# REALIZABLE EQUALIZERS FOR FREQUENCY SELECTIVE MIMO CHANNELS WITH COCHANNEL INTERFERENCE

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## ABSTRACT

We consider realizable linear and decision feedback equalization (DFE) of frequency selective multiple-input multiple-output (MIMO) channels in the presence of cochannel interference (CCI). Equalizers that are optimal in the minimum mean square error (MMSE) sense are derived with and without zero forcing (ZF) constraint. It is shown that all problems can be reduced to  $\mathcal{H}_2$  optimal deconvolution, for which a novel algorithm is presented.

*Index Terms*—MIMO systems, Intersymbol interference, Cochannel interference, Equalizers, Deconvolution

#### 1. INTRODUCTION

Recently, there has been interest in so called realizable equalization filters for frequency selective MIMO channels. Realizable filters obey causality and stability constraints, though they are not necessarily finite impulse response (FIR) filters. Instead it is required that there is a finite dimensional realization, e.g. as a state space system. Realizable linear as well as decision feedback equalizers (DFE) for frequency selective MIMO channels without cochannel interference (CCI) have already been derived [1, 2, 3]. CCI results from multiple transmitters using the same frequency, a situation which routinely occurs, e.g. at cell edges of adjacent cells in mobile networks [4]. The goal of this paper is to derive realizable linear equalizers that take CCI into account. This can be seen as an generalization of the well known interference rejection combining (IRC) for flat fading channels [5, 4]. Our main result is the optimal realizable linear equalizer in the minimum mean square error (MMSE) sense with and without zero forcing (ZF) constraint. When applied to the usual linearization of the DFE, our results directly extend to realizable DFEs for channels with CCI. We point out that various generalizations of IRC to frequency selective channels, usually termed spatio-temporal interference rejection combining, have been derived under FIR restrictions (see e.g. [6] and the references therein). Other approaches to CCI mitigation include space-time block coding and non-linear equalization [6, 7, 8]. However, there seem to be no results on realizable equalization in the presence of CCI.

The paper is structured as follows. In Section 2, we introduce the system model and problem statement together with some other preliminaries. In Sections 3, 4 and 5 we consider the four different equalization problems of the optimal realizable equalizer with and without ZF constraint in channels without CCI ( $\mathcal{H}_2$ -ZF and MMSE), the optimal realizable ZF equalizer in channels with CCI (ZF-IRC)



**Fig. 1.** System model at time k with transmitted signal  $u_k$ , interference  $u_{I,k}$ , noise  $n_k$ , estimation  $\hat{u}_k$ , demodulated signal  $\tilde{u}_k$ , and delay L. The optional feedback path of the DFE is dashed.

and the optimal realizable equalizer in channels with CCI (MMSE-IRC). We finish the paper with a numerical example that illustrates the performance of these algorithms and their DFE counterparts in Section 6.

#### 2. PRELIMINARIES

#### 2.1. Notation and Basic Definitions

We denote the complex numbers by  $\mathbb{C}$ , the naturals by  $\mathbb{N}$ , complex conjugation by  $(\bar{\cdot})$ , matrices over  $\mathbb{C}$  by  $\mathbb{C}^{m \times n}$  and set  $\mathbb{C}^m := \mathbb{C}^{m \times 1}$ .  $k, j \in \mathbb{N}$  are time indices. For any matrix,  $\operatorname{tr}\{\cdot\}$  denotes trace,  $(\cdot)^*$  the conjugate transpose and  $(\cdot)^{-1}$  the inverse. The zero matrix is denoted by 0, the identity matrix by *I*. If necessary, the dimensions of zero and identity matrix are added as subscripts.  $\mathbb{E}[\cdot]$  is the expectation operator. Transfer functions are labeled with bold letters. The  $\mathcal{H}_2$  norm of a transfer function  $\mathbf{X}(z) = \sum_{k=-\infty}^{\infty} X_k z^{-k}$  is defined by  $\|\mathbf{X}\|_2^2 := \frac{1}{2\pi i} \int_{|z|=1} \operatorname{tr} \{\mathbf{X}(z)\mathbf{X}(\bar{z}^{-1})^*\} \frac{dz}{z}$ .  $\mathbf{X}$  is causal if  $X_k = 0$  for k < 0. We further say that  $\mathbf{X}$  is *stable*, if all poles of  $\mathbf{X}$  are contained inside the complex unit circle.  $\mathbf{X}$  is called *realizable*, if it is stable, causal, and the transfer function of a state space system (see Section 2.4 for details). We denote the set of realizable  $\mathbf{G}$  such that  $\mathbf{G}(z)\mathbf{X}(z) = z^{-L}I$  by  $\mathcal{I}(\mathbf{X}, L)$ .

### 2.2. System Model

The system model is depicted in Fig. 1. We adopt the standard model of a MIMO channel with additional interferer (the extension of our results to multiple interferers is straight forward). Both channel and interferer are realizable MIMO systems with p inputs and  $q \ge p$ 

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outputs. The input-output relation is

$$y_j = \sum_{k=0}^{\infty} H_k u_{j-k} + \sum_{k=0}^{\infty} H_k^I u_{j-k}^I + n_j \qquad (j \in \mathbb{N}).$$
(1)

Here,  $\{u_k\} \subset \mathbb{C}^p$  and  $\{u_{I,k}\} \subset \mathbb{C}^p$  denote transmitted signal vectors of transmitter and interferer, respectively,  $\{n_k\} \subset \mathbb{C}^q$  denotes noise, and  $\{y_k\} \subset \mathbb{C}^q$  denotes received signals.  $\{H_k\} \subset \mathbb{C}^{q \times p}$  and  $\{H_{I,k}\} \subset \mathbb{C}^{q \times p}$  are the channel impulse responses of transmitter and interferer, respectively. We assume transmitted signals and noise are independent spatially and temporally white Gaussian random variables with zero mean. The variance of the transmitted signals and noise is one and  $\sigma^2 > 0$ , respectively. Using Z-transform, we obtain the equivalent Z-domain formulation

$$\mathbf{y}(z) = \mathbf{H}(z)\mathbf{u}(z) + \mathbf{H}_I(z)\mathbf{u}_I(z) + \mathbf{n}(z)$$

of (1), where  $\mathbf{H}(z) = \sum_{k=0}^{\infty} H_k z^{-k}$ ,  $\mathbf{u}(z) = \sum_{k=0}^{\infty} u_k z^{-k}$ , etc. After transmission through the channel, the delayed estimations

After transmission through the channel, the delayed estimations  $\{\hat{u}_{k-L}\}$  of  $\{u_k\}$ , which are obtained with a linear equalizer **G**, are demodulated. One has the option that the demodulated signals are fed back in the next time slot, passed through the feedback filter **B**, and then added to the estimated signals.

#### 2.3. Problem Statement

In this paper we aim to compute realizable equalization filters  $\mathbf{G}(z) = \sum_{k=0}^{\infty} G_k z^{-k}$  that minimize the asymptotic expectation of the mean square error, i.e.

$$J(\mathbf{G}) := \lim_{K \to \infty} \mathbb{E}\left[\frac{1}{K+1} \sum_{k=0}^{K} \|e_k\|_2^2\right].$$

Here,  $\mathbf{e}(z) = \sum_{k=0}^{\infty} e_k z^{-k} := \hat{\mathbf{u}}(z) - z^{-L} \mathbf{u}(z)$  is the equalization error for the estimation  $\hat{\mathbf{u}}(z) := \mathbf{G}(z)\mathbf{y}(z)$  of the transmitted signals.  $L \in \mathbb{N}$  is an estimation delay. We address the following linear equalization problems.

 $\mathcal{H}_2$ -ZF: Find G realizable, such that  $\mathbf{G}(z)\mathbf{H}(z) = z^{-L}I_p$  and  $\|\mathbf{G}\|_2^2 = \min$ .

**MMSE:** Find **G** realizable, such that  $J = \min$  for  $\mathbf{H}_I = 0$ .

**ZF-IRC:** Find **G** realizable, such that  $\mathbf{G}(z)\mathbf{H}(z) = z^{-L}I_p$  and  $J = \min$ .

**MMSE-IRC:** Find **G** realizable, such that  $J = \min$ .

These linear filters can directly be used to compute the corresponding optimal DFEs. Similar to [9], the DFE can be modeled by replacing **H** and  $\mathbf{H}_I$  with the equivalent channel and interferer

$$\mathbf{H}^{DFE}(z) = \begin{bmatrix} \mathbf{H}(z) \\ \mu z^{-(L+1)} I_p \end{bmatrix}, \quad \mathbf{H}_I^{DFE}(z) = \begin{bmatrix} \mathbf{H}_I(z) \\ \mathbf{0}_p \end{bmatrix}.$$

Here,  $\mu > 0$  is a parameter that measures the reliability of the feedback path  $z^{-(L+1)}I_p$ . If correct past decisions are assumed,  $\mu \to \infty$ is optimal. From any linear equalizer  $\mathbf{G}^{DFE}$  for  $\mathbf{H}^{DFE}$ , the according feed-forward and feed-backward filters  $\mathbf{G}$  and  $\mathbf{B}$  are obtained as the first q columns and the last p columns of  $\mathbf{G}^{DFE}$ , respectively.

#### 2.4. State Space Systems

A *state space system* is a linear and time-invariant (LTI) system with a time-domain description

$$\begin{bmatrix} x_{k+1} \\ y_k \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_k \\ u_k \end{bmatrix} \qquad (k \in \mathbb{N}),$$

where again  $\{u_k\} \subset \mathbb{C}^p$  and  $\{y_k\} \subset \mathbb{C}^q$  denote inputs and outputs of the system, while the  $\{x_k\} \subset \mathbb{C}^n$  are the *states* of the system. If the initial state  $x_0$  is zero, the input-output behavior of a state space system is described in the  $\mathbb{Z}$ -domain by its transfer function

$$\Gamma(z) = D + C(zI - A)^{-1}B$$
  
=  $D + \sum_{k=1}^{\infty} CA^{k-1}Bz^{-k} =: \left[\frac{A \mid B}{C \mid D}\right](z).$ 

Thus, every state space system is causal. Conversely, every causal rational matrix can be realized with a state space system [10, Ch. 6.1]. This, of course, in particular includes all FIR MIMO filters. The system is called *stable* if all eigenvalues of A have modulus less than one, which implies that its transfer function is stable. On the other hand, every causal rational matrix that is stable also has a stable state space realization [10, Ch. 6.2].

# 3. $\mathcal{H}_2$ -ZF AND MMSE

The  $\mathcal{H}_2$ -ZF problem seems to be the simplest of the problems considered in this paper. The more general MMSE problem, which may be easily approached using the Kalman filter [3], can be used to solve  $\mathcal{H}_2$ -ZF by sending  $\sigma^2 \rightarrow 0$ . However, this approach can run into numerical problems [11]. Therefore, direct approaches to  $\mathcal{H}_2$ -ZF are of interest. For the special case of no estimation delay, i.e. L = 0, direct algorithms that solve  $\mathcal{H}_2$ -ZF have been given in [12, 13]. For the general case, an optimization approach using linear matrix inequalities (LMIs) was given in [14]. Also, there exist various direct algorithms for realizable zero forcing equalizers with estimation delay [15, 16]. However, none of them minimizes the  $\mathcal{H}_2$  norm.

Our algorithm for  $\mathcal{H}_2$ -ZF extends the approach from [12] for the delay free case. Due to space limitations, we give no proof.

**Input:** 
$$\mathbf{H} = \begin{bmatrix} A & B \\ \hline C & D \end{bmatrix}$$
 with  $\mathcal{I}(\mathbf{H}, L) \neq \emptyset$ , where A is  $n \times n$ , D is  $q \times p$  with rank  $p, q > p$ ,<sup>1</sup> and  $L > 0$ .

**Outp.:** Solution G of  $\mathcal{H}_2$ -ZF.

 Set D<sup>+</sup> := (D<sup>\*</sup>D)<sup>-1</sup>D<sup>\*</sup> and compute the q × (q − p) matrix D<sub>⊥</sub> that satisfies D<sub>⊥</sub>D<sup>\*</sup><sub>⊥</sub> = I<sub>q</sub> − DD<sup>+</sup>. Set D<sup>+</sup><sub>⊥</sub> := D<sup>\*</sup><sub>⊥</sub>, and compute the singular value decomposition

$$D_{\perp}^{+} = U \begin{bmatrix} S & 0_{q-p \times p} \end{bmatrix} \begin{bmatrix} V_{1}^{*} \\ V_{2}^{*} \end{bmatrix}$$

- 2) Compute a  $n \times (q-p)$  matrix  $B_{\perp}$  such that  $A_0 := A BD^+C B_{\perp}D_{\perp}C$  is stable. Set  $B_0 := -BD^+ B_{\perp}D_{\perp}$ .
- Set E := A − BD<sup>+</sup>C, F := B<sub>0</sub>V<sub>2</sub>V<sub>2</sub><sup>\*</sup>B<sub>0</sub><sup>\*</sup>, G := C<sup>\*</sup>V<sub>1</sub>V<sub>1</sub><sup>\*</sup>C and compute the stabilizing solution Y<sub>11</sub> ≥ 0 to the discrete-time algebraic Riccati equation

$$Y_{11} = F + E(I_n + Y_{11}G)^{-1}Y_{11}E^*.$$

4) Define

$$y_1 := B_0 V_2 V_2^* (D^+)^* + E(I_n + Y_{11}G)^{-1} Y_{11}C^* (D^+)^*, y_k := E(I_n + Y_{11}G)^{-1} y_{k-1} \quad (k = 2, \dots, L).$$

<sup>&</sup>lt;sup>1</sup>The case q = p is trivial because then inverses are unique.

5) Set

$$\begin{split} \hat{A} &:= \begin{bmatrix} A_0 \\ D^+C \\ I_{(L-1)p} & 0_{(L-1)p \times p} \end{bmatrix}, \\ \hat{B} &:= \begin{bmatrix} B_0 \\ D^+ \\ 0_{(L-1)p \times q} \end{bmatrix}, \\ T &:= V_1(I_{q-p} + V_1^*CY_{11}C^*V_1)^{-1}V_1^*, \\ R &:= \begin{bmatrix} A_0Y_{11}C^* + B_0 \\ D^+CY_{11}C^* + D^+ \\ [y_1 & \dots & y_{L-1} ]^*C^* \end{bmatrix} T, \\ S &:= -y_L^*C^*T, \\ \Phi &:= \begin{bmatrix} RC & 0_{(n+Lp) \times (L-1)p} & 0_{(n+Lp) \times p} \end{bmatrix} \\ \Psi &:= \begin{bmatrix} SC & 0_{p \times (L-1)p} & I_p \end{bmatrix}, \end{split}$$

and return

$$\mathbf{G} = \begin{bmatrix} \hat{A} & & \hat{B} \\ \Phi & \hat{A} - \Phi & R \\ \hline \Psi & -\Psi & S \end{bmatrix}.$$

We point out that the complexity of our algorithm grows *only linearly* with the estimation delay L.

Now, with  $\mathcal{H}_2$ -ZF solved, the solution to MMSE follows easily.

**Proposition 1.** Let  $\mathbf{G}^{aug}(z) = \begin{bmatrix} \mathbf{G}(z) & \mathbf{G}_2(z) \end{bmatrix}$  be the solution to  $\mathcal{H}_2$ -ZF applied to the augmented channel

$$\mathbf{H}^{aug}(z) = \begin{bmatrix} \mathbf{H}(z) \\ \sigma I_p \end{bmatrix}$$

Then  $\mathbf{G}(z)$  is the solution to MMSE.

*Proof.* Note that  $\mathbf{G}^{aug}(z)\mathbf{H}^{aug}(z) = z^{-L}I_p$  if and only if  $\sigma \mathbf{G}_2(z) = z^{-L}I_p - \mathbf{G}(z)\mathbf{H}(z)$ . Since  $J(\mathbf{\tilde{G}}) = \sigma^2 \|\mathbf{\tilde{G}}\|_2^2 + \|z^{-L}I_p - \mathbf{\tilde{G}H}\|_2^2$  by [17, Prop. 3.1], we see that

$$J(\mathbf{G}) = \sigma^{2} \|\mathbf{G}\|_{2}^{2} + \|z^{-L}I_{p} - \mathbf{GH}\|_{2}^{2}$$
  
$$= \sigma^{2} (\|\mathbf{G}\|_{2}^{2} + \|\mathbf{G}_{2}\|_{2}^{2}) = \sigma^{2} \|\mathbf{G}^{aug}\|_{2}^{2}$$
  
$$= \sigma^{2} \min_{[\tilde{\mathbf{G}} \tilde{\mathbf{G}}_{2}] \in \mathcal{I}(\mathbf{H}^{aug}, L)} (\|\tilde{\mathbf{G}}\|_{2}^{2} + \|\tilde{\mathbf{G}}_{2}\|_{2}^{2})$$
  
$$= \sigma^{2} \min_{\tilde{\mathbf{G}} \text{ realizable}} (\|\tilde{\mathbf{G}}\|_{2}^{2} + \sigma^{-2}\|z^{-L}I_{p} - \tilde{\mathbf{G}}\mathbf{H}\|_{2}^{2})$$
  
$$= \min_{\tilde{\mathbf{G}} \text{ realizable}} J(\tilde{\mathbf{G}}).$$

## 4. ZF-IRC

The ZF-IRC problem is a hybrid problem between  $\mathcal{H}_2$ -ZF and MMSE-IRC. In contrast to  $\mathcal{H}_2$ -ZF, the Kalman filter cannot be applied, because the interference is temporally and spatially correlated. Before we can give the solution to ZF-IRC, we have to review the *spectral factorization*. Consider a positive definite hermitian transfer function X, i.e.  $\mathbf{X}(z) = \mathbf{X}(\bar{z}^{-1})^*$ . Then, S is a *spectral factor* of X if S is causal, stable, minimum phase (i.e. with all zeros inside the unit circle), and it holds  $\mathbf{X}(z) = \mathbf{S}(z)\mathbf{S}(\bar{z}^{-1})^*$ . The factorization. We refer to [18] and the references therein for further details.

We now derive the solution to ZF-IRC. The following proposition shows, that ZF-IRC is in fact a special case of  $\mathcal{H}_2$ -ZF.

**Proposition 2.** Let  $\mathbf{S}(z)$  be a spectral factor of  $\mathbf{H}_I(z)\mathbf{H}_I(\bar{z}^{-1})^* + \sigma^2 I_q$ , and let  $\mathbf{G}_S(z)$  be the solution to  $\mathcal{H}_2$ -ZF applied to the channel  $\mathbf{S}^{-1}(z)\mathbf{H}(z)$ . Then the solution to ZF-IRC is given by

$$\mathbf{G}(z) := \mathbf{G}_S(z)\mathbf{S}^{-1}(z)$$

*Proof.* Obviously  $\mathbf{G}(z)\mathbf{H}(z) = z^{-L}I_p$ . Similar to [17, Prop. 3.1], one shows  $J(\tilde{\mathbf{G}}) = \|\tilde{\mathbf{G}}\mathbf{H}_I\|_2^2 + \sigma^2 \|\tilde{\mathbf{G}}\|_2^2$ . Then,

$$J(\mathbf{G}) = \|\mathbf{G}\mathbf{H}_{I}\|_{2}^{2} + \sigma^{2}\|\mathbf{G}\|_{2}^{2}$$

$$= \int_{|z|=1} \operatorname{tr} \left\{ \mathbf{G}(z) \left(\mathbf{H}_{I}(z)\mathbf{H}_{I}(\bar{z}^{-1})^{*} + \sigma^{2}I\right) \times \mathbf{G}(\bar{z}^{-1})^{*} \right\} \frac{dz}{2\pi i z}$$

$$= \int_{|z|=1} \operatorname{tr} \left\{ \mathbf{G}(z)\mathbf{S}(z)\mathbf{S}(\bar{z}^{-1})^{*}\mathbf{G}(\bar{z}^{-1})^{*} \right\} \frac{dz}{2\pi i z} = \|\mathbf{G}\mathbf{S}\|_{2}^{2}$$

$$= \|\mathbf{G}_{S}\|_{2}^{2} = \min_{\tilde{\mathbf{G}} \in \mathcal{I}(\mathbf{S}^{-1}\mathbf{H},L)} \|\tilde{\mathbf{G}}\|_{2}^{2} = \min_{\tilde{\mathbf{G}} \in \mathcal{I}(\mathbf{H},L)} \|\tilde{\mathbf{G}}\mathbf{S}\|_{2}^{2}$$

$$\stackrel{(\dots)}{=} \min_{\tilde{\mathbf{G}} \in \mathcal{I}(\mathbf{H},L)} (\|\tilde{\mathbf{G}}\mathbf{H}_{I}\|_{2}^{2} + \sigma^{2}\|\tilde{\mathbf{G}}\|_{2}^{2}) = \min_{\tilde{\mathbf{G}} \in \mathcal{I}(\mathbf{H},L)} J(\tilde{\mathbf{G}}).$$

### 5. MMSE-IRC

The MMSE-IRC problem seems to be much more general than  $\mathcal{H}_2$ -ZF. However, combining the ideas from the last two sections, we can again treat it as a special case of  $\mathcal{H}_2$ -ZF.

**Proposition 3.** Let  $\mathbf{S}(z)$  be a spectral factor of  $\mathbf{H}_{I}(z)\mathbf{H}_{I}(\overline{z}^{-1})^{*} + \sigma^{2}I_{q}$ , and let  $\mathbf{G}^{aug}(z) = \begin{bmatrix} \mathbf{G}_{S}(z) & \mathbf{G}_{2}(z) \end{bmatrix}$  be the solution to  $\mathcal{H}_{2}$ -ZF applied to the augmented channel

$$\mathbf{H}^{aug}(z) = \begin{bmatrix} \mathbf{S}^{-1}(z)\mathbf{H}(z) \\ I_p \end{bmatrix}$$

Then  $\mathbf{G}(z) := \mathbf{G}_S(z)\mathbf{S}^{-1}(z)$  is the solution to MMSE-IRC.

We omit the proof. Next, we want to give an alternative approach to MMSE-IRC, that does not require the spectral factorization. If the causality constraint from MMSE-IRC is dropped, the optimal stable, but generally non-causal equalizer is readily obtained as

$$\mathbf{H}(\bar{z}^{-1})^{*}(\mathbf{H}(z)\mathbf{H}(\bar{z}^{-1})^{*} + \mathbf{H}_{I}(z)\mathbf{H}_{I}(\bar{z}^{-1})^{*} + \sigma^{2}I_{q})^{-1}.$$
 (2)

Since stable equalizers form a superset of realizable equalizers, this solution can be used as an upper performance bound for MMSE-IRC. The next proposition gives an equalizer that is asymptotically optimal because it converges to (2) with growing estimation delay.

**Proposition 4.** Let, for arbitrary estimation delay L > 0,

$$\mathbf{G}_{L}^{aug}(z) = \begin{bmatrix} \mathbf{G}_{1,L}(z) & \mathbf{G}_{2,L}(z) & \mathbf{G}_{3,L}(z) \end{bmatrix}$$

denote the solution to  $\mathcal{H}_2$ -ZF for the augmented channel

$$\mathbf{H}^{aug}(z) = \begin{bmatrix} \mathbf{H}(\bar{z})^* \\ \mathbf{H}_I(\bar{z})^* \\ \sigma I_q \end{bmatrix}.$$

Then the realizable equalizer  $\mathbf{G}_L(z) := \mathbf{G}_{1,L}(\bar{z})^*$  approaches (2) for  $L \to \infty$ , i.e.  $z^L \mathbf{G}_L(z) \to (2)$ .



Fig. 2. Simulation results for L = 12,  $\mu = 10^3$ .

*Proof.* The optimal right inverse with delay L for the channel

As  $L \to \infty$ ,  $z^L \mathbf{G}_L^{aug}(\bar{z})^*$  converges towards the pseudoinverse

$$\begin{aligned} \mathbf{H}^{aug}(z^{-1}) \left( \mathbf{H}^{aug}(\bar{z})^* \mathbf{H}^{aug}(z^{-1}) \right)^{-1} &= \\ \mathbf{H}(\bar{z}^{-1})^* \left( \mathbf{H}(z) \mathbf{H}(\bar{z}^{-1})^* + \mathbf{H}_I(z) \mathbf{H}_I(\bar{z}^{-1})^* + \sigma^2 I_q \right)^{-1} \\ \mathbf{H}_I(\bar{z}^{-1})^* \left( \mathbf{H}(z) \mathbf{H}(\bar{z}^{-1})^* + \mathbf{H}_I(z) \mathbf{H}_I(\bar{z}^{-1})^* + \sigma^2 I_q \right)^{-1} \\ \sigma^2 \left( \mathbf{H}(z) \mathbf{H}(\bar{z}^{-1})^* + \mathbf{H}_I(z) \mathbf{H}_I(\bar{z}^{-1})^* + \sigma^2 I_q \right)^{-1} \end{aligned}$$

Thus, in particular  $z^L \mathbf{G}_{1,L}(\bar{z})^* \rightarrow (2)$  holds.

#### 6. NUMERICAL EXAMPLE

We performed Monte-Carlo simulations for the 3×2 four tap channel

$$\mathbf{H}(z) = \begin{bmatrix} 0.83z^{-3} - 1.24z^{-2} - 0.4z^{-1} - 0.91 \\ -0.64z^{-3} - 0.23z^{-2} - 0.52z - 1.36 \\ -0.23z^{-3} - 0.37z^{-2} - 0.27z + 0.68 \end{bmatrix}$$
$$\begin{bmatrix} -2.7z^{-3} - 0.19z^{-2} - 0.53z^{-1} + 0.16 \\ -0.97z^{-3} + 0.86z^{-2} - 0.27z^{-1} - 0.29 \\ -1.34z^{-3} + 0.94z^{-2} + 0.41z^{-1} - 0.38 \end{bmatrix}$$

with interferer

$$\mathbf{H}_{I}(z) = \frac{2}{5} \begin{bmatrix} -0.41z^{-3} - 1.24z^{-2} + 1.58z^{-1} - 0.67 \\ 1.08z^{-3} + 0.73z^{-2} - 0.08z^{-1} + 0.14 \\ -0.01z^{-3} + 0.52z^{-2} - 0.3z^{-1} - 2.41 \\ 1.94z^{-3} + 0.91z^{-2} + 0.43z^{-1} - 1.49 \\ 0.61z^{-3} + 0.02z^{-2} + 0.17z^{-1} - 0.59 \\ -1.66z^{-3} - 1.15z^{-2} - 0.02z^{-1} + 0.65 \end{bmatrix}$$

Fig. 2 compares the performance of the various equalizers derived in this paper. For each SNR,  $10^7$  random binary phase shift keying (BPSK) modulated signal vectors have been transmitted. The equalizers that take CCI into account clearly outperform the CCI unaware equalizers. Note that the asymptotically optimal equalizer from Proposition 4 performs as good as the optimal MMSE-IRC.

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