SEMI-BLIND CHANNEL ESTIMATION FOR MIMO SINGLE CARRIER WITH FREQUENCY DOMAIN EQUALIZATION SYSTEMS *

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Abstract

We propose a semi-blind channel estimation method for multiple-input multiple-output (MIMO) single carrier with frequency domain equalization systems. By taking advantage of periodic precoding and the block circulant channel model after cyclic prefix removal, we obtain the channel product matrices by solving a series of decoupled linear systems, which is gained from the covariance matrix of the received data. Then the channel impulse response matrix is obtained by computing the positive eigenvalues and eigenvectors of a Hermitian matrix formed from the channel product matrices. We also propose an optimal design of the precoding sequence which minimizes the noise effect and numerical error in covariance matrix estimation. Simulations are used to demonstrate the performance of the proposed method.

Keywords : MIMO channel, blind estimation, frequency domain equalization, cyclic prefix, periodic precoding

1 Introduction

In recent years, multiple-input multiple-output (MIMO) communication systems have received much attention due to the potential improvement in data transmission rate resulted from their enormous channel capacity gains. However, the effect of multi-path of such systems is more complex than that of single-input single-output (SISO) systems. Hence how to choose or design equalizers with low complexity for MIMO systems becomes an important issue. For SISO systems, Falconer *et al.* [1] proposes a low complexity equalization approach, namely, single carrier with frequency domain equalization (SC-FDE), which is an important transmission system because it provides a low complexity equalization process as orthogonal frequency division multiplexing (OFDM), and it avoids some shortcomings of OFDM, e.g. subcarrier frequency offset [1]-[2]. This technology is generalized to MIMO systems [2]. However, to realize the benefit of MIMO SC-FDE systems, accurate channel state information is required. In this paper, we propose a method, using periodic precoding, to estimate the time domain channel coefficients of MIMO SC-FDE systems. We formulate the problem in the time domain and exploits the block circulant channel model to compute the channel product matrices by solving a series of decoupled linear systems. Then the channel impulse response matrix is obtained by computing the positive eigenvalues and eigenvectors of a Hermitian matrix formed from the channel product matrices. The design of the precoding sequence which minimizes the noise effect and numerical error in covariance matrix estimation is proposed. Simulations are used to demonstrate the performance of the proposed method.

This paper is organized as follows. Section 2 is the system model and problem statement. In Section 3, we derive the method for MIMO SC-FDE block transmission systems. Simulation results are given in Section 4. Section 5 concludes this paper.

Notations used in this paper are quite standard: $\mathbf{A}^T$ represents transpose of the matrix $\mathbf{A}$, and $\mathbf{A}^*$ represents conjugate transpose of the matrix $\mathbf{A}$. $\mathbf{I}_M$ is the identity matrix of dimension $M \times M$, and $\mathbf{A} \otimes \mathbf{B}$ is the Kronecker product of matrices $\mathbf{A}$ and $\mathbf{B}$. In addition, we define the following operations that will be used in the derivation of the main result. First, for any $m \times m$ matrix $\mathbf{A} = [a_{k,l}]_{0 \le k,l \le m-1}$, define $\Gamma_j(\mathbf{A}) = [a_{0,j} \ a_{1,j+1} \ \cdots \ a_{m-1-j,m-1}]^T$ for $0 \le j \le m-1$, i.e., $\Gamma_j(\mathbf{A})$ is the vector formed from the jth super-diagonal of $\mathbf{A}$. Second, for any $Jn \times Jn$ matrix $\mathbf{B} = [\mathbf{B}_{k,l}]_{0 \le k,l \le n-1}$, where $\mathbf{B}_{k,l}$ is a block matrix of dimension $J \times J$, define $\Upsilon_j(\mathbf{B}) = [\mathbf{B}_{0,j}^T \ \mathbf{B}_{1,j+1}^T \ \cdots \ \mathbf{B}_{n-1-j,n-1}^T]^T$ for $0 \le j \le n-1$, i.e., $\Upsilon_j(\mathbf{B})$ is the matrix formed from the jth block super-diagonal of $\mathbf{B}$. $\kappa(\mathbf{A})$ is the condition number of a matrix $\mathbf{A}$.

2 System Model and Problem Statement

Consider a K-input J-output discrete time SC-FDE baseband system. At the transmitter, the input signal vector $\mathbf{v}(n) = [v_1(n) \ v_2(n) \ \cdots \ v_K(n)]^T \in \mathbb{C}^K$ is first multiplied by an M-periodic sequence, $p(n) \in \mathbb{R}$, to obtain $\mathbf{s}(n) = p(n)\mathbf{v}(n)$, where $p(n + M) = p(n)$, $\forall \, n$.

Then $\mathbf{s}(n)$ is stacked as a block $\bar{\mathbf{s}}(i) = [\mathbf{s}(iM)\ \mathbf{s}(iM+1)\ \cdots\ \mathbf{s}(iM+M-1)]^T \in \mathbb{C}^{KM}$ and a cyclic prefix (CP), which is the last P block symbols (i.e. PK symbols) of $\bar{\mathbf{s}}(i)$ is inserted in front of $\bar{\mathbf{s}}(i)$ to form $\bar{\mathbf{u}}(i) \in \mathbb{C}^{KN}$, where $N = M + P$. Finally, the resulting data block $\bar{\mathbf{u}}(i)$ is parallel-to-serial converted to $\mathbf{u}(n)$ and then transmitted through the MIMO FIR channel.

At the receiver, the received signal vector is $\mathbf{x}(n) = \sum_{l=0}^{L}\mathbf{H}(l)\mathbf{u}(n-l) + \mathbf{w}(n) \in \mathbb{C}^J$, where $\mathbf{x}(n) = [x_1(n)\ x_2(n)\ \cdots\ x_J(n)]^T \in \mathbb{C}^J$, $\mathbf{w}(n) \in \mathbf{C}^J$ is the channel noise vector which is similarly defined as $\mathbf{x}(n)$. $\mathbf{H}(l) = [h_{jk}(l)] \in \mathbb{C}^{J\times K}$ is the channel coefficient matrix for $l = 0, 1, \cdots, L$. Here $\{h_{jk}(0)\ h_{jk}(1)\ \cdots\ h_{jk}(L_{jk})\}$ is the impulse response from the kth transmitter to the jth receiver with order L_{jk}, and $L = \max_{j,k}\{L_{jk}\}$ is the order of the MIMO channel.

We stack the received signal vector as a block of dimension JN and then remove the first $P \geq L$ block symbols (i.e., PJ symbols) in the block to form $\bar{\mathbf{x}}(i) = [\mathbf{x}(iM)^T\ \mathbf{x}(iM+1)^T \cdots \mathbf{x}(iM+M-1)^T]^T \in \mathbb{C}^{JM}$. Then the input-output relation of the system can be described as follows [4]:

$$\bar{\mathbf{x}}(i) = \mathbf{H}_{BC}\bar{\mathbf{s}}(i) + \bar{\mathbf{w}}(i) \qquad (2.1)$$

where $\bar{\mathbf{s}}(i) \in \mathbb{C}^{KM}$ and $\bar{\mathbf{w}}(i) \in \mathbb{C}^{JM}$ are similarly defined as $\bar{\mathbf{x}}(i)$, and $\mathbf{H}_{BC} \in \mathbb{C}^{JM\times KM}$ is a block circulant matrix with $[\mathbf{H}(0)^T\ \mathbf{H}(1)^T \cdots \mathbf{H}(L)^T\ \mathbf{0}\cdots\mathbf{0}]^T \in \mathbb{C}^{JM\times K}$ being its first block column.

The problem we study in this paper is blind identification of the MIMO channel coefficient matrices $\mathbf{H}(m)$, $0 \leq m \leq L$ using second-order statistics of the received data based on the following assumptions.

(i) The source signal $\mathbf{v}(n)$ is a zero mean white sequence with $E[\mathbf{v}(m)\mathbf{v}(n)^*] = \delta(m-n)\mathbf{I}_K \in \mathbb{R}^{K\times K}$, where $\delta(\cdot)$ is the Kronecker delta function. The noise is white zero mean with $E[\mathbf{w}(m)\mathbf{w}(n)^*] = \delta(m-n)\sigma_w^2\mathbf{I}_J \in \mathbb{R}^{J\times J}$. In addition, the source signal is uncorrelated with the noise $\mathbf{w}(n)$, i.e., $E[\mathbf{v}(m)\mathbf{w}(n)^*] = \mathbf{0}_{K\times J}, \forall\, m, n$.

(ii) The channel impulse response matrix $\mathbf{H} = [\mathbf{H}(0)^T\ \mathbf{H}(1)^T\cdots\mathbf{H}(L)^T]^T \in \mathbb{C}^{J(L+1)\times K}$ is full column rank, i.e., rank$(\mathbf{H}) = K$. In fact, this assumption is the identifiability condition of the channel.

3 Semi-blind Channel Estimation

In this section, we derive the proposed method under the assumptions (i) and (ii). We first derive the proposed method for the case where noise is absent in Section 3.1. The optimal design of the precoding sequence when noise is present is discussed in Section 3.2.

3.1 The Estimation Method

Let $\mathbf{J} \in \mathbb{R}^{M\times M}$ be a circulant matrix with the first row equal to $[0\ 0\cdots 0\ 1] \in \mathbb{R}^{1\times M}$. Then $\mathbf{H}_{BC}$ can be expressed as $\mathbf{H}_{BC} = \sum_{l=0}^{L}\mathbf{J}^l \otimes \mathbf{H}(l)$, and thus for noiseless case, (2.1) can be written as

$$\bar{\mathbf{x}}(i) = \left(\sum_{l=0}^{L}\mathbf{J}^l \otimes \mathbf{H}(l)\right)\bar{\mathbf{s}}(i) \qquad (3.1)$$

Due to precoding, $\bar{\mathbf{s}}(i) = (\mathbf{G} \otimes \mathbf{I}_K)\bar{\mathbf{v}}(i)$, where $\bar{\mathbf{v}}(i) \in \mathbb{C}^{KM}$ is similarly defined as $\bar{\mathbf{s}}(i)$ and $\mathbf{G} = \mathtt{diag}[p(0)\ p(1)\ \cdots\ p(M)] \in \mathbb{R}^{M\times M}$ is a diagonal matrix. Then (3.1) can be written as

$$\bar{\mathbf{x}}(i) = \left(\sum_{l=0}^{L}\mathbf{J}^l \otimes \mathbf{H}(l)\right)(\mathbf{G} \otimes \mathbf{I}_K)\bar{\mathbf{v}}(i) \qquad (3.2)$$

Taking expectation of $\bar{\mathbf{x}}(i)\bar{\mathbf{x}}(i)^*$, we get the covariance matrix $\mathbf{R}$ as follows.

$$\begin{aligned}\mathbf{R} &= E[\bar{\mathbf{x}}(i)\bar{\mathbf{x}}(i)^*]\\ &= \left(\sum_{k=0}^{L}\mathbf{J}^k \otimes \mathbf{H}(k)\right)(\mathbf{G}^2 \otimes \mathbf{I}_K)\left(\sum_{l=0}^{L}(\mathbf{J}^T)^l \otimes \mathbf{H}(l)^*\right)\\ &= \sum_{k=0}^{L}\sum_{l=0}^{L}(\mathbf{J}^k\mathbf{G}^2(\mathbf{J}^T)^l) \otimes (\mathbf{H}(k)\mathbf{H}(l)^*)\end{aligned} \qquad (3.3)$$

For a given $\mathbf{R}$, (3.3) defines a set of $(L+1)$ nonlinear equations in the unknowns $\mathbf{H}(0), \mathbf{H}(1), \cdots, \mathbf{H}(L)$. However, if we consider the channel product matrices of the form $\mathbf{H}(k)\mathbf{H}(l)^*$ as unknowns, then we can formulate a set of $(L+1)$ linear equations between $\mathbf{H}(k)\mathbf{H}(l)^*$ and the covariance matrix of the received signal, which offer a key to estimate the channel product matrices $\mathbf{H}(k)\mathbf{H}(l)^*$, $0 \leq k, l \leq L$. As a result, we first compute the channel product matrices instead of directly solving $\mathbf{H}(0), \mathbf{H}(1), \cdots, \mathbf{H}(L)$. To this end, we need the following proposition to further rearrange the $(L+1)$ equations defined in (3.3) in a more tractable expression.

Proposition 3.1: Let $0 \leq k, l \leq L$ be two non-negative integers. For $l = k + j$, where $0 \leq k \leq L - j$, the upper triangular part of $\mathbf{J}^k\mathbf{G}^2(\mathbf{J}^T)^l$ is zero with only the respectively jth upper diagonals nonzero, i.e.,

$$\Gamma_j\left(\mathbf{J}^k\mathbf{G}^2(\mathbf{J}^T)^l\right) = \mathbf{q}_k(1:M-j,1) \in \mathbb{R}^{M-j} \qquad (3.4)$$

where $\mathbf{q}_k = \mathbf{J}^k\mathbf{p}$, and $\mathbf{p} = [p(0)^2\ p(1)^2\ \cdots\ p(M-1)^2]^T \in \mathbb{R}^M$.

Proof: See [4].

Since

$$\Upsilon_j\left(\left(\mathbf{J}^k\mathbf{G}^2(\mathbf{J}^T)^l\right) \otimes (\mathbf{H}(k)\mathbf{H}(l)^*)\right) = \Gamma_j\left(\mathbf{J}^k\mathbf{G}^2(\mathbf{J}^T)^l\right) \otimes \mathbf{H}(k)\mathbf{H}(l)^*, \qquad (3.5)$$

it follows from (3.3), (3.4) and (3.5) that $\Upsilon_j(\mathbf{R})$ can be derived as follows.

$$\begin{aligned}&\Upsilon_j(\mathbf{R})\\ &= \sum_{k=0}^{L}\sum_{l=0}^{L}\Upsilon_j\left((\mathbf{J}^k\mathbf{G}^2(\mathbf{J}^T)^l) \otimes (\mathbf{H}(k)\mathbf{H}(l)^*)\right)\\ &= \sum_{k=0}^{L}\sum_{l=0}^{L}\Gamma_j\left(\mathbf{J}^k\mathbf{G}^2(\mathbf{J}^T)^l\right) \otimes \mathbf{H}(k)\mathbf{H}(l)^*\\ &= \sum_{k=0}^{L-j}\mathbf{q}_k(1:M-j,1) \otimes \mathbf{H}(k)\mathbf{H}(k+j)^*\\ &= \sum_{k=0}^{L-j}(\mathbf{q}_k(1:M-j,1) \otimes \mathbf{I}_J)\mathbf{H}(k)\mathbf{H}(k+j)^*\end{aligned} \qquad (3.6)$$

If we define, for $0 \leq j \leq L$, $\mathbf{M}_j = [\mathbf{q}_0(1:M-j,1)\ \mathbf{q}_1(1:M-j,1)\ \cdots\ \mathbf{q}_{L-j}(1:M-j,1)] \otimes \mathbf{I}_J \in \mathbb{R}^{J(M-j)\times J(L+1-j)}$ and $\mathbf{F}_j = [(\mathbf{H}(0)\mathbf{H}(j)^*)^T\ (\mathbf{H}(1)\mathbf{H}(1+j)^*)^T\ \cdots\ (\mathbf{H}(L-j)\mathbf{H}(L)^*)^T]^T \in \mathbb{C}^{J(L+1-j)\times K}$, then (3.6) can be written as

$$\Upsilon_j(\mathbf{R}) = \mathbf{M}_j\mathbf{F}_j, \qquad \forall 0 \leq j \leq L \qquad (3.7)$$

Since $M > L+1$, the $(L+1)$ equations in (3.7) are overdetermined and consistent. We note that the matrix $\mathbf{M}_j$, $j = 0, 1, \cdots, L$ is completely determined by the precoding sequence. By appropriately selecting the precoding sequence, we can make each $\mathbf{M}_j$ full column rank. Then the solution $\mathbf{F}_j$ can be obtained as

$$\mathbf{F}_j = (\mathbf{M}_j^T\mathbf{M}_j)^{-1}\mathbf{M}_j^T\Upsilon_j(\mathbf{R}), \qquad j = 0, 1, \cdots, L \qquad (3.8)$$

Then we use the channel product matrices $\mathbf{H}(k)\mathbf{H}(l)^*$, $0 \leq k,l \leq L$, obtained from (3.8) to form a Hermitian matrix $\mathbf{Q} = \mathbf{HH}^*$. Since rank$(\mathbf{H}) = K$ by assumption (ii), rank$(\mathbf{Q}) = K$. Since $\mathbf{Q}$ is Hermitian and positive semidefinite, $\mathbf{Q}$ has K positive eigenvalues and the associated unit-norm eigenvectors, say, $\lambda_1, \cdots, \lambda_K$ and $\mathbf{d}_1, \cdots, \mathbf{d}_K$, respectively. We can thus choose the channel impulse response matrix to be

$$\widehat{\mathbf{H}} = [\sqrt{\lambda_1}\mathbf{d}_1 \ \ \sqrt{\lambda_2}\mathbf{d}_2 \ \ \cdots \ \ \sqrt{\lambda_K}\mathbf{d}_K] \in \mathbb{C}^{J(L+1)\times K} \quad (3.9)$$

up to a unitary matrix ambiguity $\mathbf{U} \in \mathbb{C}^{K\times K}$, which can be resolved using a short pilot sequence [5].

3.2 Optimal Precoding Design

When the noise is present, the covariance matrix $\mathbf{R}$ contains the contribution of noise. Thus (3.3) becomes

$$\mathbf{R} = \sum_{k=0}^{L}\sum_{l=0}^{L} \left(\mathbf{J}^k\mathbf{G}^2(\mathbf{J}^T)^l\right) \otimes (\mathbf{H}(k)\mathbf{H}(l)^*) + \sigma_w^2\mathbf{I}_{JM} \quad (3.10)$$

From (3.3) and (3.10), we see that the noise has only contribution to the main (block) diagonal entries of $\mathbf{R}$. Hence the $(L+1)$ linear equations in (3.7) remain unchange, except for the $j = 0$ case, which becomes

$$\Upsilon_0(\mathbf{R}) = \mathbf{M}_0\mathbf{F}_0 + \mathbf{Y} \quad (3.11)$$

where $\mathbf{Y} = \sigma_w^2[\mathbf{I}_J \ \cdots \ \mathbf{I}_J]^T \in \mathbb{R}^{JM\times J}$. Hence from (3.8), the least square approximation solution of $\mathbf{F}_0$ can be written as

$$\widehat{\mathbf{F}}_0 = (\mathbf{M}_0^T\mathbf{M}_0)^{-1}\mathbf{M}_0^T(\mathbf{M}_0\mathbf{F}_0 + \mathbf{Y}) = \mathbf{F}_0 + \mathbf{Z} \quad (3.12)$$

Equation (3.12) is the actual solution $\mathbf{F}_0$ plus a perturbation term $\mathbf{Z} = (\mathbf{M}_0^T\mathbf{M}_0)^{-1}\mathbf{M}_0^T\mathbf{Y}$ due to noise. We note that from (3.12), $\widehat{\mathbf{F}}_0 = \mathbf{F}_0$ if and only if the column space of $\mathbf{M}_0$ is orthogonal to that of $\mathbf{Y}$, i.e., $\mathbf{M}_0^T\mathbf{Y} = \mathbf{0}$. Also we know $\mathbf{M}_0 = [\mathbf{q}_0 \ \mathbf{q}_1 \ \cdots \ \mathbf{q}_L] \otimes \mathbf{I}_J$ and $\mathbf{Y} = \sigma_w^2[\mathbf{I}_J \ \cdots \ \mathbf{I}_J]^T = \sigma_w^2\mathbf{b} \otimes \mathbf{I}_J$, where $\mathbf{b} = [1 \ 1\cdots 1]^T \in \mathbb{R}^M$. Then the product of $\mathbf{M}_0^T$ and $\mathbf{Y}$ can be written as

$$\begin{aligned}\mathbf{M}_0^T\mathbf{Y} &= \left([\mathbf{q}_0 \ \mathbf{q}_1 \ \cdots \ \mathbf{q}_L]^T \otimes \mathbf{I}_J\right)\left(\sigma_w^2\mathbf{b} \otimes \mathbf{I}_J\right) \\ &= \sigma_w^2\left([\mathbf{q}_0 \ \mathbf{q}_1 \ \cdots \ \mathbf{q}_L]^T\mathbf{b}\right) \otimes \mathbf{I}_J.\end{aligned} \quad (3.13)$$

Thus the constraint $\mathbf{M}_0^T\mathbf{Y} = \mathbf{0}$ is equivalent to

$$[\mathbf{q}_0 \ \mathbf{q}_1 \ \cdots \ \mathbf{q}_L]^T\mathbf{b} = \mathbf{0}. \quad (3.14)$$

If the precoding sequence can be selected to achieve the orthogonality condition (3.14), the effect of noise is completely eliminated. But this is impossible since the entries of $\mathbf{q}_0 \ \mathbf{q}_1 \ \cdots \ \mathbf{q}_L$ and $\mathbf{b}$ are positive. Therefore we seek to choose the precoding sequence such that $\mathbf{b}$ is as close to being orthogonal to $\mathbf{q}_i$ as possible, for $i = 0, 1, \cdots, L$. Define the following correlation coefficients

$$\gamma_i = \frac{\mathbf{q}_i^T\mathbf{b}}{\|\mathbf{q}_i\|_2\|\mathbf{b}\|_2}, \quad i = 0, 1, \cdots, L. \quad (3.15)$$

Since γ_i is nonnegative and by Cauchy-Schwarz inequality, $0 \leq \gamma_i \leq 1$. In order to let $\mathbf{b}$ be as close to being orthogonal to each $\mathbf{q}_i$ as possible, we choose the precoding sequence so that the correlation coefficient γ_i is as small as possible for $i = 0, 1\cdots, L$. However, since for $i = 0, 1, \cdots, L$, $\mathbf{q}_i^T\mathbf{b} = (\mathbf{J}^i\mathbf{p})^T\mathbf{b} = \mathbf{p}^T(\mathbf{J}^i)^T\mathbf{b} = \mathbf{p}^T\mathbf{b} =$

$\sum_{m=0}^{M-1} p(m)^2 = \mathbf{q}_0^T\mathbf{b}$ and $\|\mathbf{q}_i\|_2 = \sqrt{\sum_{m=0}^{M-1} p(m)^4} = \|\mathbf{q}_0\|_2$, each γ_i assumes the same value. Thus we only need to consider γ_0. Based on this point of view, we formulate the optimal selection problem as minimizing γ_0 subject to the following two constraints:

$$\frac{1}{M}\sum_{m=0}^{M-1} p(m)^2 = 1, \quad (3.16)$$

$$p(m)^2 \geq \tau > 0, \quad \forall 0 \leq m \leq M-1. \quad (3.17)$$

Roughly, constraint (3.16) normalizes the power gain of the precoding sequence of each transmitter to 1 and constraint (3.17) requires that at each instant, the power gain is no less than τ with $0 < \tau < 1$. Note that the problem of selecting the optimal precoding sequence is identical to the SISO case considered in [4], and the resulting optimal sequence is given by, for any fixed $0 \leq m \leq M-1$,

$$p(n) = \begin{cases} \sqrt{M(1-\tau)+\tau}, & n = m \\ \sqrt{\tau}, & n \neq m, \quad 0 \leq n \leq M-1 \end{cases} \quad (3.18)$$

In addition, with the optimal solution in (3.18), the corresponding $\gamma_0 = \frac{1}{\sqrt{M(1-\tau)^2+\tau(2-\tau)}}$. Note that γ_0 decreases as τ decreases, and thus the noise effect in the estimation of the covariance matrix $\mathbf{R}$ is reduced and hence estimation performance improves.

From the solution in (3.18), we know the optimal precoding sequence $p(n)$ is a two-level sequence with a single peak in one period. However, the peak location, m, does significantly affect the numerical condition of the linear equation (3.7) since different choices of m result in different matrices $\mathbf{M}_j$ and the associated condition numbers. If the condition number of $\mathbf{M}_j \in \mathbb{R}^{J(M-j)\times J(L+1-j)}$, $\kappa(\mathbf{M}_j)$, is large, then the solution $\mathbf{F}_j$ is sensitive to data error. Define $\mu = \max_{0\leq j\leq L} \kappa(\mathbf{M}_j)$. Then according to the result given in [3], we know $m = 0$, or $m = 1, \cdots$, or $m = M - L - 1$ will result in the minimum condition number of μ. Hence the optimal precoding sequence has a peak which locates at one of the first $(M-L)$ positions.

4 Simulation

In this section, we generate 100 2-input 2-output random channels with $L = 2$ for each simulation to demonstrate the performance of the proposed method. We use $M = 18$ and $P = 2$. Each channel coefficient in the channel matrix is generated according to the independent complexed-valued Gaussian distribution with zero mean and unit variance. The normalized mean-square-error (NMSE) of the channel impulse response matrix is defined as NMSE $= (1/I)\sum_{i=1}^{I} \|\widehat{\mathbf{H}}^{(i)} - \mathbf{H}\|_F^2 \cdot \|\mathbf{H}\|_F^{-2}$, where $I = 100$ is the number of independent trials, and $\|\cdot\|_F$ denotes the Frobenius norm. $\widehat{\mathbf{H}}^{(i)} = [\widehat{\mathbf{H}}^{(i)}(0)^T \ \widehat{\mathbf{H}}^{(i)}(1)^T \ \widehat{\mathbf{H}}^{(i)}(2)^T]^T$ is the ith estimate of the channel impulse response matrix $\mathbf{H}$ after removing the unitary matrix ambiguity by the least squares method. The input source symbols are i.i.d. QPSK signals. The channel noise is zero mean, temporally and spatially white Gaussian.

For simulation 1, we use 5 precoding sequences which all satisfy (3.16) and (3.17) to illustrate the effect of the precoding sequences on the estimation performance. This simulation demonstrates the results for the four optimal sequences chosen based on (3.18) with $\tau = 0.7$, but with $m = 0$, 15, 16 and 17, respectively, and a nonoptimal sequence chosen as $p(n)^2 = 0.7$ for $0 \le n \le 9$ and $p(n)^2 = 1.3$ for $10 \le n \le 18$. Figure 1(a) shows that NMSE decreases as SNR increases for every precoding sequence. In this figure, we also see that the optimal precoding sequence (3.18) with $m = 0$ or $m = M - L - 1 = 15$ yields the smallest NMSE because the peak locates in one of the first $(M - L - 1)$ position of the sequence, which results in a smaller condition number $\mu = 1.6387$ than $\mu = 3.2198$ which is the result of the precoding sequence (3.18) with $m = M - 1 = 17$ or $m = N - L = 16$. For Figure 1(b), we use the optimal precoding sequences that satisfy (3.18) with $m = 0$, but with different τ to test the effect of τ on the estimation performance. Figure 1(b) shows that the estimation performs better for smaller τ, which is consistent with our analysis in Section 3.2. In simulation 2, we compare the estimation performance of the proposed method with that of another precoding method [5]. The precoding sequence is chosen based on (3.18) with $\tau = 0.5$ for the proposed method; while the precoding matrix for the method in [5] is chosen as $\frac{1}{\sqrt{M}}\mathbf{D}_d\mathbf{E}_j$ for $j = 1, 2, 3$, where $\mathbf{D}_d = \mathtt{diag}[\sqrt{(M-1)p+1}, \ , \sqrt{1-p}, \cdots, \sqrt{1-p}] \in \mathbb{R}^{M \times M}$ is a diagonal matrix with $p = 0.5$, and $\mathbf{E}_1$, $\mathbf{E}_2$, and $\mathbf{E}_3$ are $M \times M$ Hadamard matrix, normalized IDFT matrix, and identity matrix, respectively. We set $M = 16$ instead of $M = 18$ such that the Hadamard matrix can be defined. Figure 2 shows that the estimation performance of the proposed method is better than that of the method in [5].

5 Conclusions

In this paper, we propose a method for blind channel estimation of MIMO SC-FDE systems. By exploiting the block circulant channel matrix structure and periodic precoding, the channel product matrices can be solved from a series of decoupled linear equations obtained form the covariance matrix of the received data. Then the channel impulse response matrix can be obtained by taking eigen-decomposition of a Hermitian matrix formed from the channel product matrices. We propose a two-level optimal precoding scheme that minimizes the noise effect and numerical error in the estimation of the covariance matrix of the received data. Simulation results are used to demonstrate the performance of the proposed method and compare it with a linear precoding approach.

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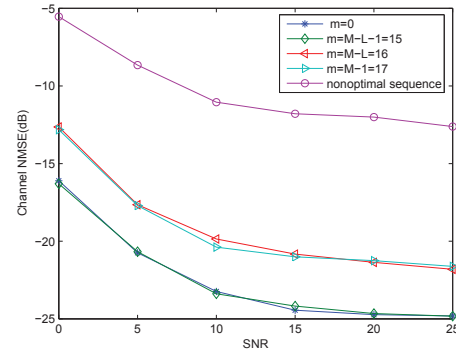
(a) Channel NMSE versus output SNR
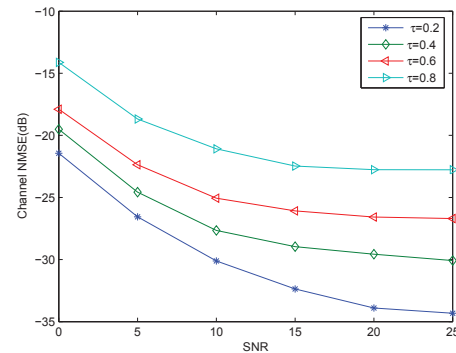
(b) Channel NMSE versus output SNR
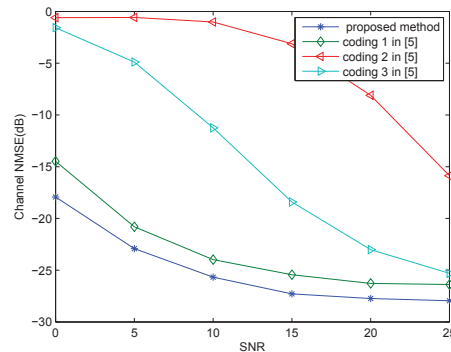
Figure 1: Simulation results for optimal selection of precoding sequences

Figure 2: Comparison with the method in [5]