf{R}_{yy}(n) := E\left\{\mathbf{y}_f(n)\,\mathbf{y}_f^H(n)\right\} = \sigma_s^2\mathbf{H}(n)\,\mathbf{H}^H(n) + \sigma_v^2\mathbf{I}_{Nl_f}$$

where $\mathbf{\Phi} := \begin{bmatrix}\mathbf{0}_{(l_b+1)\times d} & \mathbf{I}_{l_b+1} & \mathbf{0}_{(l_b+1)\times(l_f+L-d-l_b-1)}\end{bmatrix}$.

Let $\mathbf{f}(n)$ and $\mathbf{b}(n)$ denote the vectors of time-varying taps of FF and FB filters,

$$\mathbf{f}(n) := \begin{bmatrix}\mathbf{f}_0^T(n) & \mathbf{f}_1^T(n) & \cdots & \mathbf{f}_{l_f-1}^T(n)\end{bmatrix}^T,$$
$$\mathbf{b}(n) := [1 \quad b_1(n) \quad b_2(n) \quad \cdots \quad b_{l_b}(n)]^T.$$

Assuming the decisions $\{\check{s}(n)\}$ are correct and equal to $\{s(n)\}$, the FF and the FB time-varying filters of the MMSE-DFE are given by [12]

$$\mathbf{b}_{\text{MMSE}}(n) = \mathbf{R}_\delta^{-1}\mathbf{e}_0/\mathbf{e}_0^T\mathbf{R}_\delta^{-1}\mathbf{e}_0, \qquad (14)$$
$$\mathbf{f}_{\text{MMSE}}(n) = \mathbf{R}_{yy}^{-1}(n)\,\mathbf{R}_{sy}^H(n)\,\mathbf{b}_{\text{MMSE}}(n), \qquad (15)$$

where $\mathbf{e}_0 := [1 \quad 0 \quad 0 \quad \cdots \quad 0]^T$,

$$\mathbf{R}_\delta := \mathbf{R}_{ss}(n) - \mathbf{R}_{sy}(n)\,\mathbf{R}_{yy}^{-1}(n)\,\mathbf{R}_{sy}^H(n)$$
$$= \mathbf{\Phi}\left[\frac{1}{\sigma_v^2}\mathbf{H}(n)\,\mathbf{H}^H(n) + \frac{1}{\sigma_s^2}\mathbf{I}_{Nl_f}\right]^{-1}\mathbf{\Phi}^H.$$

Using (14) and (15) in (12), we have the symbol estimate $\{\hat{s}(n-d)\}$. Since the "true" channel response $\{\mathbf{h}(n;l)\}$ is not available at the receiver, we use the channel estimates $\{\hat{\mathbf{h}}(n;l)\}$ obtained by (11) to design the MMSE-DFE. In order to compensate for the channel estimation errors in (13), for the simulations presented in Sec. 4 we increased the variance of $\mathbf{v}(n)$ in (13) from $\sigma_v^2$ to $\sigma_v^2 + 0.01\sigma_s^2$.

## 4. SIMULATION EXAMPLES

We assume $\mathbf{h}(n;l)$ is zero-mean, complex Gaussian, and spatially white with $E\left\{\mathbf{h}(n;l)\,\mathbf{h}^H(n;l)\right\} = \sigma_h^2\mathbf{I}_N$. We take $L = 2$ (3 taps) in (1), and $\sigma_h^2 = 1/(L+1)$. For different $l$'s, $\mathbf{h}(n;l)$'s are mutually independent and satisfy Jakes' model. To this end, we simulate each single tap following [13]. We consider a communication system with carrier frequency of 2GHz, data rate of 40kBd (kilo-Bauds), therefore $T_s = 25\,\mu$s, and a Doppler spread $f_d = 400\,$Hz ($f_dT_s = 0.01$). The additive noise is zero-mean complex white Gaussian. The (receiver) SNR refers to the average energy per symbol over one-sided noise spectral density.

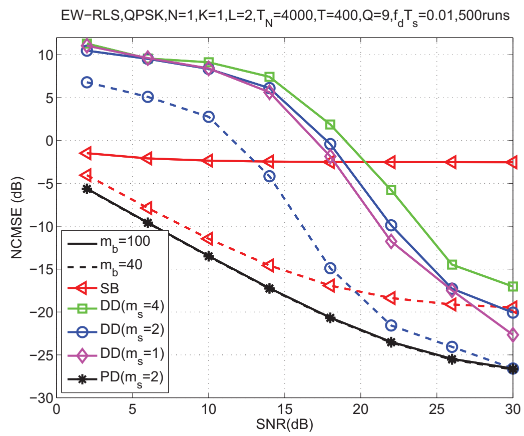


**Fig. 1**. NCMSE vs SNR, under $f_dT_s = 0.01$, $m_b = 100$ or 40 with QPSK information symbols.

We evaluate the performances of various schemes by considering their normalized channel mean square error (NCMSE) and bit error rate (BER). For $T_N$ received symbols, the NCMSE is defined as

$$\text{NCMSE} := \frac{\sum_{i=1}^{M_r}\sum_{n=0}^{T_N-1}\sum_{l=0}^{L}\left\|\hat{\mathbf{h}}^{(i)}(n;l) - \mathbf{h}^{(i)}(n;l)\right\|^2}{\sum_{i=1}^{M_r}\sum_{n=0}^{T_N-1}\sum_{l=0}^{L}\left\|\mathbf{h}^{(i)}(n;l)\right\|^2}$$

where $\mathbf{h}^{(i)}(n;l)$ is the true channel and $\hat{\mathbf{h}}^{(i)}(n;l)$ is the estimated channel at the $i$-th Monte Carlo run, among total $M_r$ runs. In each run, an "initialization" training mode of 200 BPSK symbols is followed by a decision-directed mode of 4000 QPSK symbols ($T_N = 4000$). All the simulation results are based on 500 runs, and we consider the performances during the decision-directed mode only. In the decision-directed mode, training sessions are also periodically sent to facilitate the EW-RLS tracking. The TM training scheme of [9], which is optimal for channels satisfying critically-sampled CE-BEM representations, is adopted. In [9] the block of $T_B$ symbols is segmented into subblocks of equal length of $m_b$ symbols consisting of an information session of $m_d$ symbols and a succeeding training session of $m_t$ symbols ($m_b = m_d + m_t$). The training session contains an impulse guarded by zeros (silent periods), which for a channel with $L+1$ taps has the structure $[\mathbf{0}_{1\times L} \quad \gamma \quad \mathbf{0}_{1\times L}]$, $\gamma > 0$; therefore, $m_t = 2L + 1 = 5$ symbols are devoted for training and the remaining $m_d = m_b - m_t$ are available for information symbols. We assume that each information symbol has unit power, while in every training session, we set $\gamma = \sqrt{2L+1} = \sqrt{5}$ so that the average power per symbol in the training sessions is equal to that in the information sessions.

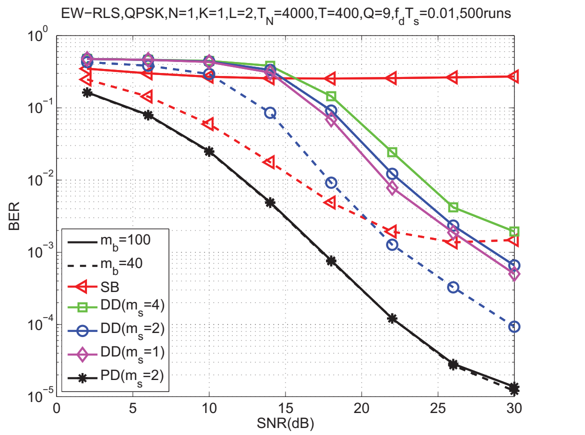


**Fig. 2**. BER vs SNR, under $f_d T_s = 0.01$, $m_b = 100$ or 40 with QPSK information symbols.

We compare the following three schemes:

1. Subblock-wise EW-RLS algorithm of [4] with $\beta = 1$. For $m_b = 40$ and 100, we take $\lambda = 0.5$ (see [4]). In the figures, this scheme is denoted by "SB".
2. The proposed decision-directed tracking scheme with step size $m_s$ in the EW-RLS tracking with $\beta = 1$. We take the forgetting factor $\lambda = 0.92, 0.96$ and 0.98 for $m_s = 4, 2$ and 1 respectively. In the figures, our approach is denoted by "DD".
3. Perfect symbol decisions are used as training for RLS channel tracking with step size $m_s = 2$ and $\beta = 1$. This scheme provides the baseline for decision-directed tracking, denoted by "PD" in the figures.

For each of the above schemes, an MMSE-DFE described in Sec. 3.4 and [3] is employed at the receiver with $l_f = 8$, $l_b = 2$, and the delay $d = 5$ symbols.

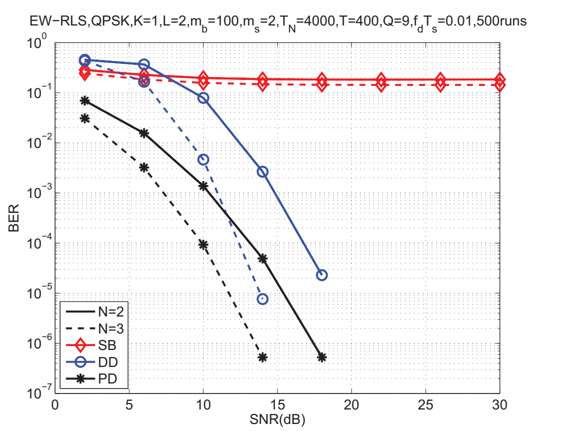


**Fig. 3**. BER vs SNR with $N$ receive antennas, under $f_d T_s = 0.01$, $m_b = 100$ with QPSK information symbols.

In Figs. 1 and 2, the performances of each scheme are compared $f_d T_s = 0.01$ and different SNR's, for the subblock size $m_b = 40$ or 100. For the subblock-wise scheme of [4], frequent training sessions are required in order to track the rapid channel variations; SB with larger subblock $m_b =$ 100 and hence less training does not work. Due to the error propagation triggered by incorrect symbol decisions in DFE, our RLS decision-directed tracking does not perform well when the SNR is low. As the SNR increases, the proposed scheme performs "closer" to the performance of the perfect decision-directed channel tracking scheme. Since the BEM coefficients are updated every $m_s$ symbols in our approach, a "finer" channel tracking can be obtained by reducing step size $m_s$ although the computational complexity increases. In Fig. 3 the three schemes are evaluated with multiple receive antennas ($N = 2$ or 3), under $f_d T_s = 0.01$ and different SNR's, for the larger subblock size $m_b = 100$. We take the step size $m_s = 2$ for the decision-directed and perfect decision-directed schemes. The subblock-wise approach of [4] is still inadequate, but our RLS decision-directed tracking scheme shows significant enhancement with higher $N$.

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