# SINGLE PARAMETER OPTIMIZATION APPROACH TO THE OPTIMAL POWER ALLOCATION OF OFDM RELAYING SYSTEM

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# ABSTRACT

In this paper, we investigate the power allocation optimization problem for dual-hop nonregenerative OFDM relaying networks, where an aggregate power constraint is imposed on all relays. We first formulate the model into a single-parameter optimization problem, in which we found the related functions are monotonically decreasing. Then based on the monotonicity, we propose a bisection algorithm to solve the problem. We further illustrate that pairing of OFDM subcarriers will not provide performance gain when the aggregate power constraint is very large.

*Index Terms*— nonregenerative relays, optimal power allocation, aggregate power constraint, OFDM

# 1. INTRODUCTION

The use of wireless relay is essential to provide broad coverage for wireless system. Deploying relays in cellular network will overcome the shadowing effect, as well as enhance the connectivity between a base station and mobile terminals at cell boundaries. Hence relay networks have been considered one of the most promising architectures for future wireless networks.

Efficient power allocation (PA) among relays can improve system performance in terms of instantaneous information rate. For the frequency-flat fading model, [1] provided the analytical optimal PA scheme where an aggregate power constraint has been imposed on all relays, [2] proposed an iterative algorithm under the condition of individual power constraint on each relay, and [3] considered a hybrid case where both aggregate and individual power constraint have been simultaneously imposed on relays. In the frequencyselective fading scenario, [4]–[6] discussed the PA problem where the network consists of *single* relay. [4] proposed the analytical solution where a transmission power constraint has been imposed on either the unique nonregenerative relay or the source node, and the result has been extended to MIMO relay link in [5]. [6] used an unified approach to approximately solve the joint PA problem for both regenerative and nonregenerative relaying scenarios, where an aggregate power constraint has been imposed on both the unique relay and the source node.

In this paper, focusing on the frequency-selective fading model, we consider the PA problem for a dual-hop OFDM relaying network that consists of *multiple* nonregenerative relays. We assume that an aggregate transmission power constraint has been imposed on all relays. Aiming at maximizing the instantaneous information rate of the network, we propose the *optimal* PA scheme via a bisection algorithm for any given PA at the source node. We further show that when the power constraint is very large, pairing of subcarriers will not improve the information rate.

The rest of the paper is organized as follows. The PA problem is introduced in section II and formulated into a single-parameter optimization problem in section III, then a bisection algorithm to solve the problem is proposed. Numerical results are presented in section IV and section V concludes the paper.

## 2. PROBLEM FORMULATION

Consider a dual-hop multi-relay network consisting of one source/destination pair and K nonregenerative relays. OFDM is used for broadband communication between nodes and the available bandwidth is divided into N subcarriers, in which the channel is assumed to be frequency-flat. In the *j*-th subcarrier, the channel from source to destination, source to the *i*-th relay, and the *i*-th relay to destination is denoted by  $h_j$ ,  $h_{ij}$  and  $g_{ij}$ , respectively. Each relay is assumed to know its own backward and forward CSI (channel state information), i.e., the *i*-th relay has access to  $h_{ij}$  and  $g_{ij}$  of all subcarriers, thus coherent amplify-and-forward relays [3] are used. Moreover, all relays are supposed to work in half-duplex mode.

In the first time slot, the source sends parallel signal streams to the rest nodes over all subcarriers. In the second time slot, each relay transmits a scaled version of the received signal to the destination. We assume that the signals of the source transmitted over the j-th subcarrier are scaled by each relay and also retransmitted in the j-th subcarrier [4]. Hence

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the scaling factor for the received data of the i-th relay in the j-th subcarrier is

$$b_{ij} = a_{ij}g_{ij}^*h_{ij}^*,$$

where non-negative factor  $a_{ij}$  is used to control the transmission power [3] and  $(\cdot)^*$  denotes complex conjugate.

For the sake of simplicity, we presume each of the parallel signal streams at source has the same power  $P_s$ , and each noise component involved in the entire transmission procedure has the same variance  $\sigma_n^2$ . Hence in the *j*-th subcarrier, the output SNR of using a temporal maximum-ratio combiner [4], which combines both received signals at destination in two consecutive time slots, is given by

$$\gamma_j = \left[ |h_j|^2 + \frac{\left(\sum_{i=1}^K a_{ij} |g_{ij}h_{ij}|^2\right)^2}{1 + \sum_{i=1}^K a_{ij}^2 |g_{ij}|^4 |h_{ij}|^2} \right] \cdot \frac{P_s}{\sigma_n^2},$$

and the instantaneous information rate of the communication between source and destination over all subcarriers is

$$C = 0.5 \sum_{j=1}^{N} \log_2(1 + \gamma_j)$$
 (1)

where 0.5 accounts for half-duplex mode.

Imposing an aggregate transmission power constraint  $P_{sum}$  on all relays, we must find out the optimal power control factors  $\{\hat{a}_{ij}\}$  so as to maximize the instantaneous information rate C in (1). For the *i*-th relay, the transmission power in the *j*-th subcarrier is

$$P_{ij} = |g_{ij}h_{ij}|^2 \left( P_s |h_{ij}|^2 + \sigma_n^2 \right) \cdot a_{ij}^2, \tag{2}$$

therefore we may formulate the PA problem as

$$\{\hat{a}_{11}, \cdots, \hat{a}_{KN}\} = \arg \max_{\substack{a_{ij} \\ \text{s.t. } \sum_{i=1}^{K} \sum_{j=1}^{N} P_{ij} \le P_{sum}}} C.$$
 (3)

Once  $\hat{a}_{ij}$  is obtained, the optimal  $\hat{P}_{ij}$  can be achieved according to (2).

### 3. OPTIMIZATION VIA BISECTION ALGORITHM

We first make the following definitions to simplify the expression of problem (3). Let

$$\mathbf{x}_j = [x_{1j}, \cdots, x_{Kj}],$$

$$x_{ij} = |g_{ij}|^2 |h_{ij}| a_{ij}, \quad c_{ij} = \frac{P_s |h_{ij}|^2 + \sigma_n^2}{|g_{ij}|^2 P_{sum}},$$
$$d_j = 1 + |h_j|^2 \frac{P_s}{\sigma_n^2}, \quad p_{ij} = \sqrt{\frac{P_s}{\sigma_n^2}} |h_{ij}|.$$

Now problem (3) can be reformulated as

$$\{\hat{\mathbf{x}}_1, \cdots, \hat{\mathbf{x}}_N\} = \arg \max_{\substack{\mathbf{x}_j, \ j=1, \cdots, N\\ \text{s.t. } \sum_{j=1}^N \sum_{i=1}^K c_{ij} x_{ij}^2 \le 1}} \sum_{j=1}^N f_j(\mathbf{x}_j) \quad (4)$$

where

$$f_j(\mathbf{x}_j) = 0.5 \log_2 \left[ d_j + \frac{\left(\sum_{i=1}^K p_{ij} x_{ij}\right)^2}{1 + \sum_{i=1}^K x_{ij}^2} \right].$$

### 3.1. Optimization in each subcarrier

In the optimal PA scheme,  $P_{sum}$  is optimally allocated among subcarriers. Furthermore, the power for each subcarrier is also optimally distributed among all relays. Therefore, we may first consider the PA problem for any given subcarrier w.r.t. an aggregate power constraint being imposed to all relays in that subcarrier.

Referring to (4), the PA problem for the j-th subcarrier can be formulated as

$$\tilde{\mathbf{x}}_{j} = \arg \max_{\mathbf{x}_{j}} f_{j}(\mathbf{x}_{j}),$$
(5)  
s.t.  $\sum_{i=1}^{K} c_{ij} x_{ij}^{2} \leq \mu_{j}$ 

in which  $\mu_j \in [0, 1]$  and represents a given aggregate power constraint on all relays in that subcarrier. Actually, problem (5) is equivalent to the PA problem in the frequency-flat scenario. It has been solved in [1] and we rewrite the result as

$$\tilde{x}_{ij} = \sqrt{\frac{1}{\sum_{i=1}^{K} \frac{\mu_j c_{ij} p_{ij}^2}{(\mu_j + c_{ij})^2}}} \cdot \frac{\mu_j p_{ij}}{\mu_j + c_{ij}}$$
(6)

which satisfies  $\sum_{i=1}^{K} c_{ij} \tilde{x}_{ij}^2 = \mu_j$ . The corresponding maximum of  $f_j(\mathbf{x}_j)$  is

$$f_j(\tilde{\mathbf{x}}_j) = \xi_j(\mu_j) = 0.5 \log_2 \left[ d_j + \sum_{i=1}^K \frac{\mu_j p_{ij}^2}{\mu_j + c_{ij}} \right].$$
 (7)

#### 3.2. Optimization among all subcarriers

Referring to (7), now problem (4) degrades to

$$\{\hat{\mu}_1, \cdots, \hat{\mu}_N\} = \arg\max_{\substack{\mu_j \\ \text{s.t. } \sum_{j=1}^N \mu_j \le 1}} \sum_{j=1}^N \xi_j(\mu_j)$$
 (8)

which means finding out the optimal PA scheme *only* among subcarriers. Since  $\xi_j(\mu_j)$  is monotonically increasing for each *j*, the optimal  $\{\hat{\mu}_1, \dots, \hat{\mu}_N\}$  should satisfy

$$\sum_{j=1}^{N} \hat{\mu}_j = 1.$$
 (9)

Taking the method of Lagrange multipliers to calculate  $\{\hat{\mu}_1, \cdots, \hat{\mu}_N\}$  by setting

$$L(\mathbf{x}_1,\cdots,\mathbf{x}_N,\lambda) = \sum_{j=1}^N f_j(\mathbf{x}_j) - \lambda \left(\sum_{i=1}^K \sum_{j=1}^N c_{ij} x_{ij}^2 - 1\right),$$

the necessary condition  $\frac{\partial L(\mathbf{x}_1, \cdots, \mathbf{x}_N, \lambda)}{\partial x_{ij}} = 0$  for maximizing  $L(\mathbf{x}_1, \cdots, \mathbf{x}_N, \lambda)$  will result in

$$\lambda = \frac{p_{ij}\phi_j\varphi_j - x_{ij}\phi_j^2}{x_{ij}c_{ij}\varphi_j(d_j\varphi_j + \phi_j^2)},\tag{10}$$

in which  $\phi_j = \sum_{i=1}^{K} p_{ij} x_{ij}$  and  $\varphi_j = 1 + \sum_{i=1}^{K} x_{ij}^2$ . In the *j*-th subcarrier, the solution  $\hat{\mathbf{x}}_j$  of problem (4) and the solution  $\hat{\mu}_j$  of problem (8) must satisfy equation (6), thus substituting (6) into (10) will yield

$$\lambda = G_j(\hat{\mu}_j) \tag{11}$$

where

$$G_j(\hat{\mu}_j) = \frac{\sum_{i=1}^K \frac{c_{ij} p_{ij}^2}{(\hat{\mu}_j + c_{ij})^2}}{d_j + \sum_{i=1}^K \frac{\hat{\mu}_j p_{ij}^2}{\hat{\mu}_j + c_{ij}}}.$$
 (12)

If denoting the inverse function of  $G_j(\cdot)$  by  $G_j^{-1}(\cdot)$ , we have

$$\hat{\mu}_j = G_j^{-1}(\lambda). \tag{13}$$

Inserting (13) into (9) will yield

$$\sum_{j=1}^{N} G_j^{-1}(\lambda) = 1,$$
(14)

which is an equation with only one unknown parameter  $\lambda$ . Once  $\lambda$  is solved from (14),  $\hat{\mu}_j$  and  $\hat{\mathbf{x}}_j$  can be obtained according to (13) and (6) respectively.

Although analytical expression of the function  $G_j^{-1}(\cdot)$  is not available, it is not difficult to prove that for each j, the function  $\lambda = G_j(\hat{\mu}_j)$  (or  $\hat{\mu}_j = G_j^{-1}(\lambda)$ , resp.) is monotonically decreasing w.r.t.  $\hat{\mu}_j$  (or  $\lambda$ , resp.). Thus  $\sum_{j=1}^N G_j^{-1}(\lambda)$ is also monotonically decreasing which guarantees that the bisection algorithm can be used to solve (14).

The value range of  $\lambda$  is determined as follows. According to the monotonicity of  $\lambda = G_j(\hat{\mu}_j)$ , we have

$$\lambda \in \bigcap_{j \in \{1, \cdots, N\}} \left[ G_j(1), G_j(0) \right]$$

which leads to  $\lambda_{\min} = \max \{G_1(1), \cdots, G_N(1)\}$  and  $\lambda_{\max} = \min \{G_1(0), \cdots, G_N(0)\}$ .

# **3.3.** Bisection algorithm to calculate $\lambda$ and $\{\hat{\mu}_1, \dots, \hat{\mu}_N\}$

We state the bisection algorithm as Algorithm 1 and we explain it as follows. In each iteration, we first set  $\lambda'$  as a

temporary value of  $\lambda$ . Then we calculate the corresponding  $\{\mu'_1, \dots, \mu'_N\}$  according to  $\mu'_j = G_j^{-1}(\lambda')$ . If  $\sum_{j=1}^N \mu'_j > 1$ , which indicates  $\lambda'$  is less than the desired  $\lambda$ , we need to increase  $\lambda'$  for next iteration, otherwise, we should decrease  $\lambda'$ . The iterative procedure is terminated when  $|\sum_{j=1}^N \mu'_j - 1| \le \epsilon_1$ , in which  $\epsilon_1$  is a prescribed error threshold.

Algorithm 1 Bisection algorithm to compute  $\lambda$  and  $\{\hat{\mu}_1, \dots, \hat{\mu}_N\}$ 

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1:	$\{\mu'_1,\cdots,\mu'_N\} \Leftarrow \{1,\cdots,1\};$
2:	while $ \sum_{j=1}^{N} \mu'_{j} - 1  > \epsilon_{1}$ do
3:	$\lambda' \Leftarrow \frac{\lambda_{\min} + \lambda_{\max}}{2};$
4:	for $j = 1$ to N do
5:	$\mu'_j \Leftarrow G_j^{-1}(\lambda');$
6:	end for
7:	if $\sum_{j=1}^{N} \mu'_j > 1$ then
8:	$\lambda_{\min} \leftarrow \lambda';$
9:	else
10:	$\lambda_{\max} \Leftarrow \lambda';$
11:	end if
12:	end while
13:	$\lambda \Leftarrow \lambda';$
14:	$\{\hat{\mu}_1,\cdots,\hat{\mu}_N\} \Leftarrow \{\mu'_1,\cdots,\mu'_N\};$

In the 5-th line of Algorithm 1,  $\mu'_j = G_j^{-1}(\lambda')$  is calculated according to Algorithm 2. Its key idea is very similar to that of Algorithm 1 and we omit the explanation.

Algorithm 2 Bisection algorithm to compute $\mu'_j = G_j^{-1}(\lambda')$			
1:	$\mu_{\max} \Leftarrow 1;  \mu_{\min} \Leftarrow 0;$	$\lambda'' \Leftarrow \lambda' + 1;$	
2:	while $ \lambda'' - \lambda'  > \epsilon_2$ do		
3:	$\mu_i'' \leftarrow \frac{\mu_{\min} + \mu_{\max}}{2};$		
4:	$\lambda^{\prime\prime\prime} \Leftarrow G_j(\bar{\mu_j^{\prime\prime}});$		
5:	if $\lambda'' > \lambda'$ then		
6:	$\mu_{\min} \leftarrow \mu_j'';$		
7:	else		
8:	$\mu_{\max} \Leftarrow \mu_i'';$		
9:	end if		
10:	end while		
11:	$\mu'_j \Leftarrow \mu''_j;$		

#### 3.4. Pairing of subcarriers

In previous sections, we have assumed that the signals of the source transmitted over the *j*-th subcarrier are scaled by each relay and also retransmitted through the *j*-th subcarrier. A higher performance in terms of instantaneous information rate can be achieved if the subcarriers of both the backward and forward channels can be paired [4]. Altogether, there are  $(N!)^K$  paring possibilities which imposes a prohibitive computation burden on the system. However, when  $P_{sum} \rightarrow \infty$ , the channels from relays to destination can support reliable communication at any rate, thus both the first hop and the direct link bottleneck the performance of the entire network. Under this situation, pairing of subcarriers *cannot* improve system performance and the network is equivalent to N parallel point-topoint SIMO systems. The *j*-th equivalent SIMO channel is  $[h_j, h_{1j}, \dots, h_{Kj}]$ , hence we have

$$\lim_{P_{sum} \to \infty} C = \sum_{j=1}^{N} 0.5 \log_2 \left[ 1 + \left( |h_j|^2 + \sum_{i=1}^{K} |h_{ij}|^2 \right) \frac{P_s}{\sigma_n^2} \right].$$
(15)

### 4. NUMERICAL RESULT

In this section we present the performance of the optimal PA scheme. We adopt the same assumption about the channels as that in [4], hence the *n*-th complex channel coefficient  $h_n$  in the time domain is distributed as

$$h_n \sim \mathcal{CN}\left(0, \frac{1}{L(1+d)^{\alpha}}\right)$$

in which d is the distance between two nodes and  $\alpha$  is the path loss exponent. We set the distance between source and destination to  $d_0 = 1000$  meters. We further assume that all relays are placed randomly within the middle range between source and destination, thus the distance between source and each relay is about d = 500 meters. The number of channel taps is set to L = 4 and other parameters are given by  $\alpha = 3$ , N = 16, K = 3,  $\sigma_n^2 = 1$ .

In Fig.1, we show the performance of the optimal PA scheme by varying  $P_{sum}$  where  $\rho_0 = \frac{P_{sum}}{N\sigma_n^2(1+d_0)^{\alpha}}$  and  $P_s = \sigma_n^2(1+d_0)^{\alpha}$ . The pairing strategy we adopted here is the same as that in [4], i.e., the best source to relay channel is paired with the best relay to destination channel. The average of (15) is also plotted in Fig.1 which is denoted by "asymptotic result". We further illustrate the performance of a suboptimal scheme, where  $P_{sum}$  is uniformly allocated not only among all subcarriers but also within each subcarrier, and we use "uniform PA" to denote this scheme.

Obviously, the performance of the optimal PA scheme is better than that of the "uniform PA" scheme. We also observe that pairing of subcarriers *cannot* provide performance gain when  $\rho_0 > 30$ dB. Furthermore, the performances of both optimal PA schemes (with pairing and without pairing) coinciding with "asymptotic result" proves (15) numerically.

### 5. CONCLUSION

Aiming at maximizing the instantaneous information rate of a dual-hop nonregenerative relaying network in the frequencyselective scenario, we have investigated the PA problem under the condition of an aggregate transmission power being



**Fig. 1**. Instantaneous information rate comparison among different PA schemes.

imposed on all relays. We have provided a bisection algorithm to obtain the optimal PA scheme. Furthermore we have illustrated that pairing of OFDM subcarriers will not provide performance gain when the imposed power constraint is very large.

#### 6. REFERENCES

- P. Larsson and H. Rong, "Large-scale cooperative relaying network with optimal coherent combining under aggregate relay power constraints," in *Proc. of Future Telecomm. Conf.*, 2003.
- [2] Y. Jing and H. Jafarkhani, "Network beamforming using relays with perfect channel information," in *Proc. ICASSP*, April 2007, vol. 3, pp. III–473–III–476.
- [3] T.Q.S. Quek, M.Z. Win, H. Shin, and M. Chiani, "Optimal power allocation for amplify-and-forward relay networks via conic programming," in *Proc. ICC*, June 2007, pp. 5058–5063.
- [4] I. Hammerström and A. Wittneben, "On the optimal power allocation for nonregenerative ofdm relay links," in *Proc. ICC*, June 2006, vol. 10, pp. 4463–4468.
- [5] I. Hammerström and A. Wittneben, "Power allocation schemes for amplify-and-forward mimo-ofdm relay links," *IEEE Trans. Wireless Commun.*, vol. 6, no. 8, pp. 2798–2802, August 2007.
- [6] Y. Li, W. Wang, J. Kong, W. Hong, X. Zhang, and M. Peng, "Power allocation and subcarrier pairing in ofdm-based relaying networks," in *Proc. ICC*, May 2008, pp. 2602–2606.