ROBUST 2-D CHANNEL ESTIMATION FOR MULTI-CARRIER SYSTEMS WITH FINITE DIMENSIONAL PILOT GRID

M. Danish Nisar [1,2], *Wolfgang Utschick* [1], *Thomas Hindelang* [2]

[1]Technical University Munich, Associate Institute for Signal Processing, Munich, Germany
[2]Nokia Siemens Networks, Radio Access, Algorithms & Simulations, Munich, Germany

ABSTRACT

Pilot-aided channel estimation for multi-carrier systems can be significantly improved by exploiting time and frequency correlations between the channel frequency response coefficients. But in practice, the knowledge of channel correlation function is not accurately available, thereby necessitating the need of an estimator that employs a fixed correlation function and is robust to mismatches with the actual one. While the maximally robust channel estimator for multi-carrier systems for the case of an infinite number of observations is well known for almost a decade, the one for the case of a finite number of observations has been only recently proposed. We extend the proposed maximally robust estimator to the practical case of small finite dimensional pilot grids by taking into account the grid edge effects. This paves the way for application of the estimator to practical systems such as 3G LTE. Simulation results for an LTE uplink system under different transmission scenarios demonstrate the superiority of the proposed maximally robust estimator over the heuristic one by as much as 1.35 dB in terms of the coded BER.

***Index Terms*—** Robust channel estimation, Minimax optimization, Multi-carrier systems

1. INTRODUCTION

Maximum Likelihood (ML) based channel estimation is one of the simplest pilot-aided channel estimation technique in multi-carrier systems. Improved estimation performance can be obtained via 2-D MMSE channel estimation by taking into account the correlations along time and frequency that exist among the neighbouring *Channel Frequency Response (CFR)* coefficients. However, in most practical wireless/cellular multi-carriers systems, a 2-D MMSE channel estimator based on the true underlying channel correlation sequence is hard to realize because of at least one of the following three reasons: First, the MAC layer scheduler often changes the assigned block of sub-carriers to each user, thereby complicating the task of channel correlation function estimation. Second, the estimation of the correlation function itself incurs additional computational complexity and last but not the least, an inaccurate estimate of the correlation function may lead to an uncontrolled degradation in terms of channel estimation MSE with no bound on the worst case performance.

Owing to the challenges associated with the 2-D MMSE channel estimation based on the true channel correlation function, it is often desirable to have the estimator based on a fixed, robust choice of channel correlation function. However, any mismatch between the true and the assumed correlation sequence would lead to degradation in estimation MSE, so a natural candidate for the fixed correlation sequence is the one that minimizes this degradation. Such an estimator promises the least worst case estimation MSE [1, 2] and therefore gets the name of *Maximally Robust (MR)* channel estimator.

While the MR channel estimator for multi-carrier systems for the case of an infinite number of observations is well known for almost a decade now [3, 4], the 2-D MR estimator or its underlying *Least Favorable (LF)* correlation sequence for the case of a finite number of observations has been proposed only recently in our paper [5]. The proposal is based on the minimax optimization over channel correlation sequences belonging to a particular uncertainty class.

In this paper, we apply the proposed MR 2-D MMSE channel estimator to a practical multi-carrier system, extend it to the case of small finite dimensional pilot grids by taking into account the grid edge effects, and compare the performance of the MR estimator with that of heuristic robust estimator [3]. After an intuitive formulation of the problem under consideration in Section 2, we briefly review the robust 2-D MMSE estimator for an infinite number of pilot observations in Section 3 and show that an estimator based on a rectangular *Doppler Spectrum (DS)* and a uniform *Power Delay Profile (PDP)* leads to the best worst-case performance in this scenario. In contrast, we show in Section 4 that being constrained on a finite number of observations along time and frequency, the sinc based heuristic correlation function, mentioned above, is no longer maximally robust, rather we end up in a semidefinite optimization problem in terms of pilot grid and estimation parameters. The solution of the semidefinite program leads to the LF 2-D correlation sequence that can be finally used to compute the *maximally robust 2-D MMSE estimator*. We conclude the paper in Section 5 with a performance analysis of the two robust estimators in terms of BER and throughput for a practical multi-carrier system.

Notation. The operators $\mathrm{E}[\bullet]$, $|\bullet|^2$, $(\bullet)^*$, $(\bullet)^{\mathrm{H}}$, $\mathrm{vec}(\bullet)$ stand for expectation, absolute value square, complex conjugate, hermitian and vectorization respectively, while ι denotes the imaginary unit.

2. PROBLEM FORMULATION

We consider a typical multi-carrier system with pilot symbols distributed over the time-frequency grid in a periodic manner as shown in Fig. 1. The total number of system sub-carriers is denoted by N. Pilot spacings along time and frequency are labeled with Δ_{T} and Δ_{F} respectively. It is interesting to point out here, that such rectangular aligned pilot grids are found in many cellular standards like WiMax and LTE systems [6].

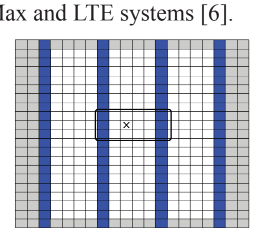

Fig. 1. Example pilot grid with time along horizontal and frequency along vertical direction. Blue squares represent pilot positions so that $\Delta_{\mathrm{F}} = 1$ and $\Delta_{\mathrm{T}} = 5$ in this illustration.

As a special case of estimation, the problem that we consider here is the one of finding a robust 2-D MMSE channel interpolation filter given the channel estimates at pilot positions.[1] Thus, we assume the knowledge of LS channel estimates at pilot positions and, intuitively speaking, look for a 2-D linear filter that performs an optimum weighted combination of the available pilot channel estimates to form an estimate of the channel at the selected data positions of interest. The prime challenge, as already explained, is the lack of knowledge of the channel correlation sequence. Hence we pursue a search to find the LF correlation sequence that shall be used for the robust estimation performance.

We now characterize the uncertainty class $\mathcal{U}_{r_H}$, to which a candidate correlation sequence belongs, in terms of practical transmission parameters. We note that the uncertainty class contains only the set of correlation sequences having band-limited spectra for both the time and the frequency correlation. The highest frequency component in the time correlation spectrum is given by the maximum Doppler frequency $f_{\text{d,max}}$. Normalizing it w.r.t the symbol duration $T_{\text{b}} = 1/f_{\text{b}}$, we note that the maximum angular frequency in the spectrum of the time correlation sequence is

$$\omega_{\text{t,max}} = 2\pi \left(f_{\text{d,max}}/f_{\text{b}}\right), \tag{1}$$

Arguing under the duality concepts of Doppler spread and coherence time v.s. delay spread and coherence bandwidth, the analogous relationship for the frequency correlation spectrum can be given as

$$\omega_{\text{f,max}} = 2\pi \left(L/N\right), \tag{2}$$

where L denotes the maximum length of channel impulse response in taps, which in our context corresponds to the cyclic prefix length. An important distinction from the time correlation spectrum is that the frequency correlation spectrum is one-sided.

3. ROBUST ESTIMATOR FOR AN INFINITE NUMBER OF OBSERVATIONS

Given an infinite dimensional pilot grid with $H_{f,t}$ denoting CFR at f-th frequency and t-th time index and $H_{0,0}$ being the reference pilot CFR, the available pilot CFR estimates can be written as

$$\tilde{H}_{i\Delta_{\text{F}},j\Delta_{\text{T}}} = H_{i\Delta_{\text{F}},j\Delta_{\text{T}}} + \eta_{i\Delta_{\text{F}},j\Delta_{\text{T}}} \tag{3}$$

for all integers i, j. The observation noise $\eta_{i,j}$ is assumed to be white gaussian with variance σ_η^2. The estimate of data CFRs $H_{f,t}$ can be expressed as

$$\hat{H}_{f,t} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} c_{f-i\Delta_{\text{F}},t-j\Delta_{\text{T}}} \tilde{H}_{i\Delta_{\text{F}},j\Delta_{\text{T}}} \tag{4}$$

where $c_{i,j}$ denote the 2-D estimator coefficients. Under the orthogonality principle to minimize the estimation MSE, the estimator coefficients shall be chosen such that

$$\mathcal{C}(\omega_{\text{f}}, \omega_{\text{t}}) = \frac{\mathcal{R}(\omega_{\text{f}}, \omega_{\text{t}})}{\mathcal{R}(\omega_{\text{f}}, \omega_{\text{t}})/\Delta + \sigma_\eta^2} \tag{5}$$

where $\Delta = \Delta_{\text{F}}\Delta_{\text{T}}$ and $\mathcal{C}(\omega_{\text{f}}, \omega_{\text{t}}) = \sum_i \sum_j c_{i,j} e^{\iota\omega_{\text{f}} i} e^{\iota\omega_{\text{t}} j}$ denotes the filter spectrum, while $\mathcal{R}(\omega_{\text{f}}, \omega_{\text{t}}) = \sum_i \sum_j r_H(i,j) e^{\iota\omega_{\text{f}} i} e^{\iota\omega_{\text{t}} j}$ denotes the channel correlation spectrum, with the CFR correlation sequence[2] $r_H(i,j) = \text{E}[H_{f,t} H^*_{f+i,t+j}]$. Let $\varepsilon(\mathcal{C}_{\text{MMSE}}(\mathcal{R}), \tilde{\mathcal{R}})$ denote the MSE attained in case of a mismatch between the assumed spectrum $\mathcal{R}(\omega_{\text{f}}, \omega_{\text{t}})$ and the actually encountered correlation spectrum $\tilde{\mathcal{R}}(\omega_{\text{f}}, \omega_{\text{t}})$, then this MSE can be decomposed as [3, 4],

$$\varepsilon(\mathcal{C}_{\text{MMSE}}(\mathcal{R}), \tilde{\mathcal{R}}) = \varepsilon(\mathcal{C}_{\text{MMSE}}(\tilde{\mathcal{R}}), \tilde{\mathcal{R}}) + \varepsilon_\Delta(\mathcal{R}, \tilde{\mathcal{R}}) \tag{6}$$

where $\varepsilon_\Delta(\mathcal{R}, \tilde{\mathcal{R}})$ term can be interpreted as the additional MSE penalty due to mismatch and is given as,

$$\varepsilon_\Delta(\mathcal{R}, \tilde{\mathcal{R}}) = \frac{\sigma_\eta^4 S \Delta}{(2\pi)^2} \left[\frac{1}{S} \int\int_{\mathcal{D}} \frac{\frac{\tilde{\mathcal{R}}(\omega_{\text{f}},\omega_{\text{t}})}{\Delta} + \sigma_\eta^2}{\frac{\mathcal{R}(\omega_{\text{f}},\omega_{\text{t}})}{\Delta} + \sigma_\eta^2} \, \text{d}\omega_{\text{f}} \text{d}\omega_{\text{t}} - 1 \right] \tag{7}$$

with $S = \int\int_{\mathcal{D}} 1 \ \text{d}\omega_{\text{f}}\text{d}\omega_{\text{t}}$. It can be easily shown now that if the estimator is designed w.r.t a rectangular (flat) correlation spectrum, i.e. $\mathcal{R}(\omega_{\text{f}}, \omega_{\text{t}}) = \mathcal{R}_{\text{rect}}$ being $(2\pi)^2/S$ for $(\omega_{\text{f}}, \omega_{\text{t}}) \in \mathcal{D}$ and zero elsewhere, then for every bandlimited correlation spectrum $\tilde{\mathcal{R}}(\omega_{\text{f}}, \omega_{\text{t}})$ that satisfies the L_1 bound: $1/(2\pi)^2 \int\int_{\mathcal{D}} \tilde{\mathcal{R}}(\omega_{\text{f}}, \omega_{\text{t}}) \ \text{d}\omega_{\text{f}}\text{d}\omega_{\text{t}} = 1$, the additional MSE penalty disappears i.e. $\varepsilon_\Delta(\mathcal{R}_{\text{rect}}, \tilde{\mathcal{R}}) = 0$. This astonishing result implies that given an infinite number of observations, a 2-D MMSE estimator based on the rectangular correlation spectrum not only leads to robust performance in case of a mismatch but also the degradation in MSE due to mismatch stays at zero.

4. ROBUST ESTIMATOR FOR A FINITE NUMBER OF OBSERVATIONS

In search for the MR estimator, we now restrict ourselves to the case of a finite number of pilot observations. Let N_{T} and N_{F} denote the number of pilot channel estimates, along the time and frequency direction respectively, to be employed for estimation and assume further that these observations lie equally on both sides of the current data position of interest. Thus, we have a fixed size rectangular sliding window around the current position encompassing all relevant *Channel Frequency Response (CFR)* coefficients. The window is shown in Fig. 1 for $N_{\text{T}} = 2$ and $N_{\text{F}} = 3$ around the crossed data position. We label this window of CFR coefficients via a matrix $\boldsymbol{H} \in \mathbb{C}^{((N_{\text{F}}-1)\Delta_{\text{F}}+1)\times((N_{\text{T}}-1)\Delta_{\text{T}}+1)}$. Using left and right selection matrices $\boldsymbol{S}_{\text{f}}$ and $\boldsymbol{S}_{\text{t}}$ of appropriate dimensions, a matrix containing only pilot CFRs can be extracted from $\boldsymbol{H}$ and then we make use of the vec($\bullet$) operator to get $\boldsymbol{h}_{\text{p}} = \text{vec}\left(\boldsymbol{S}_{\text{f}}\boldsymbol{H}\boldsymbol{S}_{\text{t}}\right) \in \mathbb{C}^{N_{\text{T}}N_{\text{F}}}$ as the pilot CFR vector. Taking into account the pilot channel estimation errors, the observation vector i.e. the LS estimate of pilot CFR coefficients reads as

$$\tilde{\boldsymbol{h}}_{\text{p}} = \boldsymbol{h}_{\text{p}} + \boldsymbol{\eta}, \tag{8}$$

where $\boldsymbol{\eta} \in \mathbb{C}^{N_{\text{T}}N_{\text{F}}}$ denotes the pilot channel estimation error vector. Now with $\boldsymbol{w}^{\text{H}} \in \mathbb{C}^{1\times N_{\text{T}}N_{\text{F}}}$ denoting the vector containing 2-D filter coefficients[3], we express the estimation MSE, $\text{E}[|H_{f,t} - \hat{H}_{f,t}|^2]$, as

$$\varepsilon(\boldsymbol{w}, \{r_H(i,j)\}) = r_H(0,0) + \boldsymbol{w}^{\text{H}}\big(\boldsymbol{R}_{h_{\text{p}}} + \boldsymbol{R}_\eta\big)\boldsymbol{w} - \boldsymbol{w}^{\text{H}}\boldsymbol{r}_{h_{\text{p}}} - \boldsymbol{r}_{h_{\text{p}}}^{\text{H}}\boldsymbol{w}, \tag{9}$$

with $\boldsymbol{R}_{h_{\text{p}}} = \text{E}\big[\boldsymbol{h}_{\text{p}}\boldsymbol{h}_{\text{p}}^{\text{H}}\big]$, $\boldsymbol{R}_\eta = \sigma_\eta^2 \boldsymbol{I}_{N_{\text{T}}N_{\text{F}}}$ and $\boldsymbol{r}_{h_{\text{p}}} = \text{E}\big[\boldsymbol{h}_{\text{p}} H^*_{f,t}\big] \in \mathbb{C}^{N_{\text{T}}N_{\text{F}}}$. Finally, the minimization of the MSE w.r.t the filter coefficients yields the well known MMSE solution,

$$\boldsymbol{w}_{\text{MMSE}} = \left(\boldsymbol{R}_{h_{\text{p}}} + \boldsymbol{R}_\eta\right)^{-1} \boldsymbol{r}_{h_{\text{p}}}, \tag{10}$$

leading to the minimum MSE attained (cf. Eq. 9)

$$\varepsilon(\boldsymbol{w}_{\text{MMSE}}, \{r_H(i,j)\}) = r_H(0,0) - \boldsymbol{r}_{h_{\text{p}}}^{\text{H}}\big(\boldsymbol{R}_{h_{\text{p}}} + \boldsymbol{R}_\eta\big)^{-1}\boldsymbol{r}_{h_{\text{p}}} \tag{11}$$

4.1. Minimax optimization setup

We are ready now to pose the problem of finding the *Maximally Robust (MR)* 2-D estimator in a minimax optimization framework. Intuitively speaking, we first maximize the MSE (cf. Eq. 9) over the set of all valid 2-D correlation sequences $\{r_H(i,j)\} \in \mathcal{U}_{r_H}$ to arrive

[1] Nevertheless, as we will see in Section 4, the underlying principle of finding the least favorable 2-D correlation sequence can also be obviously applied to the problems of robust MMSE channel estimation and prediction.

[2] Wide sense stationarity of the random process $H_{f,t}$ is assumed through out the paper so that the correlation function is independent of index f and t.

[3] To be more precise, the notation $\boldsymbol{w}_{f,t}$ should be used since the problem structure make $\boldsymbol{w}$ a function of the current position for $f = 0, 1, \ldots \Delta_{\text{F}} - 1$ and $t = 0, 1, \ldots \Delta_{\text{T}} - 1$. But we omit the subscripts for notational convenience and remark that $\boldsymbol{w}$ is indeed shift-variant.

at the worst case scenario and then minimize the resultant MSE via optimization for the filter to finally arrive at the MR estimator, i.e.

$$\min_{\boldsymbol{w}\in\mathbb{C}^{N_\mathrm{T}N_\mathrm{F}}} \max_{\{r_H(i,j)\}\in\mathcal{U}^1_{r_H}} \varepsilon(\boldsymbol{w},\{r_H(i,j)\}), \quad (12)$$

where $\mathcal{U}^1_{r_H} \subset \mathcal{U}_{r_H}$ denotes the set of all 2-D channel correlation sequences with bandwidth restrictions of $\omega_{\mathrm{t,max}}$ and $\omega_{\mathrm{f,max}}$ on their time and frequency spectra, respectively and additionally with a bounded L_1 norm, i.e. $r_H(0,0) \leq \beta$. We now employ a theorem [7, 8] on the equivalence of finite dimensional minimax and max-min problems.

Theorem 1. *Given the estimate of $\boldsymbol{\theta}$ from observation $\boldsymbol{\gamma}$ as $\hat{\boldsymbol{\theta}} = \boldsymbol{w}\boldsymbol{\gamma}$, with the unknown covariance matrix $\boldsymbol{K} = \begin{bmatrix}\boldsymbol{R}_{\gamma\gamma} & \boldsymbol{R}_{\gamma\theta}\\ \boldsymbol{R}_{\theta\gamma} & \boldsymbol{R}_{\theta\theta}\end{bmatrix}$ belonging to a convex & compact uncertainty class $\mathcal{K}$, then the two problems*

$$\min_{\boldsymbol{w}} \max_{\boldsymbol{K}\in\mathcal{K}} \mathrm{E}[\|\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}\|_2^2]$$

$$\max_{\boldsymbol{K}\in\mathcal{K}} \min_{\boldsymbol{w}} \mathrm{E}[\|\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}\|_2^2],$$

have identical solutions, i.e. the Least Favorable covariance matrix $\boldsymbol{K}_* \in \mathcal{K}$ and the corresponding estimate $\hat{\boldsymbol{\theta}}_*$ form a saddle point.

Consequently, given that the prerequisites are fulfilled, the original minimax problem in (12) can be reformulated into the equivalent max-min problem,

$$\max_{\{r_H(i,j)\}\in\mathcal{U}^1_{r_H}} \min_{\boldsymbol{w}\in\mathbb{C}^{N_\mathrm{T}N_\mathrm{F}}} \varepsilon(\boldsymbol{w},\{r_H(i,j)\}). \quad (13)$$

In essence, the problem of finding the maximally robust estimator is casted into the one of finding the *Least Favorable (LF)* 2-D correlation sequence. Note that the minimization problem in (13) is nothing else than the conventional MMSE optimization problem leading to the following residual problem (cf. Eqs. 10 and 11),

$$\max_{\{r_H(i,j)\}\in\mathcal{U}^1_{r_H}} r_H(0,0) - \boldsymbol{r}^\mathrm{H}_{h_\mathrm{p}}\left(\boldsymbol{R}_{h_\mathrm{p}}+\boldsymbol{R}_\eta\right)^{-1}\boldsymbol{r}_{h_\mathrm{p}}. \quad (14)$$

Thus, we arrived from a minimax optimization problem down to a pure maximization problem. Since the objective function in (14) is monotonically increasing in $r_H(0,0)$, the maximization is reached once $r_H(0,0)=\beta$, so that we actually need to minimize the subtractor over $\{r_H(i,j)\} \in \tilde{\mathcal{U}}^1_{r_H}$ where $\tilde{\mathcal{U}}^1_{r_H}$ is identical to $\mathcal{U}^1_{r_H}$ except that $r_H(0,0)=\beta$. Furthermore, transforming the problem into epigraph notation [9, p. 75] by introduction of a slack variable t and then employing the Schur complement positive semidefiniteness theorem, the optimization problem reduces to [5],

$$\min_{t,\{r_H(i,j)\}\in\tilde{\mathcal{U}}^1_{r_H}} t \quad \text{s.t.} \quad \begin{bmatrix} t & \boldsymbol{r}^\mathrm{H}_{h_\mathrm{p}}\\ \boldsymbol{r}_{h_\mathrm{p}} & \boldsymbol{R}_{h_\mathrm{p}}+\boldsymbol{R}_\eta\end{bmatrix} \succeq 0. \quad (15)$$

4.2. Unfolding the uncertainty class constraints

Next we decompose the $\{r_H(i,j)\} \in \tilde{\mathcal{U}}^1_{r_H}$ constraint into individual analytical constraints. The positive semidefiniteness property of the finite length correlation sequence can be expressed in terms of positive semidefiniteness of the channel correlation matrix, $\boldsymbol{R}_H = \mathrm{E}\left[\mathrm{vec}(\boldsymbol{H})\,\mathrm{vec}(\boldsymbol{H})^\mathrm{H}\right]$. In order to incorporate constraints on the bandwidths of time and frequency correlation sequence, we use a theorem [10] on the existence and uniqueness of band-limited positive semidefinite extensions.

Theorem 2. *A positive semidefinite sequence $\{x(m)\}_{m=-M}^{M}$ has a positive semidefinite extension band-limited to $[\omega_l\ ,\omega_h]$, iff*

$$\check{x}(m) = e^{\iota(\omega_h+\omega_l)/2}x(m-1) - 2\cos((\omega_h-\omega_l)/2)x(m) + e^{-\iota(\omega_h+\omega_l)/2}x(m+1)$$

evaluated at $m=0,1,\ldots,M-1$ forms a $M\times M$ positive semidefinite Toeplitz matrix.

To this end, bandlimitedness of time and frequency correlation sequences can be assured by positive semidefinite constraints on the Toeplitz matrices $\boldsymbol{R}_T$ and $\boldsymbol{R}_F$ constructed from the filtered sequences [5]. Thus the uncertainty class constraint can be equivalently described by following positive semidefiniteness constraints,

$$\{r_H(i,j)\} \in \tilde{\mathcal{U}}^1_{r_H} \Leftrightarrow \boldsymbol{R}_H \succeq 0, \boldsymbol{R}_T \succeq 0, \boldsymbol{R}_F \succeq 0, r_H(0,0)=\beta \quad (16)$$

4.3. Final semidefinite optimization problem

The overall optimization problem can now be posed as (cf. Eq. 15,16)

$$\boxed{\begin{aligned}\min_{t,\{r_H(i,j)\}} t \quad \text{s.t.} \quad &\begin{bmatrix} t & \boldsymbol{r}^\mathrm{H}_{h_\mathrm{p}}\\ \boldsymbol{r}_{h_\mathrm{p}} & \boldsymbol{R}_{h_\mathrm{p}}+\boldsymbol{R}_\eta\end{bmatrix} \succeq 0, \quad r_H(0,0)=\beta,\\ &\boldsymbol{R}_H \succeq 0, \quad \boldsymbol{R}_T \succeq 0, \quad \boldsymbol{R}_F \succeq 0. \quad (17)\end{aligned}}$$

Thus, we arrive at a minimization problem with a linear cost function, an equality constraint, and a few positive semidefiniteness constraints. As such, the problem can be solved via any semidefinite problem solver like SeDuMi [11]. The solution of this problem yields the LF 2-D CFR correlation sequence $\{r_H^{\mathrm{LF}}(i,j)\}$ with respective bandwidth constraints. This LF correlation sequence can then be used for the computation of the *maximally robust (MR) 2-D MMSE estimation filter* coefficients, i.e.

$$\boldsymbol{w}^{\mathrm{MR}}_{\mathrm{MMSE}} = \left(\boldsymbol{R}^{\mathrm{LF}}_{h_\mathrm{p}}+\boldsymbol{R}_\eta\right)^{-1}\boldsymbol{r}^{\mathrm{LF}}_{h_\mathrm{p}}, \quad (18)$$

with $\boldsymbol{r}^{\mathrm{LF}}_{h_\mathrm{p}}$ and $\boldsymbol{R}^{\mathrm{LF}}_{h_\mathrm{p}}$ as defined earlier. The superscripts $(\bullet)^{\mathrm{LF}}$ emphasize that they are based on the optimized LF correlation sequence.

4.4. Computational complexity

Unlike the case of heuristic robust estimator [3], the maximally robust estimator does not have an explicit expression for the underlying least favorable 2-D correlation sequence, rather the sequence results from the solution of the semidefinite optimization problem in (17). This means an added computational burden, but the solution needs to be obtained only once for a given set of pilot grid and estimation parameters. Precisely speaking, the solution depends only on the pilot spacings Δ_T and Δ_F, the number of observations N_T and N_F, and the observation noise power. Thus, the LF correlation sequence or the corresponding filter coefficients can easily be precomputed offline and stored with parameterization in terms of SNR.

4.5. Incorporating edge effects

Aiming at a practically feasible robust estimator, we constrained ourselves in Section 4 to the case of a finite number of observations along both the time and the frequency direction. A related but not identical constraint that must be additionally taken into account to ensure applicability of the estimator to practical systems, is the finite dimensionality of the pilot grid. This implies the existence of edge effects, i.e. there are regions on the corners of time-frequency grid (shaded portions in Fig. 1) where we no longer have enough observations on one side of the data position of interest and all such grid edge positions require a special treatment. This calls for the formulation of separate optimization problems for each of the unique edge positions, which renders the entire process of determining the robust estimator or the underlying LF correlation sequence, cumbersome and computationally expensive.

A simulation based analysis however, suggests a simpler yet an efficient solution to handle these special edge positions. To this end, we change the problem formulation for the edge positions to have an unequal number of observations on both sides such that still the total number of observations i.e. N_F and N_T remain fixed. Next rather than optimizing individually for the LF correlation sequence of the edge problems, we propose to simply reuse the LF correlation sequence optimized for the central position on the grid.

5. SIMULATION RESULTS

We consider a multi-carrier system with a regular rectangular pilot grid having $\Delta_{\mathrm{T}} = 7$ and $\Delta_{\mathrm{F}} = 1$. This in fact corresponds to the LTE Uplink [6] transmission format. More specifically, we choose the 20 MHz band for various other parameters. The MAC layer scheduler is assumed to operate on sub-frame level, so we constrain ourselves further to a small finite dimensional grid consisting of two slots only, i.e. we have $N_{\mathrm{T}} = 2$, and we choose $N_{\mathrm{F}} = 5$. We employ the standard frequency domain MMSE equalizer and a rate 1/3 turbo code to show the overall system performance.

For the sake of clarity, we reiterate that the *heuristic estimator* refers to a 2-D MMSE estimator based on rectangular *Doppler Spectrum (DS)* and uniform *Power Delay Profile (PDP)* as obtained in Section 3, while the *Maximally Robust (MR) estimator* is based on the LF 2-D correlation sequence obtained as a result of the finite observation minimax optimization in Section 4. As a reference, we also consider the performance with ideal channel estimation. The channel time correlation spectra that we use in our simulations include the well known Jakes DS or a Bandpass DS with evenly distributed power at the extreme 10% Doppler frequencies. For the channel frequency correlation spectra we use the Vehicular-A (Veh-A) and Typical Urban (TU) standard power delay profiles.

Fig. 2 shows the comparison in terms of coded BER under different transmission scenarios. For QPSK modulation at a speed of 120 kmph with Bandpass DS and Veh-A PDP, we observe a gain of 1.05 dB at the coded BER of 10^{-3} by employing the maximally robust channel estimator instead of the heuristic estimator. Similarly for 16-QAM modulation scheme at 60 kmph with TU PDP and the standard Jakes DS, a gain of 0.9 dB is achieved at the coded BER of

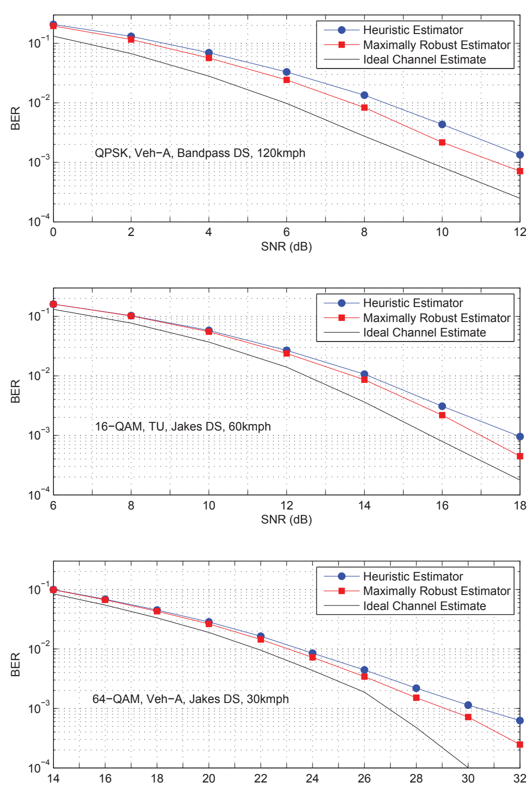

Fig. 2. Performance comparison of Heuristic and Maximally Robust estimators against the performance with ideal channel estimation in terms of coded BER for different transmission scenarios.

Table 1. Throughput gains of MR estimator over Heuristic estimator

Transmission Scenario	Gains at throughput level	
	80%	95%
QPSK, Veh-A, Bandpass DS, 120kmph	0.50 dB	0.91 dB
16-QAM, TU, Jakes DS, 60kmph	0.08 dB	0.35 dB
64-QAM, Veh-A, Jakes DS, 30kmph	0.16 dB	0.43 dB

10^{-3} by the MR estimator, while for the 64-QAM modulation with Veh-A PDP, and Jakes DS the gain increases to 1.35 dB.

In Table 1, we present the gains of MR estimator over the Heuristic estimator in terms of SNR required to reach a specified throughput level. Gains of as much as 0.9 dB can be observed at the 95% throughput level in different transmission scenarios.

6. CONCLUSION

The paper discussed the idea of robust 2-D MMSE estimation with a finite number of available observations. It has been shown that the heuristic robust estimator based on the rectangular (uniform) correlation spectrum proposed in [3], is not maximally robust if the number of observations is finite. The maximally robust estimator has been shown to be obtained as a result of a semidefinite optimization procedure for finding the least favorable 2-D correlation sequence under certain constraints. We extended this MR estimator to the case of small finite dimensional pilot grids by taking into account the grid edge effects in a computationally efficient manner. Finally the paper presented a simulation based comparison of the heuristic and the maximally robust estimator for a practical multi-carrier system with a small finite dimensional pilot grid. With different channel correlation spectra simulated, gains of around 1 dB and more in terms of coded BER and throughput have been shown to be achieved by the MR estimator for different modulation schemes and terminal velocities.

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